Chapter 5: Indeterminate Structures – Force Method

1. Introduction

- **Statically indeterminate structures** are the ones where the independent reaction components, and/or internal forces cannot be obtained by using the equations of equilibrium only. To solve indeterminate systems, we must combine the *concept of equilibrium with compatibility*.

- **Advantages**. There are several *advantages* in designing indeterminate structures. These include the design of lighter and more rigid structures. With added redundancy in the structural system, there is an increase in the overall factor of safety.

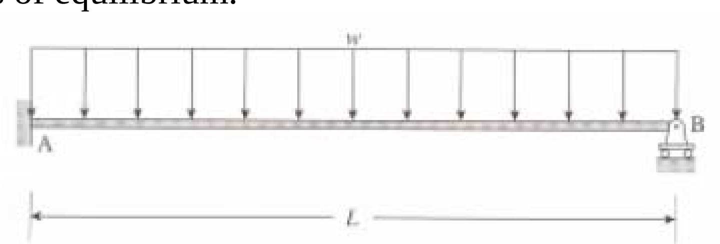
- **Several classical methods** have been developed to solve for the forces and displacements of statically indeterminate systems. In this course, we will introduce two important classical approaches: *the force method and the slope-deflection method*.

Force Method –Basic Idea.

The basic idea of this method is to identify the redundant forces first. Then using the compatibility conditions, determine the redundant forces. This way the structure is essentially reduced to a statically determinate structure. All the remaining forces can be determined using the equations of equilibrium.

2. Force Method for Beams – One Redundant Force

- **Check for indeterminacy**: # of unknowns > # of equations.
  (3m + r) > (3j + c); m = # of members, r = # of reactions, j = # of joints, c = # of internal hinges.
- Consider the beam shown below. The beam is statically indeterminate to degree one: four support reactions and three equations of equilibrium.

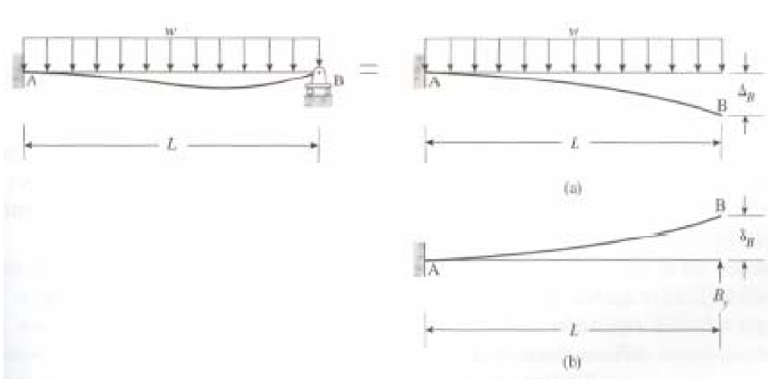


- If we remove an appropriate reaction, say R (also called the redundant), from the original indeterminate beam, the resulting beam (beam I) is stable and determinate. The deflection at the reaction point is calculated. Consider the same determinate beam without the external loads (beam II). We now apply the reaction R (which was removed from the original beam) as an external load on the beam II and determine the deflection at point B. The unknown R will be calculated from the structural compatibility as

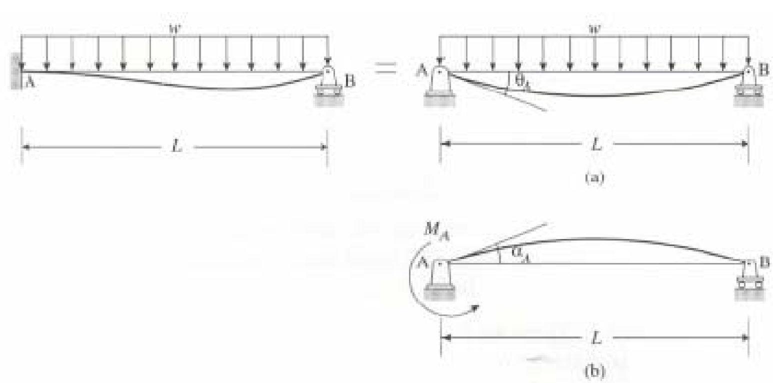
$$\Delta_B + \delta_B = 0$$

where Δ_B is the deflection at point B for the beam I, and δ_B is the deflection at point B for the beam II.

Scheme 1



Scheme 2



- We use the unit load method to compute deflections. For the current case,

$$\Delta_B = \int \frac{M(x)m(x)}{EI} dx$$

$$\delta_B = R \cdot \int \frac{m(x)m(x)}{EI} dx$$

where $M(x)$ is the bending moment distribution for the beam I, and $m(x)$ is the bending moment distribution for the beam II with reaction R a unit force.

- **General procedure for beams with a single redundant**.

Step 1: Identify the redundant. If the redundant is removed from the original structure, the resulting beam must be stable and determinate. Now create the two beams whose superposition results in the original indeterminate beam.

- Remove the redundant from the original beam but leave the external loads. This is beam DSRL (*Determinate Structure with Real Loads*) – beam I.
- Remove the redundant and all loads from the original beam. Assume a direction for the redundant. Now apply a unit force or moment along the assumed direction of the redundant. This is beam DSUL (*Determinate Structure with Unit Load*) – beam II.
- Write the single compatibility equation in the symbolic form. Select a sign convention for the associated displacements appearing in the equation. This equation should contain the redundant.

Step 2: Compute the deflection for the beam DSRL.

Step 3: Compute the deflection for the beam DSUL.

Step 4: Substitute the deflections from Steps 2 and 3 into the compatibility equation. Use the sign convention to assign the correct sign to the two displacements. Solve the compatibility equation for the redundant. If the answer is positive, the assumed direction for the redundant is correct. Otherwise, flip the direction.

Step 5: The other support reactions can now be computed using the free-body diagram of the original beam (or through superposition of the two determinate beams).

- **Example 1**: Compute the support reactions for the beam. Example 5.1.1, pages 250-252,)

- **Example 2**: Compute the support reactions of the beam. Example 5.1.3, pages 254-256.

3. Force Method for Frames – One Redundant Force

- Indeterminate frames can be solved in the same manner as indeterminate beams. If the frame is statically indeterminate to degree one, then one of the support reactions must be selected as the redundant.

- **Example 1**: Compute the support reactions of the frame. Example 5.1.6, pages 263-265.

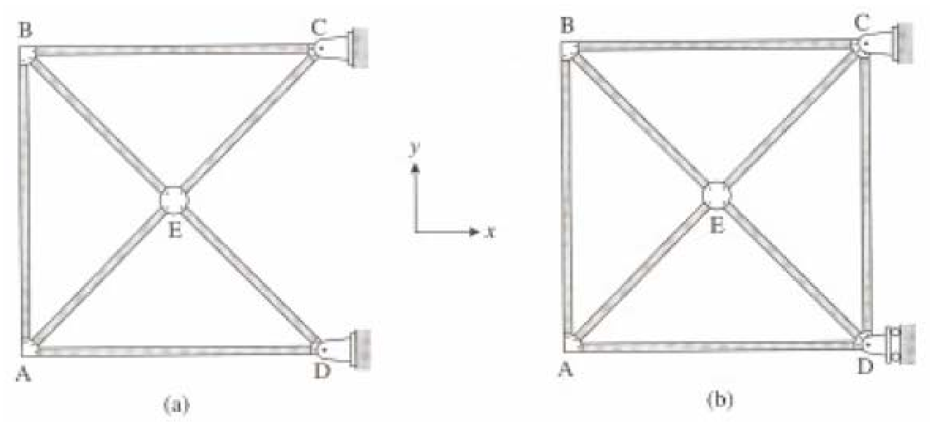
- **Example 2**: Compute the support reactions of the frame. Example 5.1.7, pages 265-267.

4. Force Method for Trusses – One Redundant Force

- A truss is a structural system that satisfies the following requirements:
  (a) The members are straight, slender, and prismatic
  (b) The joints are frictionless pins (internal hinges)
  (c) The loads are applied only at the joints
- **Check for indeterminacy**: # of unknowns > # of equations.
  $(m + r) > 2j$

- Trusses can be statically indeterminate due to a variety of reasons – redundant support reactions (externally indeterminate), redundant members (internally indeterminate), or a combination of both.

$m+r-2j=7+4-2(5)=1$ $\qquad$ $m+r-2j=8+3-2(5)=1$



- **General procedures for internally indeterminate trusses**

Step 1: Identify the redundant member (*ij*). If the member is removed from the original structure, the resulting truss must be stable and determinate. Now create the two trusses whose superposition results in the original indeterminate truss.

- Remove the redundant from the original truss but leave the external loads. This is truss DTRL.
- Remove the redundant and all loads from the original truss. Assume that the redundant member is in tension. Now apply unit tensile forces along the redundant member. This is truss DTUL.
- Write the single compatibility equation in the symbolic form. This equation should contain the redundant member force F_{ij}

Step 2: Compute the displacement along *ij* for the truss DTRL.

Step 3: Compute the displacement along *ij* for the truss DTUL.

Step 4: Now substitute the displacement from Steps 2 and 3 into the compatibility equation. Solve the compatibility equation for the redundant. If the answer is positive, the redundant is in tension. Otherwise, the member is in compression.

Step5: The other member forces can be computed through superposition of the two determinate trusses.

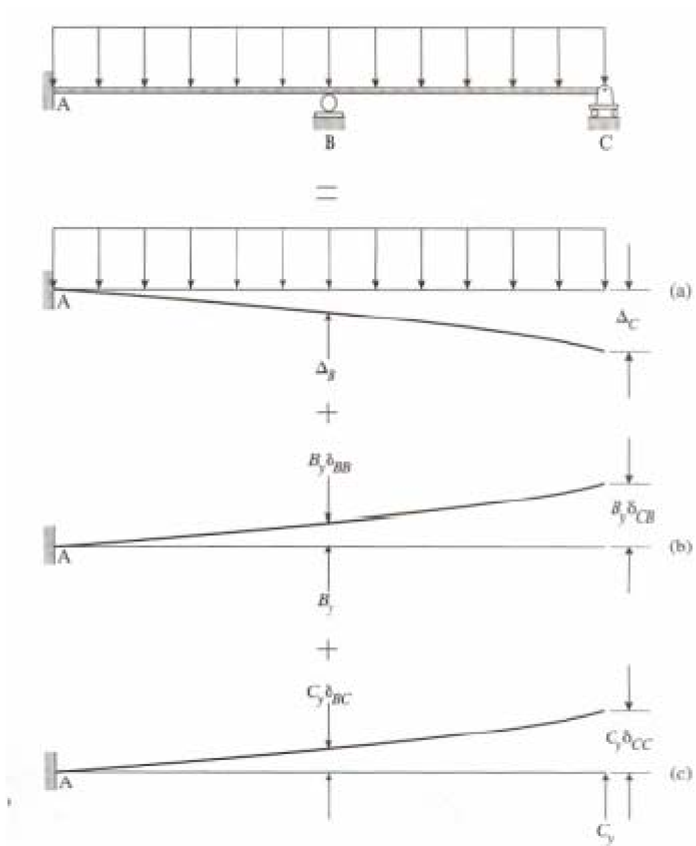
- **Example 1**: Compute the support reactions and the internal member forces for the truss. Example 5.1.8, pages 274-275.

- **Example 2**: Compute the member forces for the truss. Example 5.1.9, page 275-277.

5. Higher Degrees of Indeterminacy

- Consider the beam that is statically indeterminate to degree two, as shown below. We select the redundants as the support reactions B_y and C_y. These two unknowns can be solved by generating the two equations of compatibility.

Scheme 1

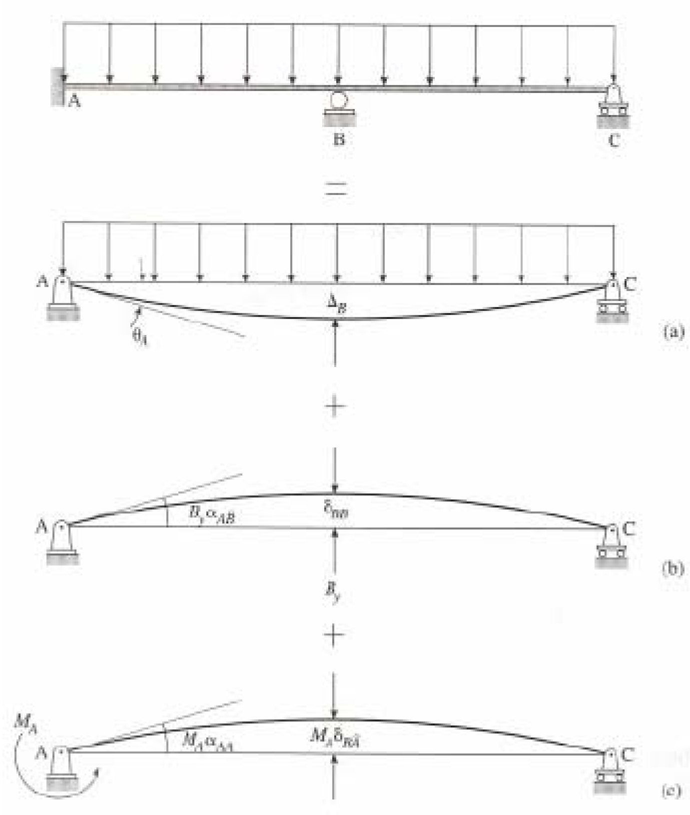


Compatibility conditions:

$$\Delta_B + B_y\delta_{BB} + C_y\delta_{BC} = 0$$
$$\Delta_C + B_y\delta_{CB} + C_y\delta_{CC} = 0$$

δ_{BC} : *deflection at B due to unit load at C*

Scheme 2



- **Example**: Compute the support reactions of the beam. Example 5.1.10, page 284-286.