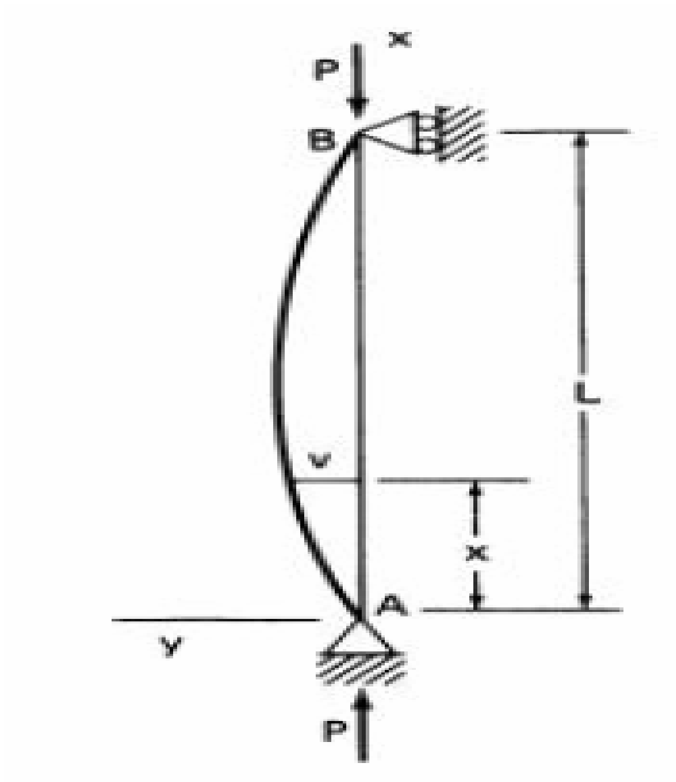
Design of Compression Members (Part 4 of AISC/LRFD)



Euler Buckling of Columns

Global buckling of a member happens when the member in compression becomes unstable due to its slenderness and load. Buckling can be elastic (longer thin members) or inelastic (shorter members). Here we shall derive the Euler buckling (critical) load for an elastic column.

Consider a long and slender compression member (hinged) as shown in the figure above. The Euler buckling formula is derived for an ideal or perfect case, where it is assumed that the column is long, slender, straight, homogeneous, elastic, and is subjected to concentric axial compressive loads. The *differential equation for the lateral displacement v* is given as:

$$EI\frac{d^2v}{dx^2} = M = -Pv$$

where E is the modulus of elasticity, I is the moment of inertia about the axis of bending in the cross section, P is the axial compressive force, and M is the bending moment at a distance x from support A. If we consider the column to be at the point of buckling, we have

$$\frac{d^2v}{dx^2} + \frac{P_{cr}}{EI}v = 0 \quad \text{or} \quad v'' + k^2 v = 0, \text{ where } k^2 = \frac{P_{cr}}{EI}$$

This is a second-order homogeneous linear differential equation with constant coefficients. The boundary conditions for the problem are also homogeneous as

$$v(0) = 0 \text{ and } v(L) = 0$$

The solution of the differential equation is

$$v = C_1 \cos kx + C_2 \sin kx$$

The integration constants C_1 and C_2 can be found by applying the following geometric boundary conditions:

At $x = 0$: $v = 0 \rightarrow C_1 = 0$
At $x = \mathrm{L}$: $v = 0 \rightarrow C_2\, sin\, kL = 0$

The above equation indicates that either C_2 = 0, which means no lateral displacement at all, or $\sin kL = 0$ with solution

$$kL = \pi, 2\pi, 3\pi \cdots = n\pi$$

Therefore, P_{cr} is

$$P_{cr} = \frac{n^2\pi^2 EI}{L^2}$$

Various values of n correspond to different buckling loads. When $n = 1$, the smallest value obtained is known as *critical load*, *buckling load*, or *Euler formula*:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Note that the critical buckling load is independent of the strength of the material (say, F_y, the yield stress). This equation was obtained for a column with hinged ends. The equation can be used for columns with other end conditions, as follows:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

where *KL* is the distance between the points of zero moment, or inflection points along the length. The length *KL* is known as the *effective length* of the column. The dimensionless coefficient K is called the *effective length factor*.

Dividing the critical load P_{cr} by the cross-sectional area of the column *A*, we can find the critical stress F_{cr}, as

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{(KL)^2 A} = \frac{\pi^2 E}{(KL/r)^2}$$

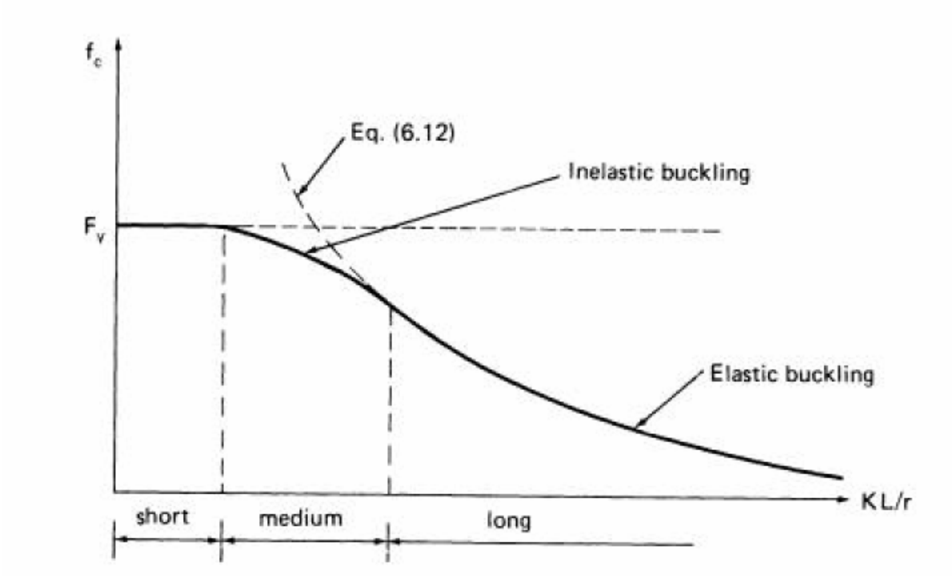
where *r* is the radius of gyration of the cross section about the axis of bending ($I = Ar^2$) and *KL/r* is called the *slenderness ratio* of the column. A thin column has small radius of gyration and a stocky column has large radius of gyration. The slenderness ratio determines elastic or inelastic mode of buckling failure. Columns with small *slenderness ratios* are called short columns.

- *Short columns (small KL/r) do not buckle* and simply *fail by material yielding*.

- *Long columns (large KL/r) usually fail by elastic buckling* mentioned above.

- Between short and long regions, the failure of the column occurs through *inelastic buckling*.

The figure shows the three types of failure modes for a column.



If we define a *slenderness parameter* as $\left(\lambda_c^2 = F_y / F_{cr}\right)$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}}$$

Then the equation of the critical stress F_{cr} is

$$F_{cr} = \frac{\pi^2 E}{(KL/r)^2} = \frac{1}{\lambda_c^2} F_y$$

Note that $\lambda_c \geq 1$.

Notations:

ϕ_c Resistance factor for compression (0.85)
A_g Gross cross-sectional area
F_y Specified minimum yield stress
P_n Nominal axial strength of the section
P_u Required axial strength
E Modulus of elasticity
K *Effective length factor*
L *Lateral unbraced length* of the member
r *Governing* radius of gyration

Design Strength:

$\phi_c P_n$ for compression members based on buckling failure mode

- The critical load is given as

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 EA}{(KL/r)^2}; \qquad I = r^2 A$$

- Buckling can take place about the strong (x) axis or the weak (y) axis.
- Larger value for *KL/r* will give smaller critical load, and thus will govern the design strength. Define

$$\lambda_x = \frac{K_x L_x}{r_x}; \qquad \lambda_y = \frac{K_y L_y}{r_y}$$

L_x = unbraced length for bending about the strong axis
L_y = unbraced length for bending about the weak axis

- If $\lambda_y > \lambda_x$, buckling about the y axis will govern the design strength; i.e.,

$$\frac{K_y L_y}{r_y} > \frac{K_x L_x}{r_x} \quad \text{or} \quad K_y L_y > \frac{K_x L_x}{r_x / r_y}$$

How to Use Manual Table 4-2:

- *Design strength* in axial compression is calculated as

$$\phi_c P_n = 0.85 F_{cr} A_g$$

- Table contains $\phi_c P_n$ for various values of $K_y L_y$, assuming buckling about y-axis.

- How to check buckling about x-axis:

  If $K_y L_y < \dfrac{K_x L_x}{r_x / r_y}$ buckling is about x-axis.

- How to read $\phi_c P_n$ if buckling is about x-axis:

  Use the length as $\dfrac{K_x L_x}{r_x / r_y}$ in Table 4-2.

Design Procedure:

1. Calculate the factored design loads P_u.

2. From the column tables, determine the effective length *KL* using

$$KL = max\left\{K_y L_y (\text{weak} - \text{axis}),\ \frac{K_x L_x}{r_x / r_y} (\text{strong} - \text{axis})\right\}$$

   and pick a section from Table 4-2.

3. Check the member thickness ratio in Table B5.1, if the member is not slender, use LRFD Chapter E2; otherwise, use LRFD Specifications Appendix E3 (reduction of design strength by factor Q given in Appendix B of Specifications).

4. Check using Table 4-2 to 4-17:

    - Calculate *KL* and enter into Table 4-2 to 4-17.
    - Find the design strength $\phi_c P_n$.

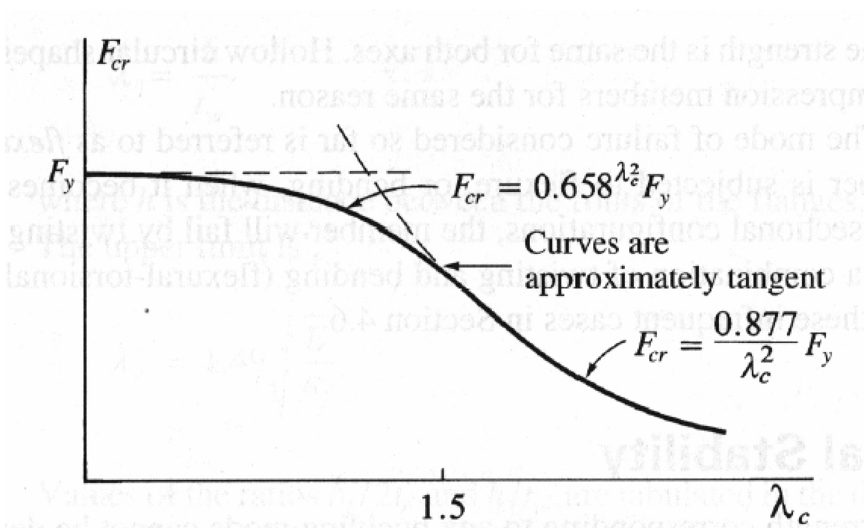
Or, using the formulas given in Chapter E2:

The slenderness parameter is calculated as

$$\lambda_c = max\left\{ \frac{K_x L_x}{r_x \pi} \sqrt{\frac{F_y}{E}},\ \frac{K_y L_y}{r_y \pi} \sqrt{\frac{F_y}{E}} \right\}$$

The critic al stress is calculated as

$$F_{cr} = \begin{cases} 0.658^{\lambda_c^2} F_y & \text{for } \lambda_c < 1.5 \quad AISC\ EQ.(E2-2) \\ \dfrac{0.877}{\lambda_c^2} F_y & \text{for } \lambda_c \geq 1.5 \quad AISC\ EQ.(E2-3) \end{cases}$$



The design strength $\phi_c P_n = \phi_c F_{cr} A_g = 0.85 F_{cr} A_g$

Required strength ≤ Design strength

$$P_u \le \phi_c P_n$$

Check for Slenderness Ratio:

Slenderness ratio (recommendation) (SPEC B7)

$$KL/r \le 200$$

Local Buckling

Local buckling is an instability due to the plates of the member becoming unstable. The local buckling of a member depends on its slenderness which is defined as the width-thickness ratio (*b/t* ratio), *b* is the width of the section and *t* is its thickness. Steel sections are classified as compact, noncompact or slender depending on the width-thickness ratio of their elements.

Compact section: is capable of developing a fully plastic stress distribution and possess rotation capacity of approximately three before the onset of local buckling; i.e., local buckling is not an issue.

Noncompact section: can develop the yield stress in compression elements before local buckling occurs, but will not resist inelastic local buckling at strain levels required for a fully plastic stress distribution. Local buckling can occur in the inelastic zone.

Compact sections have small *b/t* ratio and do not buckle locally; noncompact section can buckle locally; slender sections have a large *b/t* ratio. Let us define the width-thickness ration of an element of the cross-section (flange or web of WF shapes) as

$$\lambda = \frac{b}{t}$$

Then the members are classified as follows:

Compact section: $\lambda \le \lambda_p$ for all elements

Noncompact sections: $\lambda_p < \lambda \le \lambda_r$.

Slender: $\lambda > \lambda_r$.

The limiting values λ_p and λ_r for λ are given in Table B5.1 of the LRFD Secifications.

The strength corresponding to any buckling mode cannot be developed if the elements of the cross-section fail in *local buckling*. When *b/t* exceeds a limit λ_r (Table B5.1 of the LRFD Specifications), the member is classified as *slender*. Slender members can fail in *local buckling* resulting in reduced design strength. For slender members, Appendix B of the LRFD Specifications describes the reduction factors Q to be used for calculation of the critical stress F_{cr}.

Basically, the design strength needs to be reduced if the member is slender. Table B5.1 of the LRFD Specifications defines the following limits for sections that are not slender:

Unstiffened elements (flange): $\frac{b_f}{2t_f} \le \lambda_r; \quad \lambda_r = 0.56\sqrt{E/F_y}$

Stiffened element (web): $\frac{h}{t_w} \le \lambda_r; \quad \lambda_r = 1.49\sqrt{E/F_y}$

Flexural-Torsional Buckling:

Thin *unsymmetrical* members can fail in flexural-torsional buckling under axial loads, such as angles, tees. Calculation of design strength based on the flexural-torsional buckling failure mode is described in Section E3 and Appendix E3 of the LRFD Specifications.