Problem 1:
A semicircular cantilever beam, as shown in figure below, has a radius of curvature $R$ and a uniform circular cross-section of diameter $d$. It is subjected to loads of magnitude $P$ at points B and C.

(a) Using Castigliano's theorem, calculate the vertical deflection at C (say, $\delta_C$) in terms of $P$, modulus of elasticity $E$, shear modulus $G$, radius of curvature $R$, cross-sectional area $A$, form factor for shear $\alpha$, and moment of inertia of the cross-section $I$. Include all components of strain energy due to axial force, shear force, and bending moment.

(b) For $P = 150$ N, $R = 200$ mm, $d = 20$ mm, $E = 200$ GPa, $\alpha = 10/9$, and $G = 77.5$ GPa, calculate $\delta_C$ from the solution obtained in (a). What is the error in predicting $\delta_C$ when the components of strain energy due to axial and shear forces are ignored?

Note, the components of strain energy ($U_N$, $U_V$, and $U_M$) due to axial force ($N$), shear force ($V$), and bending moment ($M$) are:

$$U_N = \int \frac{N^2 ds}{2AE}; \quad U_V = \int \frac{\alpha V^2 ds}{2AG}; \quad \text{and} \quad U_M = \int \frac{M^2 ds}{2EI}.$$
Problem 2:
A weightless, Hookean, elastic beam ABC, shown in figure below, is fixed at point A and hinged at point C. The support at C allows rotation but prevents displacements. The beam exhibits small displacements in the x-y plane due to statically applied vertical load $P$ at point B. Assume that the beam stores strain energy $U$ only in bending, where $U = \int M^2 ds / 2EI$, $E$ is the elastic modulus, and $I$ is the moment of inertia of the beam cross-section. Using Castigliano's theorem:

(a) Determine the reactions at point C in terms of $P$.

(b) Find the vertical deflection of point B in terms of $P$ and beam parameters.