## Ken \& Larry Ice Cream

from Intro to O.R. by Hillier \& Lieberman (7th ed) p. 296
Ken and Larry, Inc., supplies its ice cream parlors with three flavors of ice cream: chocolate, vanilla, and banana.

Because of extremely hot weather and a high demand for its products, the company has run short of its supply of ingredients: milk, sugar, \& cream. Hence, they will not be able to fill all the orders received from their retail outlets, the ice cream parlors.

Owing to these circumstances, the company has decided to choose the amount of each product to produce that will maximize total profit, given the constraints on supply of the basic ingredients.

|  | Chocolate | Vanilla | Banana | Available |
| :--- | :---: | :---: | :---: | :--- |
| Milk | 0.45 | 0.5 | 0.4 | 200 gallons |
| Sugar | 0.5 | 0.4 | 0.4 | 150 pounds |
| Cream | 01 | 0.15 | 0.2 | 60 gallons |
| Profit \$/gal. | 1.00 | 0.90 | 0.95 |  |

The LP formulation for this problem has variables $\mathbf{C}, \mathbf{V}$, and $\mathbf{B}$ representing gallons of chocolate, vanilla, and banana ice cream produced, respectively.

```
MAXIMIZE C+0.9V+0.95B
ST
    0.45C + 0.50V + 0.40B <= 200 ! milk
    0.50C + 0.40V + 0.40B <= 150 ! sugar
    0.10C + 0.15V + 0.20B <= 60 ! cream
END
```



| OBJECTIVE FUNCTION VALUE |  |  |
| :---: | :---: | ---: |
| 1) | $\mathbf{3 4 1 . 2 5 0 0}$ |  |
| VARIABLE | VALUE | REDUCED COST |
| C | 0.000000 | 0.037500 |
| V | 300.000000 | 0.000000 |
| B | 75.000000 | 0.000000 |
|  |  |  |
| ROW | SLACK OR SURPLUS | DUAL PRICES |
| $2)$ | 20.000000 | 0.000000 |
| $3)$ | 0.000000 | 1.875000 |
| $4)$ | 0.000000 | 1.000000 |

```
RANGES IN WHICH THE BASIS IS UNCHANGED:
    OBJ COEFFICIENT RANGES
VARIABLE CURRENT ALLOWABLE ALLOWABLE
            COEF INCREASE
    C 1.000000 0.037500
    V 0.900000 0.050000
    B 0.950000 0.021429
    RIGHTHAND SIDE RANGES
ROW CURRENT ALLOWABLE
            RHS INCREASE
        INFINITY
        10.000000
        15.000000
        ALLOWABLE
        DECREASE
    2 200.000000
    3 150.000000
    DECREASE
        INFINITY
    0.012500
    0.050000
\begin{tabular}{cccc}
\multicolumn{4}{c}{ RIGHTHAND } \\
ROW & CURRENT & ALLOWABLE RANGES \\
& RHS & INCREASE & ALLOWABLE \\
2 & 200.000000 & INFINITY & DECREASE \\
3 & 150.000000 & 10.000000 & 30.000000 \\
4 & 60.000000 & 15.000000 & 3.7500000
\end{tabular}
```

a. What is the optimal profit and the optimal solution?
b. Suppose the profit per gallon of banana changes to $\$ 1.00$. Will the optimal solution change, and what can be said about the effect on total profit?
c. Suppose the profit per gallon of banana changes to 92 cents. Will the optimal solution changes, and what can be said about the effect on total profit?
d. Suppose the company discovers that 3 gallons of cream have gone sour and so must be thrown out. Will the optimal solution change, and what can be said about the effect on the total profit?
e. Suppose that the company has the opportunity to buy an additional 15 pounds of sugar at a total cost of $\$ 15$. Should they buy it? Explain!

| THE TABLEAU |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ROW | BASIS | C | V | B | SLK 2 | SLK 3 | SLK 4 |  |
| 1 | ART | 0.038 | 0.00 | 0.00 | 0.00 | 1.875 | 1.000 | 341.250 |
| 2 | SLK 2 | -0.350 | 0.00 | 0.00 | 1.00 | -2.000 | 2.000 | 20.000 |
| 3 | V | 3.000 | 1.00 | 0.00 | 0.00 | 10.000 | -20.000 | 300.000 |
| 4 | B | -1.750 | 0.00 | 1.00 | 0.00 | -7.500 | 20.000 | 75.000 |

Chocolate ice cream is not included in the optimal production plan. If one gallon of chocolate ice cream were to be produced, how would it change the quantity
...of vanilla ice cream produced?
...of banana ice cream produced?
...of milk used?
...of sugar used?
...of cream used?
(use substitution rates in the tableau!)

In (d), you were asked about the effect on profit of a reduction in the quantity of available cream due to spoilage. That is, the effect of an increase in the unused cream (slack in the available cream constraint). According to the substitution rates in the tableau, what would be the effect of this spoilage on the quantity
...of vanilla ice cream produced?
...of banana ice cream produced?
...of milk used?
...of sugar used?

The right-hand-side of row \#4 (available cream) was changed to zero, and then parametric analysis performed with the right-hand-side increasing to 150
gallons, with the results below.

| RIGHTHANDSIDE | PARAMETRICS | REPORT FOR ROW: | 4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| VAR | VAR | PIVOT | RHS | DUAL PRICE | OBJ |  |
| OUT | IN | ROW | VAL | BEFORE PIVOT | VAL |  |
|  |  |  |  | 0.0000 | 10.0000 | 0.000 |
| SLK | B | V | 4 | 30.0000 | 10.0000 | 300.000 |
| C | B | 3 | 56.2500 | 1.42857 | 337.500 |  |
| V | SLK | 4 | 4 | 75.0000 | 1.00000 | 356.250 |
|  |  |  | 150.000 | 0.0000 | 356.250 |  |



Using LINDO's report, indicate on the graph above the slope of each linear segment and the coordinates of each break-point (profit \& gallons of cream).

