Name ____________________________________

<><><><> 57:022 Principles of Design II <><><><>
Quiz #1 -- January 29, 1999

Under normal circumstances, 5% of the items produced by a certain process are defective. All items are routinely inspected as soon as they are produced.

===
For each random variable below, indicate the number of the probability distribution below which would best be used to model it:

Distribution	*Quantity*
_____	number of defective items among the first ten
_____	the number of the item having the first defect
_____	the number of the item having the second defect
_____	an indication of whether the first item is defective
_____	number of defective items among the first 100

Probability distributions:

1. Binomial
2. Bernouilli
3. Erlang
4. Poisson
5. Exponential
6. Geometric
7. Normal
8. Pascal

===
For each quantity below, indicate the formula which would be used to compute its probability:

Computation	*Probability*
_____	exactly one of the first ten items is defective
_____	the first and second item are both defective
_____	the second defective item is item #10
_____	two or fewer defective items are found among the first 10 inspected
_____	the tenth item is the first to be found defective

Probability calculations:

a. $(0.05)(0.95)^9 = 0.03151$

b. $(0.05)^2 = 0.0025$

c. $9\,(0.05)(0.95)^9 = 0.2836$

d. $10\,(0.05)^2(0.95)^8 = 0.0166$

e. $(0.95)^2 = 0.9025$

f. $(0.05)^1(0.95)^9 + 10\,(0.05)^1(0.95)^9 + 5\,(0.05)^2(0.95)^8 = 0.9222$

g. $9\,(0.05)^2(0.95)^8 = 0.01493$

h. $10(0.05)^1(0.95)^9 + 5\,(0.05)^1(0.95)^9 + (0.05)^2(0.95)^8 = 0.4743$

i. $(0.05)^9(0.95) = 1.855\text{x}10^{-12}$

j. $10\,(0.05)^1(0.95)^9 = 0.3151$

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Quiz #2 Solutions-- February 5, 1999

Vehicles arrive at a toll booth on the freeway at the average rate of 4 per minute in a completely random fashion. The arrival times are recorded for one hour, beginning at 8:00 am. Eighty percent of the vehicles arriving are cars, while the remainder are trucks, buses, etc.

===

For each random variable below, indicate the number of the probability distribution below which would best be used to model it:

Distribution	*Quantity*
Poisson	the number of vehicles which arrive between 8:00 & 8:03 a.m.
Exponential	time between arrivals of vehicles
Erlang *(or Gamma)*	the time at which the fourth vehicle arrives
Binomial	number of cars in the first 16 vehicles to arrive
Geometric	the number of the vehicle which is the first non-car (truck or bus)

Probability distributions:

1. Binomial
2. Erlang
3. Bernouilli
4. Exponential
5. Weibull
6. Negative binomial
7. Gamma
8. Normal
9. Geometric
10. Pascal
11. Poisson
12. None of the above

===

For each quantity below, indicate the formula which would be used to compute its probability: Note: arrival rate is λ=4/minute

Computation	*Probability*
$1-e^{-1} = 63.2\%$	the first vehicle arrives during the first 15 seconds
$1-\frac{2^0e^{-2}}{1}-\frac{2^1e^{-2}}{1} = 59.4\%$	the second vehicle arrives during the first 30 seconds
$(0.2)^2 = 4\%$	neither the first nor the second item are cars
$28\,(0.8)^6(0.2)^2 = 29.4\%$	exactly six of the first 8 vehicles are cars
$\frac{2^2e^{-2}}{2} = 27.1\%$	exactly two vehicles arrive during the first 30 seconds

Probability calculations:

a. $\frac{2^0e^{-2}}{1}+\frac{2^1e^{-2}}{1}+\frac{2^2e^{-2}}{2} = 20.9\%$

b. $(0.2)^2 = 4\%$

c. $(0.8)^6(0.2)^2 = 1\%$

d. $1-\frac{2^0e^{-2}}{1}-\frac{2^1e^{-2}}{1} = 59.4\%$

e. $1-e^{-1}-e^{-2} = 49.7\%$

f. $1-e^{-1/2} = 39.3\%$

g. $e^{-4} = 1.8\%$

h. $\frac{2^2e^{-2}}{2} = 27.1\%$

i. $1-\frac{2^2e^{-2}}{2} = 72.9\%$

j. $1-e^{-1} = 63.2\%$

k. $(0.8)(0.2) = 16\%$

l. $28\,(0.8)^6(0.2)^2 = 29.4\%$

m. $e^{-1} = 36.8\%$

n. *None of the above*

<><><><> 57:022 Principles of Design II <><><><>

Quiz #3 Solutions -- Spring 1999

The time between arrivals of forty vehicles are measured. The number of observations O_i falling within each half-minute interval is shown in the table below. The average is computed by weighting the midpoint of each interval by its number of observations: 0.25x9 + 0.75x4 + 1.25x5 + ... = 2.225 minutes. We wish to test the "goodness of fit" of the exponential distribution with mean 2.225 minutes.

i	Interval	O_i	p_i	E_i	$(E_i-O_i)^2/E_i$
1	0.0-0.5	9	0.2015	8.0594	0.1098
2	0.5-1.0	4	0.1609	6.4355	0.9217
3	1.0-1.5	5	0.1285	5.1389	0.0038
4	1.5-2.0	3	0.1026	4.1035	0.2967
5	2.0-2.5	7	0.0819	3.2767	4.2308
6	2.5-3.0	3	0.0654	2.6165	0.0562
7	> 3.0	9	0.2592	10.3696	0.1809

The sum of the values in the last column is D = 5.8.

Note: The order of the statements varied slightly in the three versions of the quiz.

Indicate whether true or false, using "+" for true, "o" for false.

False 1. The chi-square distribution for this test will have 6 "degrees of freedom". *Note: five degrees of freedom.*

False 2. The CDF of the distribution is assumed to be $F(t) = P\{T \le t\} = 1 - \lambda e^{-\lambda t}$ *Note: $F(t) = 1 - e^{-l\, t}$*

True 3. The quantity D is assumed to have the chi-square distribution.

False 4. If α=10% is used as a criterion, the exponential distribution with mean 2.225 minutes should be *rejected* as a model for the interarrival times of the vehicles. *Note: Since P{D‡9.236}=10%, we would reject this distribution only if D‡ 9.236, but D is only 5.8, i.e., it is more than 10% likely that D would be at least 5.8.*

True 5. The arrivals of the vehicles will form a Poisson process. *Note: provided, of course, that the time between arrivals has exponential distribution!*

True 6. The degrees of freedom is reduced by 2 because (i) the total number of observations is fixed, and (ii) the data was used to estimate one parameter for the distribution being tested.

True 7. The parameter of the exponential distribution is assumed to be $\lambda = 1/2.225 \approx 0.45$

True 8. The quantity E_i is the expected number of observations in interval #i

False 9. The chi-square distribution for this test will have 7 "degrees of freedom". *Note: it will have 5 degrees of freedom.*

False 10. The probability p_i that a car arrives in an interval #i, $[t_1, t_2]$, is $F(t_1) - F(t_2)$. *Note: The probability is $F(t_2) - F(t_1)$. The quantity $F(t_1) - F(t_2)$ is negative!*

False 11. The sum of several N(0,1) random variables has chi-square distribution. *Note: the sum of squares of N(0,1)-distributed random variables has a chi-square distribution.*

True 12. The smaller the value of D, the better the fit for the distribution being tested.

False 13. The quantity $(E_i-O_i)^2/E_i$ is assumed to have the normal N(0,1) distribution. *Note: It is the square of a random variable having approximately N(0,1) distribution.*

False 14. If T does in fact have the exponential distribution with mean 2.225 minutes, the probability that D exceeds 5.8 would be less than 10%. *Note: P{D‡9.236}=10%*

Chi-square Dist'n $P\{D \ge \chi^2\}$

deg.of freedom	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

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We wish to generate some random numbers having the same distribution as the inter-arrival times above (where the average is 2.225 minutes.) Suppose that a procedure for generating uniformly-distributed random numbers has yielded the value R=0.353. We want to generate a random value for T_1, i.e., the time at which the *first* car arrives.

d or h 15. Using the Inverse Transformation method, then according to the table below the *nearest* value of Y_1 should be

a. 0.25 minute	e. 1.25 minute	i. 2.5 minutes
b. 0.5 minute	f. 1.5 minutes	j. 2.75 minutes
c. 0.75 minute	g. 2 minutes	k. 3 minutes
d. 1 minute	h. 2.25 minutes	l. *greater than 3 min.*

Note: F(1) = 0.362, and it would appear by interpolation that $F^{-1}(0.353)$ *» 0.997. So the first random number is 0.997. If you used 1-R instead of R, then you would obtain* $F^{-1}(0.647)$ *» 2.335.*

_l_ 16. Suppose that the next uniformly-generated random number is 0.619. Then the corresponding arrival time of the *second* car is (choose *nearest* value):

a. 0.25 minute	b. 1.25 minute	c. 2.5 minutes
d. 0.5 minute	e. 1.5 minutes	f. 2.75 minutes
g. 0.75 minute	h. 2 minutes	i. 3 minutes
j. 1 minute	k. 2.25 minutes	l. *greater than 3 min.*

Note: F(2) = 0.593, and it would appear by interpolation that $F^{-1}(0.619)$ *» 2.333. So the second random number is 2.333. This represents the time between arrivals, and so the second simulated car arrives at time 0.997+2.333 » 3.33, i.e., greater than 3 minutes. If you used 1-R instead of R, you would obtain* $F^{-1}(0.381)$ *» 1.074 for the inter-arrival time, and so the time of arrival of the second car would be 2.335+1.074 = 3.409.*

x	P{T≤x}	Δp	P{T>x}
0	0.00000000	0.00000000	1.00000000
0.5	0.20125937	0.20125937	0.79874063
1	0.36201340	0.16075403	0.63798660
1.5	0.49041418	0.12840078	0.50958582
2	0.59297310	0.10255892	0.40702690
2.5	0.67489108	0.08191798	0.32510892
3	0.74032229	0.06543122	0.25967771

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_c_ 17. We want to generate random numbers X between 0 and 3, having the triangular distribution whose density function is shown below. What is the value of C? *(Choose nearest value.)*

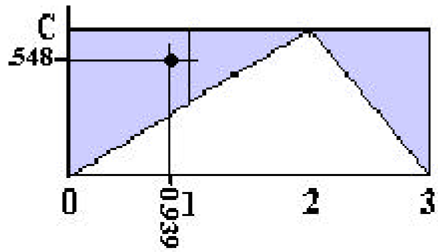
a. 0.2	d. 0.8	g. 1.4
b. 0.4	e. 1.0	h. 1.6
c. 0.6	f. 1.2	i. 1.8

Note: The area of the triangle must be (1/2)3C = 1, which requires that C= 2/3.

_g_ 18. Suppose that we generate two uniformly-distributed random numbers in the interval [0,1], namely R_1=0.313 and R_2=0.824, and apply the *rejection* method. What random number is generated from this pair of numbers? *(Choose nearest value.)*

a. 0.5	d. 2.0	g. *None*
b. 1.0	e. 2.5	
c. 1.5	f. 3.0	

Note: 0.939 is 0.313 times the base of the rectangle, and 0.548 is 0.824 times the height. As shown in the figure, the point (0.939, 0.548) does <u>*not*</u> *lie within the triangle, and so the number 0.939 is rejected and no random number is generated from this pair.*


C
.548
0
0.939
1
2
3

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Quiz #4 Solutions -- Spring 1999

True (+) or False (o)?

_o_ 1. The Weibull distribution is usually appropriate for the maximum of a large number of nonnegative random variables. *Note: appropriate for minimum of nonnegative rv's , or maximum of rv's with an upper bound.*

_o_ 2. When choosing between two different regression models, i.e., "fits" of a curve to data points, the model with the lower value of R^2 should be chosen.

_+_ 3. Given only a coefficient of variation for the Weibull distribution, the parameter k can be determined.

_+_ 4. If you use the *Minitab* program to fit a line, it will choose the straight line which minimizes the sum of the squares of the errors, i.e., the sum of the squares of the vertical distances between each data point and the line.

Multiple Choice:

_d_ 5. The "Cumulative Distribution Function" (CDF) of a random variable X is

a. $f(x) = P\{x \mid X\}$ b. $F(x) = P\{X \geq x\}$ c. $f(x) = P\{x\}$
d. $F(x) = P\{X \leq x\}$ e. $F(x) = P\{X = x\}$ f. $f(x) = P\{X \mid x\}$

_b_ 6. Suppose that a steel chain is made up of many links. The strength of the chain is, of course, the strength of its weakest link, since the chain fails whenever any link fails. A reasonable assumption for the probability distribution for the strength of the chain is

a. Normal distribution **b. Weibull distribution** c. Exponential distribution
d. Gumbel distribution e. Uniform distribution f. *None of the above*

_c_ 7. The CDF, i.e., F(x), of the Weibull distribution with parameters k and u is

a. $e^{\left(-e^{-k(x-u)}\right)}$ b. $1 - e^{-k(x-u)}$ c. $1 - e^{-(x/u)^k}$
d. $u - \frac{\ln(-\ln x)}{k}$ e. $1 - \frac{u \ln(-\ln x)}{k}$ f. *None of the above*

_d_ 8. The "coefficient of variation" of a probability distribution with mean μ and variance σ^2, is

a. $\sqrt{\mu^2 + \sigma^2}$ b. μ^2/σ^2 c. σ^2/μ^2
d. σ/μ e. μ/σ f. *None of the above*

_b_ 9. The "Gamma" function is related to the factorial function for integers by

a. $\Gamma(1-k) = k!$ d. $\Gamma(k) = (k+1)!$
b. $\Gamma(1+k) = k!$ e. $\Gamma(1+1/k) = k!$
c. $\Gamma(k) = k!$ f. *None of the above*

_f_ 10. To generate a single random number x having a Weibull distribution F(x), you should

a. obtain two random numbers (x,y) having uniform distribution in [0,1].
b. obtain a single random number y uniformly-distributed in the interval [0,1].
c. Derive the inverse of the function F, and compute $F^{-1}(y)$ where y has uniform distribution in [0,1].
d. Plot a randomly-generated point (x,y), and accept x if $y \leq f(x)$, otherwise try again.
e. Both (a) and (d) are true.
f. Both (b) and (c) are true. *Note: the inverse transformation can be used, but the rejection method cannot, because the density function cannot be "put into a box", i.e., there is no interval such that f(x)=0 outside this interval.*
g. *None of the above.*

_b_ 11. Given a set of data points (x_i,y_i), i=1,2,...n, "linear regression" is a method for determining a relationship y = f(x) which

a. minimizes the sum of the errors $\Sigma_i \{y_i - f(x_i)\}$
b. minimizes the sum of the squares of the errors: $\Sigma_i (y_i - f(x_i))^2$
c. minimizes the sum of the absolution values of the errors: $\Sigma_i |y_i - f(x_i)|$
d. minimizes the maximum error $\max \{y_i - f(x_i)\}$
e. *None of the above*

Name ______________________

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Quiz #5 -- March 1, 1999

True (+) or False (o)? (Many of the questions relate to the homework exercise in which the failure times of a set of light bulbs wereused to estimate reliability.)

___ 1. We assumed in this HW that the lifetime of a light bulb has a Weibull distribution.

___ 2. If 10 units of this bulb are installed in a light fixture, the number still functioning after 500 hours has a Weibull distribution.

___ 3. To estimate the time at which 90% of the bulbs will have failed, evaluate 1 - F(0.90).

___ 4. The quantity R_t is the expected fraction of the bulbs which have survived until time t.

___ 5. The method used in homework #5 to estimate the Weibull parameters u & k requires that the light bulbs be tested until <u>all</u> have failed.

___ 6. We assumed in this exercise that the number of failures at time t , $N_f(t)$, has a Weibull distribution.

___ 7. The Weibull CDF, i.e., F(t), gives, for each light bulb, the probability that it has failed at time t.

___ 8. The time between the failures in the batch of 500 light bulbs was assumed to have the Weibull distribution.

___ 9. The CDF of the failure time of a light bulb is assumed to be $F(t) = 1 - e^{-\left(k/u\right)^t}$ for some parameters u & k.

___ 10. When choosing between two different regression models, i.e., "fits" of a curve to data points, the model with the lower value of R^2 should be chosen.

___ 11. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

___ 12. A value of k > 1 indicates an increasing failure rate, while k<1 indicates a decreasing failure rate.

___ 13. Given a coefficient of variation for the Weibull distribution (i.e., the ratio σ/μ), the parameter k can be determined by solving a nonlinear equation (or consulting a table).

___ 14. The slope of the straight line fit by Minitab in this homework exercise was the estimate of the "shape" parameter k.

Multiple Choice:

___ 15. Given a set of data points (x_i, y_i), i=1,2,...n, "linear regression" is a method for determining a relationship y = f(x) which

a. minimizes the sum of the errors $\sum_i \{y_i - f(x_i)\}$

b. minimizes the sum of the squares of the errors: $\sum_i (y_i - f(x_i))^2$

c. minimizes the sum of the absolution values of the errors: $\sum_i |y_i - f(x_i)|$

d. minimizes the maximum error $\max \{y_i - f(x_i)\}$

e. *None of the above*

___ 16. The "Cumulative Distribution Function" (CDF) of a random variable X is

a. $f(x) = P\{x \mid X\}$ b. $F(x) = P\{X \geq x\}$ c. $f(x) = P\{x\}$

d. $F(x) = P\{X \leq x\}$ e. $F(x) = P\{X = x\}$ f. $f(x) = P\{X \mid x\}$

___ 17. The "coefficient of variation" of a probability distribution with mean μ and variance σ^2, is

a. $\sqrt{\mu^2 + \sigma^2}$ b. μ^2/σ^2 c. σ^2/μ^2

d. σ/μ e. μ/σ f. *None of the above*

Select the letter below which indicates each correct answer:

When plotting the points to fit a straight line,

___ 18. The vertical axis should represent ...

___ 19. The horizontal axis should represent ...

___ 20. The slope of the line should be approximately ...

___ 21. The vertical intercept (y-intercept) of the line should be approximately ...

a. t	f. –k ln u	k. ln ln $1/t$	o. coefficient of variation
b. ln $1/t$	g. ln u	l. ln ln $1/R_t$	p. shape parameter k
c. ln t	h. ln R_t	m. ln ln R_t	q. scale parameter u
d. ln ln t	i. ln $1/R_t$	n. standard deviation σ	r. mean value μ
e. R_t	j. u		

<><><><> 57:022 Principles of Design II <><><><>

Quiz #5 -- March 1, 1999

True (+) or False (o)? (Many of the questions relate to the homework exercise in which the failure times of a set of light bulbs wereused to estimate reliability.)

_+_ 1. We assumed in this HW that the lifetime of a light bulb has a Weibull distribution.

_o_ 2. If 10 units of this bulb are installed in a light fixture, the number still functioning after 500 hours has a Weibull distribution.

_o_ 3. To estimate the time at which 90% of the bulbs will have failed, evaluate 1 - F(0.90).

_+_ 4. The quantity R_t is the expected fraction of the bulbs which have survived until time t.

_o_ 5. The method used in homework #5 to estimate the Weibull parameters u & k requires that the light bulbs be tested until <u>all</u> have failed.

_o_ 6. We assumed in this exercise that the number of failures at time t , $N_f(t)$, has a Weibull distribution.

_+_ 7. The Weibull CDF, i.e., F(t), gives, for each light bulb, the probability that it has failed at time t.

_o_ 8. The time between the failures in the batch of 500 light bulbs was assumed to have the Weibull distribution.

_+_ 9. The CDF of the failure time of a light bulb is assumed to be $F(t) = 1 - e^{-(k/u)^t}$ for some parameters u & k.

_o_ 10. When choosing between two different regression models, i.e., "fits" of a curve to data points, the model with the lower value of R^2 should be chosen.

_+_ 11. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

_+_ 12. A value of k > 1 indicates an increasing failure rate, while k<1 indicates a decreasing failure rate.

_+_ 13. Given a coefficient of variation for the Weibull distribution (i.e., the ratio σ/μ), the parameter k can be determined by solving a nonlinear equation (or consulting a table).

_+_ 14. The slope of the straight line fit by Minitab in this homework exercise was the estimate of the "shape" parameter k.

Multiple Choice:

_b_ 15. Given a set of data points (x_i,y_i), i=1,2,...n, "linear regression" is a method for determining a relationship y = f(x) which

a. minimizes the sum of the errors $\Sigma_i \{y_i - f(x_i)\}$

b. minimizes the sum of the squares of the errors: $\Sigma_i (y_i - f(x_i))^2$

c. minimizes the sum of the absolution values of the errors: $\Sigma_i |y_i - f(x_i)|$

d. minimizes the maximum error $\max \{y_i - f(x_i)\}$

e. *None of the above*

_d_ 16. The "Cumulative Distribution Function" (CDF) of a random variable X is

a. f(x) = P{x | X} b. F(x) = P{X ≥ x} c. f(x) = P{x}

d. F(x) = P{X ≤ x} e. F(x) = P{X = x} f. f(x) = P{X | x}

_d_ 17. The "coefficient of variation" of a probability distribution with mean μ and variance σ^2, is

a. $\sqrt{\mu^2 + \sigma^2}$ b. μ^2/σ^2 c. σ^2/μ^2

d. σ/μ e. μ/σ f. *None of the above*

Select the letter below which indicates each correct answer:

When plotting the points to fit a straight line,

_l_ 18. The vertical axis should represent ...

_c_ 19. The horizontal axis should represent ...

_p_ 20. The slope of the line should be approximately ...

_f_ 21. The vertical intercept (y-intercept) of the line should be approximately ...

a. t	f. –k ln u	k. $\ln \ln 1/t$	o. coefficient of variation
b. $\ln 1/t$	g. ln u	l. $\ln \ln 1/R_t$	p. shape parameter k
c. ln t	h. $\ln R_t$	m. $\ln \ln R_t$	q. scale parameter u
d. ln ln t	i. $\ln 1/R_t$	n. standard deviation σ	r. mean value μ
e. R_t	j. u		

Name ___________________________________

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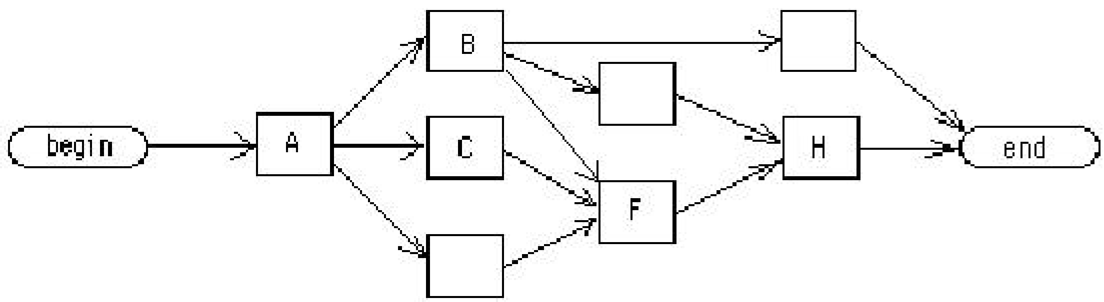
Quiz #6 -- April 9, 1999

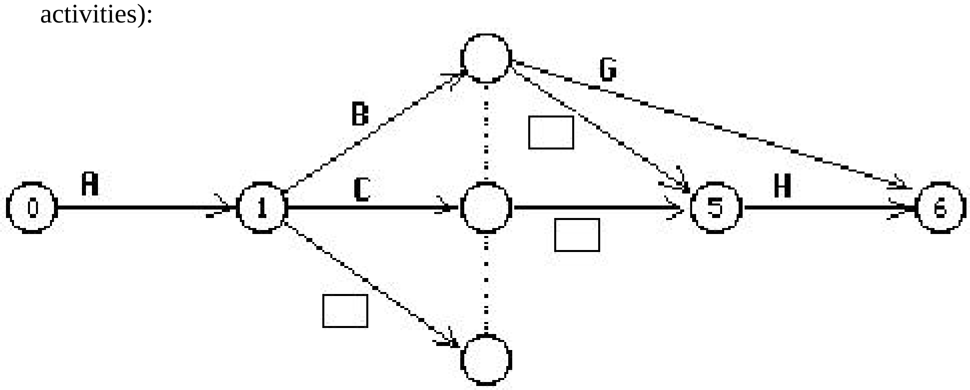
Consider the project consisting of the following activities:

Activity	Description	Predecessor(s)	Duration (days)
A	Clear & level site	none	2
B	Erect building	A	6
C	Install generator	A	4
D	Install water tank	A	2
E	Install maintenance equipment	B	4
F	Connect generator & tank to building	B,C,E	5
G	Paint & finish work on building	B	3
H	Facility test & checkout	D,F	2

1. Complete the AON network by labeling the nodes:



2. Complete the AOA network by labeling the three arrows (not including those for the "dummy" activities):

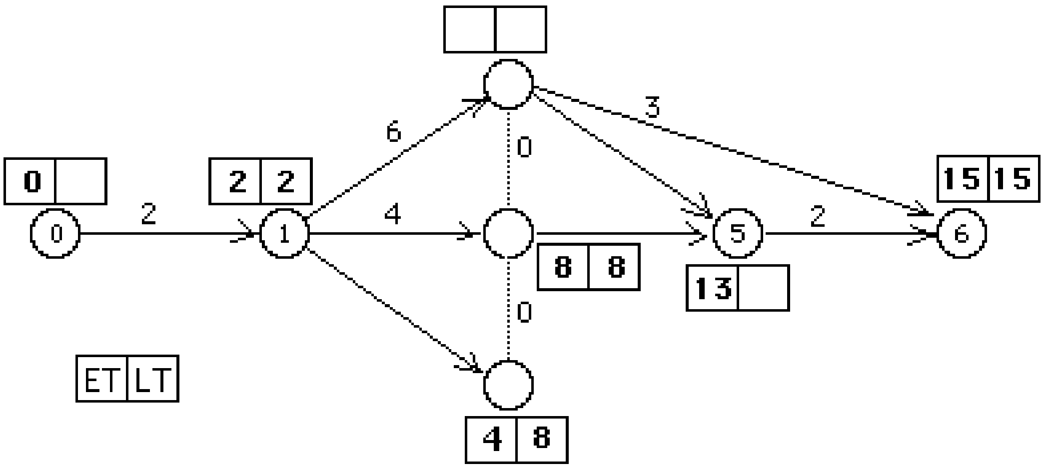


3. Two "dummy" activities in the AOA network above (vertical lines) have no directions indicated. Add directions to these two arrows.

4. Three nodes in the AOA network above are not labeled. Label them.

Name ___________________________________

5. Complete the computation of the earliest & latest times for the events (indicated in the boxes below). *There are four values to be computed!*



6. Indicate whether activity **F** is critical, and for activity **G**, compute:
   - ES = earliest start time
   - LS = latest start time
   - EF = earliest finish time
   - LF = latest finish time
   - TF = total float (slack)

Activity	Duration	ES	LS	EF	LF	TF	Critical?
A	2	0	0	2	2	0	Yes
B	6	2	2	8	8	0	Yes
C	4	2	4	6	8	2	No
D	2	2	6	4	8	4	No
E	4	8	9	12	13	1	No
F	5	8	8	13	13	0	___
G	3	___	___	___	___	___	No
H	2	13	13	15	15	0	Yes

7. What is the earliest completion time for the project? ___________

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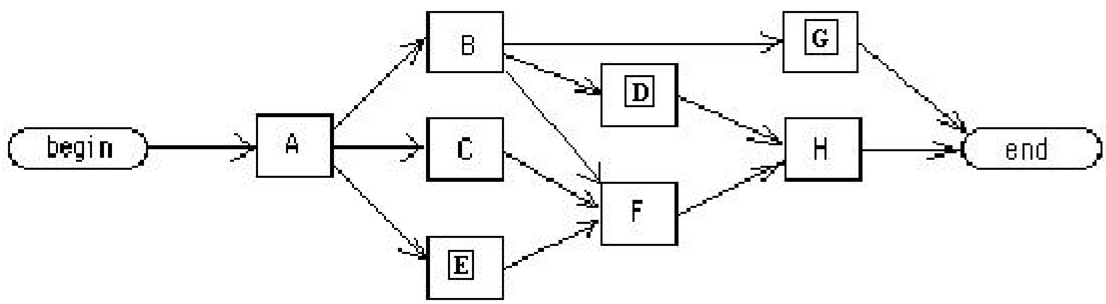
Quiz #6 Solutions -- Spring 1999

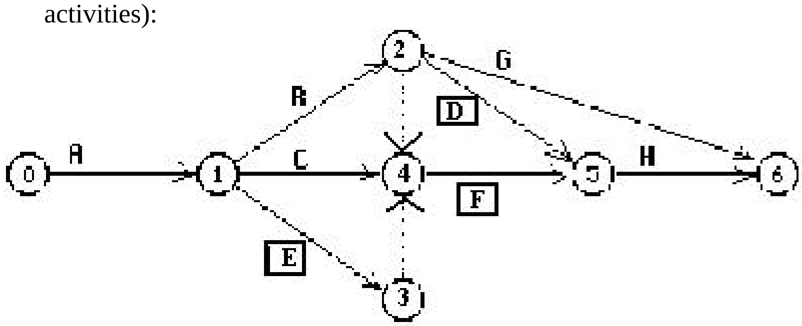
Consider the project consisting of the following activities:

Activity	Description	Predecessor(s)	Duration (days)
A	Clear & level site	none	2
B	Erect building	A	6
C	Install generator	A	4
D	Install water tank	A	2
E	Install maintenance equipment	B	4
F	Connect generator & tank to building	B,C,E	5
G	Paint & finish work on building	B	3
H	Facility test & checkout	D,F	2

1. Complete the AON network by labeling the nodes:



2. Complete the AOA network by labeling the three arrows (not including those for the "dummy" activities):

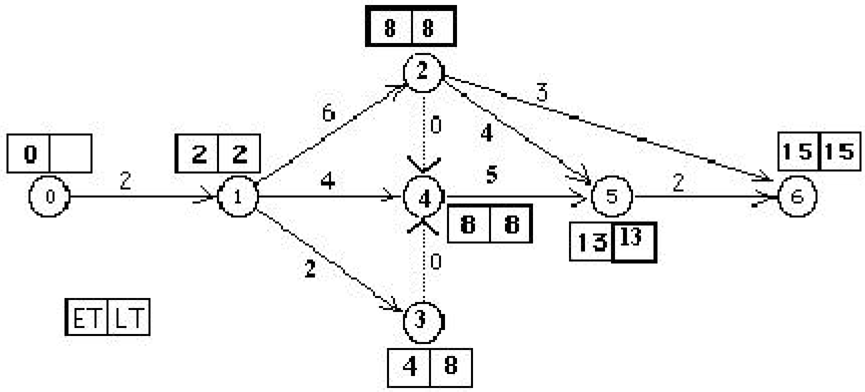


3. Two "dummy" activities in the AOA network above (vertical lines) have no directions indicated. Add directions to these two arrows.

4. Three nodes in the AOA network above are not labeled. Label them.

Note: the labels "2" & "3" may be reversed in the diagram above!

5. Complete the computation of the earliest & latest times for the events (indicated in the boxes below). *There are four values to be computed!*



6. Indicate whether activity **F** is critical, and for activity **G**, compute:
   - ES = earliest start time
   - LS = latest start time
   - EF = earliest finish time
   - LF = latest finish time
   - TF = total float (slack)

Activity	Duration	ES	LS	EF	LF	TF	Critical?
A	2	0	0	2	2	0	Yes
B	6	2	2	8	8	0	Yes
C	4	2	4	6	8	2	No
D	2	2	6	4	8	4	No
E	4	8	9	12	13	1	No
F	5	8	8	13	13	0	**YES**
G	3	**8**	**12**	**11**	**15**	**4**	No
H	2	13	13	15	15	0	Yes

Note: ES = Early Start of activity G is the ET (early time) of node 2, while LF = Late Finish of the activity is the LT (late time) of node 6. The EF (Early Finish) is then ES + duration, while LS (Late Start) = Late Finish **-** *duration. The TF (Total Float) = LS-ES = LF-EF = 4*

7. What is the earliest completion time for the project? ___**15**____

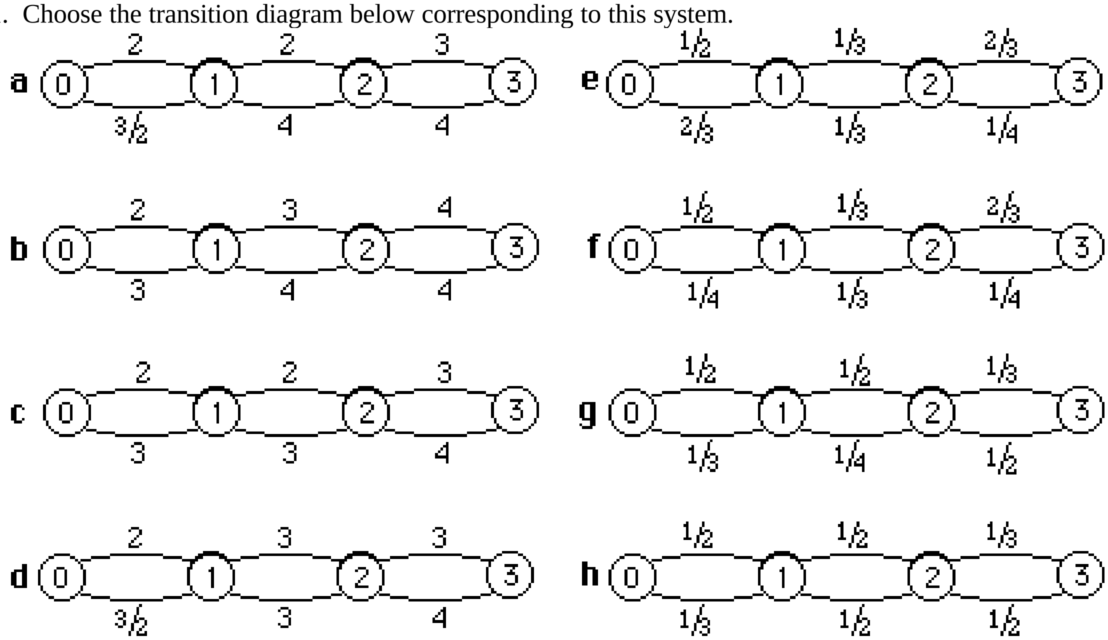
Name ___________________________________

<><><><> 57:022 Principles of Design II <><><><>
Quiz #7 -- April 16, 1999

Consider the following situation:

- Two mechanics work in an auto repair shop, with a capacity of **3** cars.
- If there are **2 or more** cars in the shop, each mechanic works individually, each completing the repair of a car in an average of **4** hours (the actual time being random with exponential distribution).
- If there is only one car in the shop, both mechanics work together on it, completing the repair in an average time of **3** hours (also exponentially distributed).
- Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when there are less than two cars in the shop, but one every **3** hours when both mechanics are busy.
- If 3 cars are already in the shop, no cars arrive.

____ 1. Choose the transition diagram below corresponding to this system.



i. *None of the above*

____ 2. The steadystate probability π_0 is computed by the formula:

a. $\frac{1}{\pi_0} = 1 + \frac{3}{2} + \frac{3}{2}\times 1 + \frac{3}{2}\times 1\times\frac{2}{3}$

b. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{4}{3}\times\frac{1}{2} + \frac{4}{3}\times\frac{1}{2}\times\frac{3}{4}$

c. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{1}{2} + \frac{3}{4}$

d. $\frac{1}{\pi_0} = 1 + \frac{2}{3} + \frac{2}{2} + \frac{3}{4}$

e. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{4}{3}\times 1 + \frac{4}{3}\times 1\times\frac{3}{4}$

f. $\frac{1}{\pi_0} = 1 + \frac{3}{2} + 1 + \frac{2}{3}$

g. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{4}{3}\times\frac{1}{2} + \frac{4}{3}\times\frac{1}{2}\times\frac{3}{4}$

h. $\frac{1}{\pi_0} = 1 + \frac{2}{3} + \frac{2}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{2}{3}\times\frac{3}{4}$

i. $\frac{1}{\pi_0} = 1 + \frac{3}{2} + \frac{3}{2}\times 2 + \frac{3}{2}\times 2\times\frac{2}{3}$

j. *None of the above*

Name ___________________________________

For ease of computation in the following questions, suppose that the steady-state probabilities for this system are *(not the actual values)*:

π_0=20%, π_1=30%, π_2=30%, & π_3=20%.

____ 3. What fraction of the day will both mechanics be idle?

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 4. What fraction of the day will both mechanics be working on the same car?

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 5. What is the average number of cars in the shop? (Choose nearest answer.)

a. 0.5	c. 1.0	e. 1.5	g. 2.0
b. 0.75	d. 1.25	f. 1.75	h. 2.5

____ 6. What is the average number of cars waiting to be serviced? (Choose nearest answer.)

a. 0.5	c. 1.0	e. 1.5	g. 2.0
b. 0.75	d. 1.25	f. 1.75	h. 2.5

____ 7. Suppose that the average arrival rate in steady state is approximately one every 2.5 *hours (not the actual value)*, i.e., 0.4/hour. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is *(choose nearest value):*

a. 2 hours	c. 3 hours	e. 4 hours	g. 5 hours
b. 6 hours	d. 7 hours	f. 8 hours	h. 9 hours

____ 8. What is the average time that a car spends in the shop waiting to be serviced? (Choose nearest answer.)

a. 0.5 hours	c. 1.0 hours	e. 1.5 hours	g. 2.0 hours
b. 0.75 hours	d. 1.25 hours	f. 1.75 hours	h. 2.5 hours

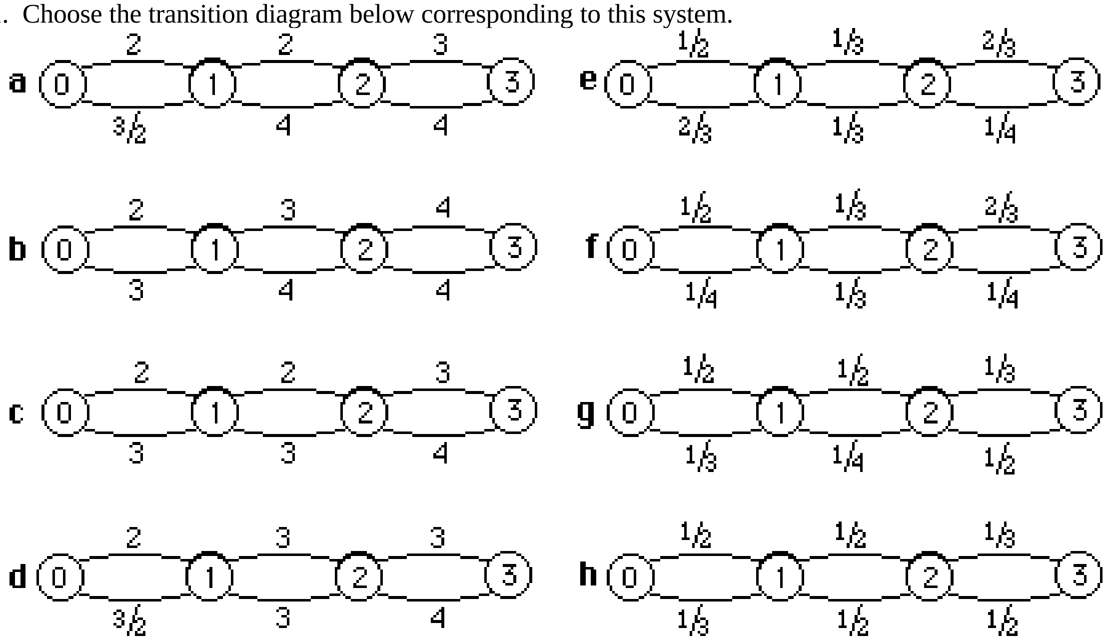
Name ____________________________________

<><><><><> 57:022 Principles of Design II <><><><><>
Quiz #7 -- April 16, 1999

Consider the following situation:

- Two mechanics work in an auto repair shop, with a capacity of **3** cars.
- If there are **2 or more** cars in the shop, each mechanic works individually, each completing the repair of a car in an average of **4** hours (the actual time being random with exponential distribution).
- If there is only one car in the shop, both mechanics work together on it, completing the repair in an average time of **3** hours (also exponentially distributed).
- Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when there are less than two cars in the shop, but one every **3** hours when both mechanics are busy.
- If 3 cars are already in the shop, no cars arrive.

__**h**_ 1. Choose the transition diagram below corresponding to this system.



i. *None of the above*

__**a**_ 2. The steadystate probability π_0 is computed by the formula:

a. $\frac{1}{\pi_0} = 1 + \frac{3}{2} + \frac{3}{2}\times 1 + \frac{3}{2}\times 1\times\frac{2}{3}$

b. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{4}{3}\times\frac{1}{2} + \frac{4}{3}\times\frac{1}{2}\times\frac{3}{4}$

c. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{1}{2} + \frac{3}{4}$

d. $\frac{1}{\pi_0} = 1 + \frac{2}{3} + \frac{2}{2} + \frac{3}{4}$

e. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{4}{3}\times 1 + \frac{4}{3}\times 1\times\frac{3}{4}$

f. $\frac{1}{\pi_0} = 1 + \frac{3}{2} + 1 + \frac{2}{3}$

g. $\frac{1}{\pi_0} = 1 + \frac{4}{3} + \frac{4}{3}\times\frac{1}{2} + \frac{4}{3}\times\frac{1}{2}\times\frac{3}{4}$

h. $\frac{1}{\pi_0} = 1 + \frac{2}{3} + \frac{2}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{2}{3}\times\frac{3}{4}$

i. $\frac{1}{\pi_0} = 1 + \frac{3}{2} + \frac{3}{2}\times 2 + \frac{3}{2}\times 2\times\frac{2}{3}$

j. *None of the above*

Name ___________________________________

For ease of computation in the following questions, suppose that the steady-state probabilities for this system are *(not the actual values)*:

π_0=20%, π_1=30%, π_2=30%, & π_3=20%.

__**b**_ 3. What fraction of the day will both mechanics be idle?

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

Solution: π_0

__**c**_ 4. What fraction of the day will both mechanics be working on the same car*?*

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

Solution: π_1

__**e**_ 5. What is the average number of cars in the shop? (Choose nearest answer.)

a. 0.5	c. 1.0	e. 1.5	g. 2.0
b. 0.75	d. 1.25	f. 1.75	h. 2.5

Solution: $L = 0\pi_0 + 1\pi_1 + 2\pi_2 + 3\pi_3 = 0+1 \times 0.3 + 2 \times 0.3+3 \times 0.2 = 0.3+0.6+0.6=1.5$

__**a**_ 6. What is the average number of cars waiting to be serviced? (Choose nearest answer.)

a. 0.5	c. 1.0	e. 1.5	g. 2.0
b. 0.75	d. 1.25	f. 1.75	h. 2.5

Solution: $L_q = 0\pi_0 + 0\pi_1 + 0\pi_2 + 1\pi_3 = 0+0 + 0+1 \times 0.2 = 0.2$, since only in state 3 will a car be waiting to be serviced!

__**e**_ 7. Suppose that the average arrival rate in steady state is approximately one every 2.5 *hours (not the actual value)*, i.e., 0.4/hour. According to Little's Formula, the average total time spent by a car in the shop (including both waiting and repair time) is *(choose nearest value):*

a. 2 hours	c. 3 hours	e. 4 hours	g. 5 hours
b. 6 hours	d. 7 hours	f. 8 hours	h. 9 hours

Solution: $L = \lambda W \Rightarrow W= L/\lambda = (1.5)/(0.4/hr) = 3.75$ hrs

__**a**_ 8. What is the average time that a car spends in the shop waiting to be serviced? (Choose nearest answer.)

a. 0.5 hour	c. 1.0 hour	e. 1.5 hours	g. 2.0 hours
b. 0.75 hour	d. 1.25 hours	f. 1.75 hours	h. 2.5 hours

Solution: $Wq = L_q/\lambda = 0.2/(0.4/hr) = 0.5$ hr.