Name ___________________________

57:022 Principles of Design II -- Quiz #1
Friday, January 31, 1997

Write the <u>number</u> corresponding to the correct probability distribution in each blank below. Note that some distributions may apply in more than one case, while others may not apply at all!

A telephone exchange (for ticket reservatons, for example) contains 10 lines. A line can be busy or available for calls and all lines act independently. Each line is busy 82% of the noon period (so that the probability that a specified line will be busy at any given time during the noon period is 82%). When I call the exchange's number, I will be connected to a free line if available, or else obtain a busy signal if all ten lines are busy. What probability distribution does each of the following random variables have?

___ a. number of free lines when I call at a specific time, e.g., 12:15.
___ b. number of times I must call before finally receiving a free line.
___ c. an indication whether the first call which I attempt obtains either a busy signal or a free line.

A certain production process has a fraction defective of 15%. All items are routinely inspected as soon as they are produced. Three good pieces are required. Pieces are produced until 3 pieces which pass inspection have been obtained.
Which probability distributions would best be used to compute the probability that...

___ d. the third item is the first good item to be produced.
___ e. the number of items required in order to obtain the third good item is 6.

Suppose that, instead of inspecting each item before producing the next, a batch of size 4 is produced, after which the items are inspected. Which probability distributions would best be used to compute the probability that...

___ f. at least three good items are found in the batch of size 4

A pair of dice is thrown, with a desire of achieving a value of either “7” or “11”.
Which probability distributions would best be used to compute the probability that...

___ g. number of throws of the dice required in order to obtain a “7” or “11” is exactly 3.
___ h. in six throws of the dice, a “7” or “11” is obtained at least twice.

<u>Probability distributions</u>:

1. Bernouilli	4. Pascal	7. Poisson
2. Binomial	5. Geometric	8. Normal
3. Erlang	6. Exponential	

Indicate with true (+) or false (o):

___ i. A random number with Pascal distribution is the sum of random variables having the Bernouilli distribution.
___ j. The geometric distribution is a special case of the Pascal distribution.
___ k. In a Bernouilli process, the number of "successes" N_n in n trials has the Poisson distribution.
___ l. If W_1 has the geometric distribution, then

$$P\{W_1=1\} \geq P\{W_1=2\} \geq P\{W_1=2\} \geq \ldots$$

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57:022 Principles of Design II
Quiz #1 Solutions
Spring 1997

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Write the number corresponding to the correct probability distribution in each blank below. Note that some distributions may apply in more than one case, while others may not apply at all!

A telephone exchange (for ticket reservatons, for example) contains 10 lines. A line can be busy or available for calls and all lines act independently. Each line is busy 82% of the noon period (so that the probability that a specified line will be busy at any given time during the noon period is 82%). When I call the exchange's number, I will be connected to a free line if available, or else obtain a busy signal if all ten lines are busy. What probability distribution does each of the following random variables have?

- Binomial a. number of free lines when I call at a specific time, e.g., 12:15.
- Geometric b. number of times I must call before finally receiving a free line.
- Bernouilli c. an indication whether the first call which I attempt obtains either a busy signal or a free line.

A certain production process has a fraction defective of 15%. All items are routinely inspected as soon as they are produced. Three good pieces are required. Pieces are produced until 3 pieces which pass inspection have been obtained.
Which probability distributions would best be used to compute the probability that...

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- Pascal e. the number of items required in order to obtain the third good item is 6.

Suppose that, instead of inspecting each item before producing the next, a batch of size 4 is produced, after which the items are inspected. Which probability distributions would best be used to compute the probability that...

- Binomial f. at least three good items are found in the batch of size 4

A pair of dice is thrown, with a desire of achieving a value of either "7" or "11".
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- Pascal g. number of throws of the dice required in order to obtain a "7" or "11" is exactly 3.
- Binomial h. in six throws of the dice, a "7" or "11" is obtained at least twice.

Indicate with true (+) or false (o):

- false i. A random number with Pascal distribution is the sum of random variables having the Bernouilli distribution.
- true j. The geometric distribution is a special case of the Pascal distribution.
- false k. In a Bernouilli process, the number of "successes" N_n in n trials has the Poisson distribution.
- true l. If T_1 has the geometric distribution, then $P\{T_1=1\} \geq P\{T_1=2\} \geq P\{T_1=2\} \geq \ldots$

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57:022 Principles of Design II

Quiz #2

7 February 1997

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I. Along highway I-80 in Iowa, the probability that each passing car stops to pick up a hitchhiker is p=**2**%, i.e, an average of one in fifty drivers will stop; different drivers, of course, make their decisions whether to stop or not independently of each other.

___ 1. If 25 cars pass the hitchhiker, the probability that *none* of them stop is

a. 25(0.02) b. $(0.02)^{25}$ c. $(0.98)^{25}$

d. $(0.98)^{24}(0.02)$ e. $(0.02)^{24}(0.98)$ f. None of the above

___ 2. For the stochastic process described above, the random variable T_1 = (# of the first vehicle to stop) has which distribution?

a. Geometric b. Bernouilli c. Binomial

d. Poisson e. Exponential f. None of the above

___ 3. Consider a stochastic process in which X_n=1 if car n stops to pick up the hitchhiker, and X_n=0 otherwise. Then $\{X_n$: n=1,2,3,...$\}$ is a

a. Binomial process b. Bernouilli process c. Poisson process

d. Markov process e. Exponential process f. None of the above

___ 4. $P\{X_n=1\}$ is equal to

a. 0.50 b. 0.2 c. 0.02

d. 0.025 e. 0.98 f. None of the above

___ 5. Given that a hitchhiker has counted 30 cars which have passed him without stopping, what is the probability that he will be picked up by the 40^{th} car *or before?*

a. $(0.98)^{40}$ b. $1-(0.98)^{10}$ c. $(0.98)^{30}(0.02)^{10}$

d. $1-(0.02)^{10}$ e. $(0.98)^{10}$ f. None of the above

Suppose that the arrivals of the cars form a Poisson process, at the average rate of 10 per minute. Define "success" for the hitchhiker to occur at time t provided that *both* an arrival occurs at t *and* that car stops to pick him up. Let T_1 be the time (in seconds) of the first "success", i.e., the time that he finally gets a ride, when he begins his wait at time t=0.

___ 6. Suppose that after 5 minutes (during which 42 cars have passed by) he is still there waiting for a ride. What is the *conditional* expected value of T_1 (expected *total* waiting time, i.e., since time 0, given that he has already waited 5 minutes).

a. 50 minutes b. 15 minutes c. 10 minutes

d. 1 minute e. 0.5 minutes f. None of the above`

___ 7. What's the probability that his waiting time is less than or equal to 5 min. ($P\{T_1 \leq 5\}$?

a. $1 - e^{-5}$ b. $1 - e^{-1}$ c. e^{-1}

d. e^{-5} e. $1 - e^{-0.1}$ f. None of the above

___ 8. What is the probability that he must wait *exactly* 5 minutes for a ride ($P\{T_1 = 5\}$?

a. $1 - e^{-5}$ b. $1 - e^{-1}$ c. e^{-1}

d. e^{-5} e. $1 - e^{-0.1}$ f. None of the above

___ 9. The arrival rate of "successes" is

a. 1/minute b. 2/minute c. 0.1/minute

d. 0.2/minute e. 0.5/minute f. None of the above

___ 10. The random variable T_1 has what distribution?

a. Poisson b. Geometric c. Exponential

d. Pascal e. Erlang f. None of the above

___ 11. What is $E(T_1)$, the expected (mean) value of T_1?

a. 10 minutes b. 5 minutes c. 2 minutes

d. 1 minute e. 0.2 minute f. None of the above

II. A bearing in a Grass Chopper mower's PTO mechanism fails randomly, with an expected lifetime of 250 hours. Assume that the lifetime of the bearing has an exponential distribution.

___ 12. If, when the bearing fails, it is replaced (with a new bearing), what is the probability distribution of the time of the second failure (T_2)?

a. Poisson b. Geometric c. Exponential
d. Pascal e. Erlang f. None of the above

___ 13. What is the probability that the bearing will fail (& be replaced)three or more times in 750 hours of mowing?

a. $\sum_{x=0}^{3} \frac{(3)^x}{x!} e^{-3}$ b. $\frac{(3)^3}{3!} e^{-3}$ c. $1 - \sum_{x=0}^{2} \frac{(3)^x}{x!} e^{-3}$

d. $1 - \sum_{x=0}^{3} \frac{(3)^x}{x!} e^{-3}$ e. $1 - \frac{(3)^2}{2!} e^{-3}$ f. None of the above

___ 14. What is the probability that the bearing lasts *longer than* 250 hours?

a. e^{-1} b. $1 - e^{-1}$ c. e^{250}
d. e^{-250} e. $1 - e^{-250}$ f. None of the above

___ 15. If the mower has already operated without failure, for 150 hours, what is the probability that the bearing will last a total of *at least* 250 hours?

a. e^{-2} b. $1 - e^{-0.1}$ c. $e^{-0.1}$
d. $1 - e^{-250}$ e. e^{-150} f. None of the above

<u>(Possibly) Useful Formulas:</u>

Binomial distribution $P\{x \text{ successes in n trials}\} = \binom{n}{x} p^x (1-p)^{n-x}$

Exponential distribution $P\{T \leq t\} = 1 - e^{-\lambda t}$

Poisson distribution $P\{N_t = x\} = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$

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57:022 Principles of Design II

Quiz #2 Solutions

Spring 1997

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I. Along highway I-80 in Iowa, the probability that each passing car stops to pick up a hitchhiker is p=**2**%, i.e, an average of one in fifty drivers will stop; different drivers, of course, make their decisions whether to stop or not independently of each other.

_c_ 1. If 25 cars pass the hitchhiker, the probability that *none* of them stop is
a. 25(0.02) b. $(0.02)^{25}$ c. $(0.98)^{25}$
d. $(0.98)^{24}(0.02)$ e. $(0.02)^{24}(0.98)$ f. None of the above

_d_ 2. For the stochastic process described above, the random variable T_1 = (# of the first vehicle to stop) has which distribution?
a. Geometric b. Bernouilli c. Binomial
d. Poisson e. Exponential f. None of the above

_b_ 3. Consider a stochastic process in which X_n=1 if car n stops to pick up the hitchhiker, and X_n=0 otherwise. Then $\{X_n: n=1,2,3,...\}$ is a
a. Binomial process b. Bernouilli process c. Poisson process
d. Markov process e. Exponential process f. None of the above

_c_ 4. $P\{X_n=1\}$ is equal to
a. 0.50 b. 0.2 c. 0.02
d. 0.025 e. 0.98 f. None of the above

_b_ 5. Given that a hitchhiker has counted 30 cars which have passed him without stopping, what is the probability that he will be picked up by the 40th car *or before?*
a. $(0.98)^{40}$ b. $1-(0.98)^{10}$ c. $(0.98)^{30}(0.02)^{10}$
d. $1-(0.02)^{10}$ e. $(0.98)^{10}$ f. None of the above

Suppose that the arrivals of the cars form a Poisson process, at the average rate of 10 per minute. Define "success" for the hitchhiker to occur at time t provided that *both* an arrival occurs at t *and* that car stops to pick him up. Let T_1 be the time (in seconds) of the first "success", i.e., the time that he finally gets a ride, when he begins his wait at time t=0.

_c_ 6. Suppose that after 5 minutes (during which 42 cars have passed by) he is still there waiting for a ride. What is the *conditional* expected value of T_1 (expected *total* waiting time, i.e., since time 0, given that he has already waited 5 minutes).
a. 50 minutes b. 15 minutes c. 10 minutes
d. 1 minute e. 0.5 minutes f. None of the above`

_b_ 7. What's the probability that his waiting time is less than or equal to 5 min. ($P\{T_1 \le 5\}$?
a. $1 - e^{-5}$ b. $1 - e^{-1}$ c. e^{-1}
d. e^{-5} e. $1 - e^{-0.1}$ f. None of the above

_e_ 8. What is the probability that he must wait *exactly* 5 minutes for a ride ($P\{T_1= 5\}$?
a. $1 - e^{-5}$ b. $1 - e^{-1}$ c. e^{-1}
d. e^{-5} e. $1 - e^{-0.1}$ f. None of the above

_d_ 9. The arrival rate of "successes" is
a. 1/minute b. 2/minute c. 0.1/minute
d. 0.2/minute e. 0.5/minute f. None of the above

_c_ 10. The random variable T_1 has what distribution?
a. Poisson b. Geometric c. Exponential
d. Pascal e. Erlang f. None of the above

_b_ 11. What is $E(T_1)$, the expected (mean) value of T_1?
a. 10 minutes b. 5 minutes c. 2 minutes
d. 1 minute e. 0.2 minute f. None of the above

II. A bearing in a Grass Chopper mower's PTO mechanism fails randomly, with an expected lifetime of 250 hours. Assume that the lifetime of the bearing has an exponential distribution.

_e_ 12. If, when the bearing fails, it is replaced (with a new bearing), what is the probability distribution of the time of the second failure (T_2)?

a. Poisson b. Geometric c. Exponential
d. Pascal e. Erlang f. None of the above

_e_ 13. What is the probability that the bearing will fail (& be replaced)three or more times in 750 hours of mowing?

a. $\sum_{x=0}^{3} \frac{(3)^x}{x!} e^{-3}$ b. $\frac{(3)^3}{3!} e^{-3}$ c. $1 - \sum_{x=0}^{2} \frac{(3)^x}{x!} e^{-3}$

d. $1 - \sum_{x=0}^{3} \frac{(3)^x}{x!} e^{-3}$ e. $1 - \frac{(3)^2}{2!} e^{-3}$ f. None of the above

_a_ 14. What is the probability that the bearing lasts *longer than* 250 hours?

a. e^{-1} b. $1 - e^{-1}$ c. e^{250}
d. e^{-250} e. $1 - e^{-250}$ f. None of the above

_f_ 15. If the mower has already operated without failure, for 150 hours, what is the probability that the bearing will last a total of *at least* 250 hours?

a. e^{-2} b. $1 - e^{-0.1}$ c. $e^{-0.1}$
d. $1 - e^{-250}$ e. e^{-150} f. None of the above

Note: the correct value is

$$P\{T \geq 250 \mid T \geq 150\} = P\{T \geq 100\} = e^{-\frac{1}{250} \times 100} = e^{-0.4}$$

<u>(Possibly) Useful Formulas:</u>

Binomial distribution $P\{x \text{ successes in n trials}\} = \binom{n}{x} p^x (1-p)^{n-x}$

Exponential distribution $P\{T \leq t\} = 1 - e^{-\lambda t}$

Poisson distribution $P\{N_t = x\} = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$

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57:022 Principles of Design II

Quiz #3 Solutions

Spring 1997

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Goodness-of-Fit test: The number of vehicles arriving during each of 25 one-minute intervals was recorded. The mean value of O_i was computed to be 1.76. We wish to test the "goodness of fit" of the Poisson distribution having mean 1.76. Denote the sum of the last column by "D", i.e., D=9.03703. *Note that we are ignoring the advice to aggregate cells to avoid very small numbers of observations.)*

i	O_i	p_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
0	4	0.17204	4.30112	0.02108
1	6	0.30280	7.56997	0.32560
2	11	0.26646	6.66158	2.82544
3	2	0.15633	3.90813	0.93163
4	0	0.06878	1.71958	1.71958
5	2	0.02421	0.60529	3.21369
sum	25	0.99063	24.76566	9.03703

Indicate "+" for true, "o" for false:

__+_ 1. The smaller the value of D, the better the fit of the distribution being tested.

__o_ 2. The quantity O_i is a random variable with approximately Poisson distribution (assuming the fitted distribution is the true distribution).

__o_ 3. The chi-square distribution for this goodness-of-fit test will have 5 "degrees of freedom".

__+_ 4. The quantity E_i is the expected number of observations of i arrivals (assuming the fitted distribution is the true distribution).

__+_ 5. The CDF of the distribution of interarrival times is assumed to be $F(t) = 1 - e^{-1.76t}$ (assuming the fitted distribution is the true distribution).

__+_ 6. In this test, the "degrees of freedom" is reduced by 2 because (i) the total number of observations is fixed at 25, and (ii) the data was used to estimate one parameter of the distribution being tested.

__+_ 7. Because the raw data is random, the number of observations O_i in interval #i is a random variable with approximately binomial distribution, with n (number of "trials") = 25 and p (probability of "success") = p_i (assuming the fitted distribution is correct).

__+_ 8. The parameter of the Poisson distribution is assumed to be $\lambda = 1.76$/minute.

__+_ 9. The chi-square distribution for this test will have 4 "degrees of freedom".

__o_ 10. The quantity D is assumed to have approximately a Normal distribution.

__+_ 11. The probability p_2 that 2 cars arrive during a one-minute interval, under the assumption that the arrival process is Poisson, is $e^{-1.76}\frac{1.76^2}{2!}$ (assuming the fitted distribution is correct).

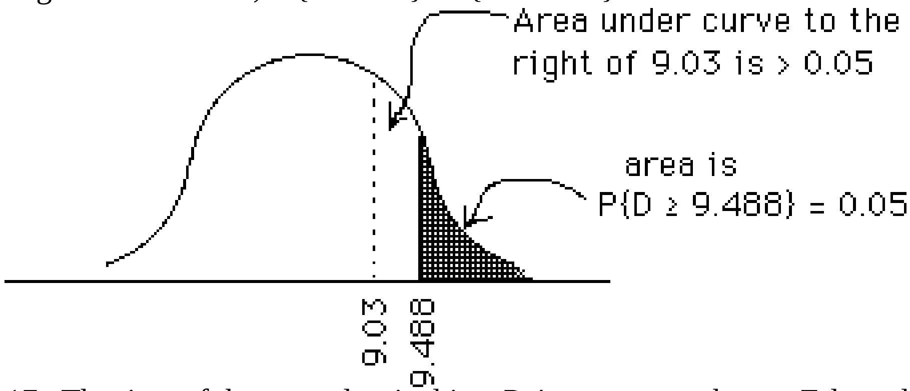
__+_ 12. The sum of the squares of several N(0,1) random variables has a chi-square distribution.

__+_ 13. If a chi-square random variable D has 7 degrees of freedom, then according to the chi-square table, the probability that D will exceed 12.017 is 10%.

__+_ 14. The quantity D is assumed to have the chi-square distribution.

__+_ 15. The time between arrivals in a Poisson process has exponential distribution.

__+_ 16. <u>If</u> it is true that this is a Poisson arrival process with rate 1.76 /minute, then the probability that D exceeds the computed value 9.03 is greater than 5%. Note: With 4 degrees of freedom, $P\{D \geq 9.03\} > P\{D \geq 9.488\} = 5\%$



__+_ 17. The time of the second arrival in a Poisson process has an Erlang distribution.

__o_ 18. If we choose $\alpha = 10\%$, the Poisson distribution with mean 1.76 may be accepted as a model for the number of the vehicles arriving per minute.

__+_ 19. If the assumed distribution is correct, the arrivals of the cars forms a Poisson process.

__o_ 20. The quantity D is assumed to have approximately a Normal distribution.

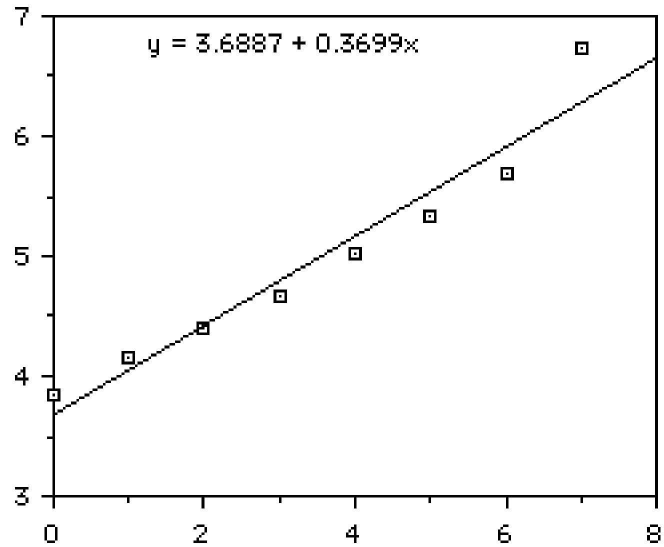
A portion of a table of the chi-square distribution is given below:

deg.of freedom	Chi-square Dist'n $P\{D \geq \chi^2\}$ 99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

Regression Analysis. The number Y of bacteria per cubic centimeter found in a tillage after T hours is given in the following table:

T (hours)	0	1	2	3	4	5	6	7
Y (#/cm^3)	47	64	81	107	151	209	298	841

It is believed that the relation between the two variables is of the form $Y = \alpha\beta^T$. In order to use linear regression to fit this nonlinear function to the data, certain transformations were performed on the original variable(s), and a plot drawn as shown below (which includes the linear fit of the data points):



Note: the linearized form of the equation $Y = \alpha\beta^T$ is $(\ln Y) = (\ln \alpha) + (\ln \beta)T$, whose graph is a line with vertical intercept equal to $(\ln \alpha)$ and slope equal to $(\ln \beta)$.

_b_ 21. What is plotted on the horizontal axis above?

a. Y b. t c. ln ln t
d. ln Y e. ln t f. *None of the above*

_d_ 22. What is plotted on the vertical axis above?

a. Y b. t c. ln ln t
d. ln Y e. ln t f. *None of the above*

_f_ 23. What is being minimized in this linear regression?

a. $\sum_{i=0}^{7} Y_i - \alpha\beta^{t_i}$

b. $\sum_{i=0}^{7} \left|Y_i - \alpha\beta^{t_i}\right|$

c. $\sum_{i=0}^{7} \left(Y_i - \alpha\beta^{t_i}\right)^2$

d. $\underset{i=0,\ldots 7}{\text{Max}} \left\{\left|Y_i - \alpha\beta^{t_i}\right|\right\}$

e. $\sum_{i=0}^{7} \left(\ln Y_i - \left[\ln \alpha + \beta \times \ln t_i\right]\right)^2$

f. $\sum_{i=0}^{7} \left(\ln Y_i - \left[\ln \alpha + t_i \times \ln\beta\right]\right)^2$

g. *None of the above*

_d_ 24. What is the nonlinear form of the function fitted by linear regression?

a. $Y = (\ln 3.6887) \times (\ln 0.3699)^t$

b. $Y = \left(e^{3.6887}\right) \times (\ln 0.3699)^t$

c. $Y = \left(e^{3.6887}\right) \times (0.3699)^t$

d. $Y = \left(e^{3.6887}\right) \times \left(e^{0.3699}\right)^t$

e. $Y = (\ln 3.6887) \times \left(e^{0.3699}\right)^t$

f. *None of the above*

Name ____________________________

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57:022 Principles of Design II

Quiz #4

Wednesday, February 19, 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Statements below refer to today's homework assignment (HW#4).
Indicate "+" for true, "o" for false:

___ 1. The Cricket Graph program fits a line through data points which minimizes the maximum error, where the error is the vertical distance between a data point and the fitted line.

___ 2. The method used in homework exercise #4 to estimate the Weibull parameters u & k requires that the motors be tested until all have failed.

___ 3. We assumed in this exercise that the number of failures at time t , $N_f(t)$, has a Weibull distribution.

___ 4. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has failed at time t.

____ 5. In this test, the "degrees of freedom" is reduced by 2 because (i) the total number of observations is fixed at 25, and (ii) the data was used to estimate one parameter of the distribution being tested.

___ 6. The Weibull distribution might be appropriate for the maximum of a large number of exponentially-distributed random variables.

____ 7. The CDF of the failure time of a motor is assumed to be $F(t) = 1 - e^{-\left(k/u\right)^t}$ for some parameters u & k.

___ 8. According to the results of this homework exercise, the failure rate of the motors is decreasing rather than increasing.

___ 9. If the assumption of Weibull distribution were correct, a plot of the data points (t_i vs. $R(t_i)$, where t_i is the i^{th} failure time and $R(t_i)$ is the fraction of survivors at this time) should lie approximately on a straight line.

____ 10. If a chi-square random variable D has 4 degrees of freedom, then according to the chi-square table, the probability that D will exceed 12.017 is 10%.

___ 11. A value of k > 1 indicates an increasing failure rate, while k<1 indicates a decreasing failure rate.

___ 12. Given a coefficient of variation for the Weibull distribution (i.e., the ratio σ/μ), the parameter k can be determined by solving a nonlinear equation.

___ 13. $\Gamma(n-1) = n!$ if n is an integer.

___ 14. The slope of the straight line fit by Cricket Graph in this homework exercise was the estimate of the "shape" parameter k.

___ 15. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

____ 16. The sum of the squares of several N(0,1) random variables has a chi-square distribution.

___ 17. We assumed in this HW that the lifetime of the device has a Weibull distribution.

___ 18. If 10 units of this motor are installed in a manufacturing system, the number still functioning after 500 hours has a Weibull distribution.

___ 19. To estimate the time at which 90% of the units will have failed, evaluate 1 - F(0.90).

___ 20. The quantity R_t is the expected fraction of the devices which have survived until time t.

___ 21. The time between the failures in the batch of 100 motors is assumed to have the Weibull distribution.

Name ______________________________

In order to estimate k & u in this HW exercise, Cricket Graph was used to fit a straight line to some plotted data points. Select the letter below which indicates each correct answer:

___ 22. The slope of the line fit by Cricket Graph should be approximately ...
___ 23. The label on the vertical axis should be ...
___ 24. The label on the horizontal axis should be ...
___ 25. The vertical intercept of the line fit by Cricker Graph should be approximately ...

a. $\ln {}^{1}/_{t}$	h. $\ln {}^{1}/_{R_t}$	o. mean value μ
b. ln ln t	i. $\ln \ln R_t$	p. standard deviation σ
c. t	j. R_t	q. shape parameter k
d. ln t	k. $\ln R_t$	r. scale parameter u
e. -k ln u	l. u ln k	s. ln k
f. $\ln \ln {}^{1}/_{t}$	m. $\ln \ln {}^{1}/_{R_t}$	t. ln u
g. k ln u	n. - u ln k	u. None of the above

___ 26. The "Cumulative Distribution Function" (CDF) of any random variable X is defined as

a. f(x) = P{x}	c. f(x) = P{X|x}	e. F(x) = P{X=x}
b. f(x) = P{x | X}	d. F(x) = P{X≥x}	f. F(x) = P{X≤x}

___ 27. The Reliability of a device with random failure time T is defined as

a. R(t) = P{t}	c. R(x) = P{T | t}	e. R(x) = P{T=t}
b. R(t) = P{t | T}	d. R(t) = P{T≥t}	f. R(x) = P{T≤t}

A portion of a table of the chi-square distribution is given below:

deg.of	Chi-square Dist'n $P\{D \geq \chi^2\}$					
freedom	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

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57:022 Principles of Design II

Quiz #4 Solutions

Spring 1997

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Statements below refer to today's homework assignment (HW#4).
Indicate "+" for true, "o" for false:

_o_ 1. The Cricket Graph program fits a line through data points which minimizes the maximum error, where the error is the vertical distance between a data point and the fitted line.

_o_ 2. The method used in homework exercise #4 to estimate the Weibull parameters u & k requires that the motors be tested until <u>all</u> have failed.

_o_ 3. We assumed in this exercise that the number of failures at time t , $N_f(t)$, has a Weibull distribution.

_+_ 4. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has failed at time t.

_o_ 5. In this test, the "degrees of freedom" is reduced by 2 because (i) the total number of observations is fixed at 25, and (ii) the data was used to estimate one parameter of the distribution being tested.

_o_ 6. The Weibull distribution might be appropriate for the maximum of a large number of exponentially-distributed random variables.

_o_ 7. The CDF of the failure time of a motor is assumed to be $F(t) = 1 - e^{-\left(k/u\right)^t}$ for some parameters u & k.

_o_ 8. According to the results of this homework exercise, the failure rate of the motors is decreasing rather than increasing.

_o_ 9. If the assumption of Weibull distribution were correct, a plot of the data points (t_i vs. $R(t_i)$, where t_i is the i^{th} failure time and $R(t_i)$ is the fraction of survivors at this time) should lie approximately on a straight line.

_+_ 10. <u>If</u> a chi-square random variable D has 5 degrees of freedom, then according to the chi-square table, the probability that D will exceed 9.236 is 10%.

_+_ 11. A value of k > 1 indicates an increasing failure rate, while k<1 indicates a decreasing failure rate.

_+_ 12. Given a coefficient of variation for the Weibull distribution (i.e., the ratio σ/μ), the parameter k can be determined by solving a nonlinear equation.

_o_ 13. $\Gamma(n-1) = n!$ if n is an integer. *Note:* $\Gamma(n+1) = n!$ *For example,* $\Gamma(1+{}^1/_k) = ({}^1/_k)!$ *if* $k={}^1/_n$ *for some integer n.*

_+_ 14. The slope of the straight line fit by Cricket Graph in this homework exercise was used to estimate the "shape" parameter k.

_+_ 15. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

_+_ 16. The sum of the squares of several N(0,1) random variables has a chi-square distribution.

_+_ 17. We assumed in this HW that the lifetime of the device has a Weibull distribution.

_o_ 18. If 10 units of this motor are installed in a manufacturing system, the number still functioning after 30 days has a Weibull distribution.

_o_ 19. To estimate the time at which 90% of the units will have failed, evaluate 1 - F(0.90).

_+_ 20. The quantity R_t is the expected fraction of the devices which have survived until time t.

_o_ 21. The time between the failures in the batch of 100 motors is assumed to have the Weibull distribution.

In order to estimate k & u in this HW exercise, Cricket Graph was used to fit a straight line to some plotted data points. Select the letter below which indicates each correct answer:

_q_ 22. The slope of the line fit by Cricket Graph should be approximately ...
_m_ 23. The label on the vertical axis should be ...
_d_ 24. The label on the horizontal axis should be ...
_e_ 25. The vertical intercept of the line fit by Cricker Graph should be approximately ...

a. $\ln {}^{1}/_{t}$	h. $\ln {}^{1}/_{R_t}$	o. mean value μ
b. ln ln t	i. $\ln \ln R_t$	p. standard deviation σ
c. t	j. R_t	q. shape parameter k
d. ln t	k. $\ln R_t$	r. scale parameter u
e. -k ln u	l. u ln k	s. ln k
f. $\ln \ln {}^{1}/_{t}$	m. $\ln \ln {}^{1}/_{R_t}$	t. ln u
g. k ln u	n. - u ln k	u. None of the above

_f_ 26. The "Cumulative Distribution Function" (CDF) of any random variable X is defined as
a. f(x) = P{x} c. f(x) = P{X|x} e. F(x) = P{X=x}
b. f(x) = P{x | X} d. F(x) = P{X≥x} f. F(x) = P{X≤x}

_d_ 27. The Reliability of a device with random failure time T is defined as
a. R(t) = P{t} c. R(x) = P{T | t} e. R(x) = P{T=t}
b. R(t) = P{t | T} d. R(t) = P{T≥t} f. R(x) = P{T≤t}
Note: Strictly speaking, the answer should have been R(t) = P{T>t}, which is the same value if the lifetime T has a continuous distribution function, so that P{T=t}=0.

A portion of a table of the chi-square distribution is given below:

deg.of	Chi-square Dist'n $P\{D \geq \chi^2\}$					
freedom	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

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57:022 Principles of Design II

Quiz #5

Wednesday, February 26, 1997

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Some statements below refer to today's homework assignment (HW#5).
Indicate "+" for true, "o" for false:

___ 1. If you use the Cricket Graph program to fit a line, it will find the straight line which minimizes the sum of the squares of the errors, i.e., the sum of the squares of the vertical distances between each data point and the line.

___ 2. In the chi-square goodness-of-fit test, the number of degrees of freedom is never more than the number of "cells" of the histogram.

___ 3. The inverse transformation method to generate a random number can be used to simulate failure times having a Weibull distribution.

___ 4. The Weibull distribution is a special case of the exponential distribution.

___ 5. If T is the time at which a unit fails, its reliability is defined as

- a. $R(x) = 1 - f(t)$
- b. $R(x) = f(t)$
- c. $R(t) = P\{T=t\}$
- d. $R(t) = P\{T\geq t\}$
- e. $R(t) = P\{T=t \mid T\geq t\}$
- f. $R(t) = P\{T\leq t\}$

___ 6. We assumed in this HW that the number of motors which have failed at time t , $N_f(t)$, has a Weibull distribution.

___ 7. If the assumption of Weibull distribution for the lifetime of the motors were correct, a plot of $N_f(t)$ vs. t should be approximately on a straight line.

___ 8. To estimate the time at which 90% of the units will have failed, we evaluate $F^{-1}(0.90)$, where F is the CDF of the failure-time distribution.

___ 9. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has survived (not failed) until time t.

___ 10. If the assumption of Weibull distribution were correct, a plot of $\ln \ln {}^{1}/_{N_{s(t)}}$ vs. ln t should be approximately a straight line, where $N_s(t)$ is the fraction of motors surviving at time t.

___ 11. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

___ 12. A value of k greater than 1.0 indicates an increasing failure rate, and k less than 1.0 indicates a decreasing failure rate.

___ 13. If ten motors are installed in a manufacturing facility, the number still functioning after 60 days has a binomial distribution.

___ 14. The expected number of machines E_i which fail in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$ where F(t) is the CDF of the failure time distribution.

___ 15. According to the results of this homework exercise, the failure rate of the motors is increasing rather than decreasing over time.

___ 16. A positive value of the shape parameter k indicates an increasing failure rate, and negative k indicates a decreasing failure rate for the Weibull probability model.

___ 17. The error is defined to be the horizontal distance between the data point and a point on the curve.

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___ 18. The Cricket Graph program fits a straight line which minimizes...

a. $\sum_t \left(\ln \ln \frac{1}{R_t} - [a+b \ln t]\right)$

e. $\sum_t \left(F_t - \exp\left[-(t/u)^k\right]\right)^2$

b. $\sum_t \left(\ln \ln \frac{1}{R_t} - [a+b \ln t]\right)^2$

c. $\sum_t \left(R_t - \exp[-(t/u)^k]\right)^2$

d. $\sum_t \left(F_t - [-(t/u)^k]\right)^2$

f. $\sum_t \left(R_t - \exp[-(t/u)^k]\right)$

g. $\sum_t \left(\ln F_t - [-(t/u)^k]\right)^2$

h. *none of the above*

___ 19. The Curve Fit program fits a curve which minimizes ...

a. $\sum_t \left(\ln \ln \frac{1}{R_t} - [a+b \ln t]\right)$

b. $\sum_t \left(\ln \ln \frac{1}{R_t} - [a+b \ln t]\right)^2$

c. $\sum_t \left(R_t - \exp[-(t/u)^k]\right)^2$

d. $\sum_t \left(F_t - [-(t/u)^k]\right)^2$

e. $\sum_t \left(F_t - \exp[-(t/u)^k]\right)^2$

f. $\sum_t \left(R_t - \exp[-(t/u)^k]\right)$

g. $\sum_t \left(\ln F_t - [-(t/u)^k]\right)^2$

h. *none of the above*

___ 20. A failure time T having Weibull distribution with parameters u & k can be randomly generated by using a uniformly-generated random variable X and computing

a. $T = e^{-(X/u)^k}$

b. $T = -u(-\ln X)^{1/k}$

c. $T = -\frac{u \ln X}{k}$

d. $T = 1 - e^{-(X/u)^k}$

e. $T = -\frac{k \ln X}{u}$

f. *None of the above*

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57:022 Principles of Design II

Quiz #5 Solutions

Spring 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Some statements below refer to today's homework assignment (HW#5).
Indicate "+" for true, "o" for false:

_+_ 1. If you use the Cricket Graph program to fit a line, it will find the straight line which minimizes the sum of the squares of the errors, i.e., the sum of the squares of the vertical distances between each data point and the line.

_+_ 2. In the chi-square goodness-of-fit test, the number of degrees of freedom is never more than the number of "cells" of the histogram.

_+_ 3. The inverse transformation method to generate a random number can be used to simulate failure times having a Weibull distribution.

_o_ 4. The Weibull distribution is a special case of the exponential distribution.

_d_ 5. If T is the time at which a unit fails, its reliability is defined as

a. $R(x) = 1 - f(t)$
b. $R(x) = f(t)$
c. $R(t) = P\{T=t\}$
d. $R(t) = P\{T \geq t\}$
e. $R(t) = P\{T=t \mid T \geq t\}$
f. $R(t) = P\{T \leq t\}$

_o_ 6. We assumed in this HW that the number of motors which have failed at time t , $N_f(t)$, has a Weibull distribution.

_o_ 7. If the assumption of Weibull distribution for the lifetime of the motors were correct, a plot of $N_f(t)$ vs. t should be approximately on a straight line.

_+_ 8. To estimate the time at which 90% of the units will have failed, we evaluate $F^{-1}(0.90)$, where F is the CDF of the failure-time distribution.

_o_ 9. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has survived (not failed) until time t.

_+_ 10. If the assumption of Weibull distribution were correct, a plot of $\ln \ln {}^{1}/_{N_s(t)}$ vs. ln t should be approximately a straight line, where $N_s(t)$ is the fraction of motors surviving at time t.

_+_ 11. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

_+_ 12. A value of k greater than 1.0 indicates an increasing failure rate, and k less than 1.0 indicates a decreasing failure rate.

_+_ 13. If ten motors are installed in a manufacturing facility, the number still functioning after 60 days has a binomial distribution.

_o_ 14. The expected number of machines E_i which fail in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$ where F(t) is the CDF of the failure time distribution.

_+_ 15. According to the results of this homework exercise, the failure rate of the motors is increasing rather than decreasing over time.

_o_ 16. A positive value of the shape parameter k indicates an increasing failure rate, and negative k indicates a decreasing failure rate for the Weibull probability model.

_o_ 17. The error is defined to be the horizontal distance between the data point and a point on the curve.

_b_ 18. The Cricket Graph program fits a straight line which minimizes...

a. $\sum_t \left(\ln \ln \frac{1}{R_t} - \left[a+b \ln t\right]\right)$

b. $\sum_t \left(\ln \ln \frac{1}{R_t} - \left[a+b \ln t\right]\right)^2$

c. $\sum_t \left(R_t - \exp\left[-\left(t/u\right)^k\right]\right)^2$

d. $\sum_t \left(F_t - \left[-\left(t/u\right)^k\right]\right)^2$

e. $\sum_t \left(F_t - \exp\left[-\left(t/u\right)^k\right]\right)^2$

f. $\sum_t \left(R_t - \exp\left[-\left(t/u\right)^k\right]\right)$

g. $\sum_t \left(\ln F_t - \left[-\left(t/u\right)^k\right]\right)^2$

h. *none of the above*

_c_ 19. The Curve Fit program fits a curve which minimizes ...

a. $\sum_t \left(\ln \ln \frac{1}{R_t} - \left[a+b \ln t\right]\right)$

b. $\sum_t \left(\ln \ln \frac{1}{R_t} - \left[a+b \ln t\right]\right)^2$

c. $\sum_t \left(R_t - \exp\left[-\left(t/u\right)^k\right]\right)^2$

d. $\sum_t \left(F_t - \left[-\left(t/u\right)^k\right]\right)^2$

e. $\sum_t \left(F_t - \exp\left[-\left(t/u\right)^k\right]\right)^2$

f. $\sum_t \left(R_t - \exp\left[-\left(t/u\right)^k\right]\right)$

g. $\sum_t \left(\ln F_t - \left[-\left(t/u\right)^k\right]\right)^2$

h. *none of the above*

_f_ 20. A failure time T having Weibull distribution with parameters u & k can be randomly generated by using a uniformly-generated random variable X and computing

a. $T = e^{-\left(X/u\right)^k}$

b. $T = -u\left(-\ln X\right)^{1/k}$

c. $T = -\frac{u \ln X}{k}$

d. $T = 1 - e^{-\left(X/u\right)^k}$

e. $T = -\frac{k \ln X}{u}$

f. *None of the above*

Note: the correct answer is $T = u\left(-\ln X\right)^{1/k}$, which I had intended to give as answer (b).

Name ________________________________

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57:022 Principles of Design II

Quiz #6

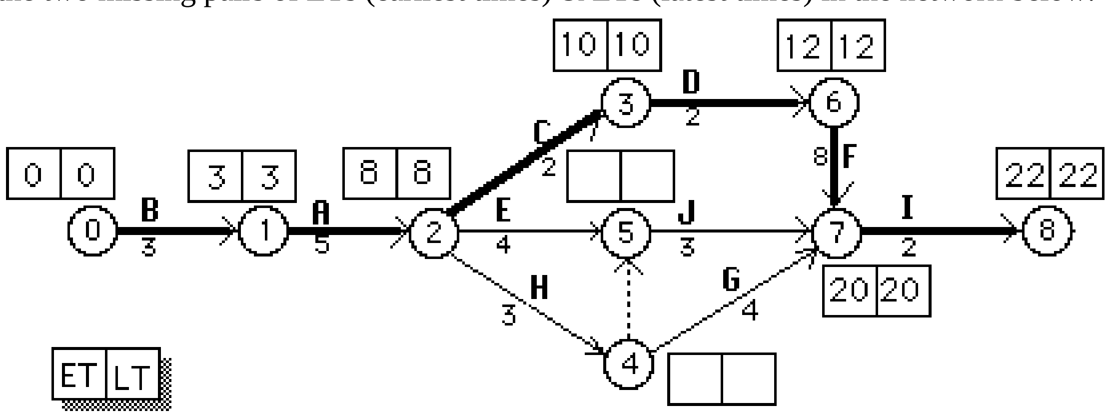
Wednesday, March 5, 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Part One: Project Scheduling. Indicate true by "+" and false by "o":

___a. The *MacProject* software used in this homework assignment requires that you enter the project network in "Activity on Node" form.
___b. The quantity LT(i) [i.e. latest time] for each node i is determined by a backward pass through the network.
___c. If an activity is represented by an arrow from node i to node j, then LF (latest finish time) for that activity is LT(j).
___d. If an activity is represented by an arrow from node i to node j, then EF (early finish time) for that activity is ET(j).
___e. If an activity is represented by an arrow from node i to node j, then that activity has zero "float" or "slack" if and only if ET(i)=LT(j).
___f. An activity is critical if and only if its total float ("slack") is zero.
___g. A "dummy" activity cannot be critical.
___h. The forward and backward pass methods for scheduling a project apply to the AON network representation of the project.
___i. PERT assumes that each activity's duration has a Normal distribution.
___j. PERT assumes that the project duration has a Normal distribution.
___k. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
___l. PERT assumes that the project duration has a beta distribution.
___m. Either the triangular or beta distributions for an activity are uniquely specified by giving the minimum & maximum durations and a most likely duration.

In the project network below, each activity is assumed to require its expected duration. Complete the two missing pairs of ETs (earliest times) & LTs (latest times) in the network below.



The critical path is shown in bold above. If the durations are random, with expected values as shown and *standard deviations all equal to 1.0,* what is
... the expected completion time of the project, according to PERT? ______ days
... the standard deviation of the project completion time, according to PERT? _______days

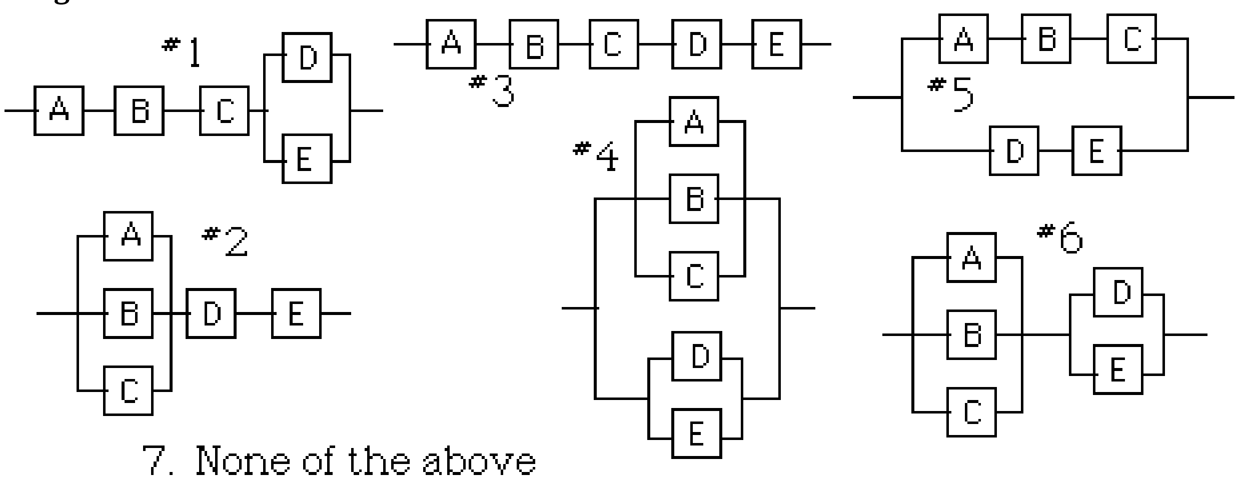
Name ________________________________

Part Two: A system consists of five components (A,B,C,D, &E). The probability that each component *fails during the first year* of operation is 10% for A, B, and C, and 20% for D and E. For each alternative of (a) through (e), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)

Diagram	Reliability	
___	___	a. The system requires that at least one of A, B, & C function, and that D & E both function.
___	___	b. The system will fail if all of A, B, and C fail or if both D and E fail.

Diagrams:



Reliabilities:

1. $[1-(0.1)^3]\,(0.8)^2 =$ 63.9%
2. $(0.9)^3[1-(0.2)^2] =$ 69.9%
3. $1-[1-(0.9)^3]\,[1-(0.8)^2] =$ 90.2%
4. $1-(0.1)^3(0.2)^2 =$ 99.9%
5. $[1-(0.1)^3]\,[1-(0.2)^2] =$ 95.9%
6. $[1-(0.1)^3]\,[1-(0.2)^2] =$ 95.9%
7. $(0.9)^3(0.8)^2 =$ 46.6%
8. *None of the above*

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57:022 Principles of Design II

Quiz #6 Solutions

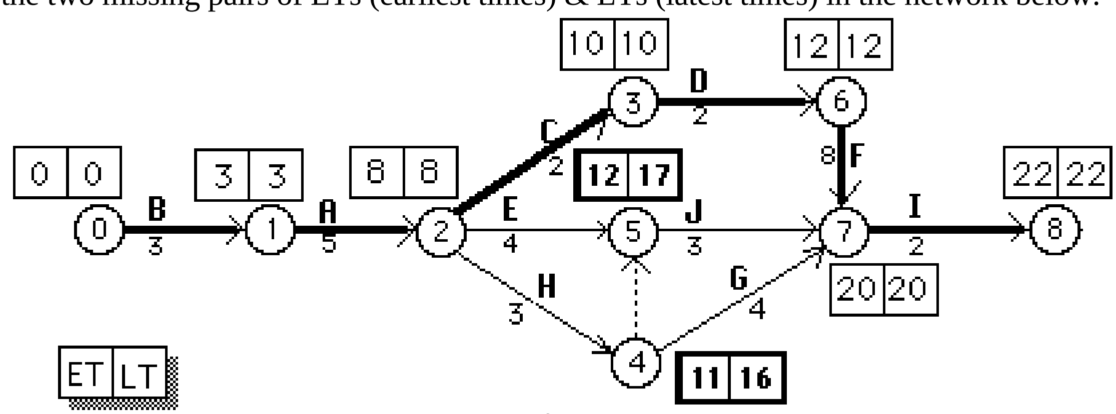
Wednesday, March 5, 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Part One: Project Scheduling. Indicate true by "+" and false by "o":

_+_a. The *MacProject* software used in this homework assignment requires that you enter the project network in "Activity on Node" form.
_+_b. The quantity LT(i) [i.e. latest time] for each node i is determined by a backward pass through the network.
_+_c. If an activity is represented by an arrow from node i to node j, then LF (latest finish time) for that activity is LT(j).
_o_d. If an activity is represented by an arrow from node i to node j, then EF (early finish time) for that activity is ET(j).
_o_e. If an activity is represented by an arrow from node i to node j, then that activity has zero "float" or "slack" if and only if ET(i)=LT(j).
_+_f. An activity is critical if and only if its total float ("slack") is zero.
_o_g. A "dummy" activity cannot be critical.
_o_h. The forward and backward pass methods for scheduling a project apply to the AON network representation of the project.
_o_i. PERT assumes that each activity's duration has a Normal distribution.
_+_j. PERT assumes that the project duration has a Normal distribution.
_+_k. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
_o_l. PERT assumes that the project duration has a beta distribution.
_+_m. Either the triangular or beta distributions for an activity are uniquely specified by giving the minimum & maximum durations and a most likely duration.

In the project network below, each activity is assumed to require its expected duration. Complete the two missing pairs of ETs (earliest times) & LTs (latest times) in the network below.



The critical path is shown in bold above. If the durations are random, with expected values as shown and *standard deviations all equal to 1.0*, what is

... the expected completion time of the project, according to PERT? __22__ days

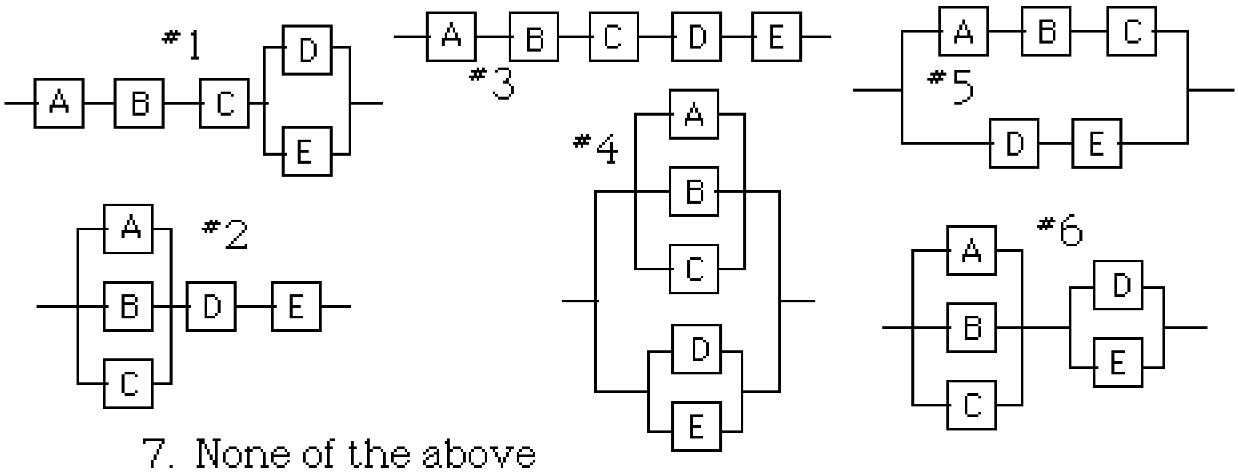
... the standard deviation of the project completion time, according to PERT? $\sqrt{6} \approx 2.45$ days

Part Two: A system consists of five components (A,B,C,D, &E). The probability that each component *fails during the first year* of operation is 10% for A, B, and C, and 20% for D and E. For each alternative of (a) through (e), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)
- the SLAM network which would simulate the system lifetime

Diagram	Reliability	
_#2_	_#1_	a. The system requires that at least one of A, B, & C function, <u>and</u> that D & E both function.
_#6_	_#5_	b. The system will fail if all of A, B, and C fail <u>or</u> if both D and E fail.

Diagrams:



Reliabilities:

1. $[1-(0.1)^3]\ (0.8)^2 =$ 63.9%
2. $(0.9)^3[1-(0.2)^2] =$ 69.9%
3. $1- [1-(0.9)^3]\ [1-(0.8)^2] =$ 90.2%
4. $1 - (0.1)^3(0.2)^2 =$ 99.9%
5. $[1-(0.1)^3]\ [1-(0.2)^2] =$ 95.9%
6. $[1-(0.1)^3]\ [1- (0.2)^2] =$ 95.9%
7. $(0.9)^3(0.8)^2 =$ 46.6%
8. *None of the above*

Note: The reliability of a component is obtained by subtracting its failure probability from 1.0.

Name ________________________

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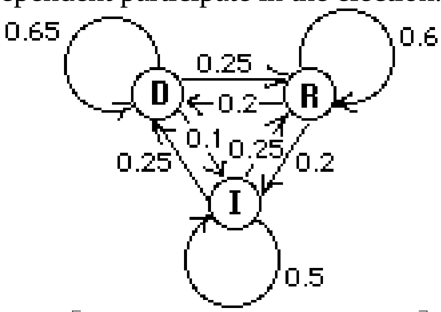
57:022 Principles of Design II

Quiz #7

Wednesday, March 12, 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Part One: In the Iowa primary elections, held every two years (in even-numbered years), voters can change parties every election year regardless of their past affiliation. At each primary election in Outaluck County, suppose that 25% of the Democrats switch to the Republican party and 10% declare themselves Independent, while 20% of the Republicans switch to the Democratic party and 20% declare themselves Independent. At each primary election, half of the Independents join a party, and they are as likely to join the Democrats as the Republicans. (All eligible voters who are not Independent participate in the election.)



$$P = \begin{bmatrix} 0.65 & 0.25 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.25 & 0.25 & 0.5 \end{bmatrix},$$

$$P^2 = \begin{bmatrix} 0.4975 & 0.3375 & 0.165 \\ 0.3 & 0.46 & 0.24 \\ 0.3375 & 0.3375 & 0.325 \end{bmatrix},$$

$$P^3 = \begin{bmatrix} 0.4321 & 0.3681 & 0.1998 \\ 0.3470 & 0.411 & 0.242 \\ 0.3681 & 0.3681 & 0.2638 \end{bmatrix}, \text{ and } \lim_{n\to\infty} P^n = \begin{bmatrix} 0.3846 & 0.3846 & 0.2308 \\ 0.3846 & 0.3846 & 0.2308 \\ 0.3846 & 0.3846 & 0.2308 \end{bmatrix}$$

___ 1. If Mrs. Smith voted in the Republican primary in 1996, what is the probability that she will vote in the Republican primary in the following election year (1998)? **(choose nearest answer!)**

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60%
g. 70% h. 80% i. 90%

___ 2. If Mrs. Smith voted in the Republican primary in 1996, what is the probability that she will vote in the Democratic primary 2 elections hence, i.e., in 2000? **(choose nearest answer!)**

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60%
g. 70% h. 80% i. 90%

___ 3. If Mr. Jones voted in the Democratic primary in 1996, what is the probability that he will vote in the primary 3 elections hence, i.e., in 2002? **(choose nearest answer!)**

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60%
g. 70% h. 80% i. 90%

___ 4. The fraction of the voters in this county who belong to the Republican party is expected to be **(choose nearest answer!)**

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60%
g. 70% h. 80% i. 90%

___ 5. Which of the following equations must be satisfied by the steadystate probability distribution?

a. $0.65\pi_1 + 0.25\pi_2 + 0.1\pi_3 = 0$ b. $0.2\pi_1 + 0.6\pi_2 + 0.2\pi_3 = 0$
c. $\pi_2 = 0.25\pi_1 + 0.6\pi_2 + 0.25\pi_3$ d. $\pi_1 = 0.65\pi_1 + 0.25\pi_2 + 0.1\pi_3$
e. $0.1\pi_1 + 0.2\pi_2 + 0.5\pi_3 = 0$ f. *None of the above*

Name ___________________________

Part Two: Consider an (s,S) inventory system in which the number of items on the shelf is checked at the end of each day. The maximum number on the shelf is 8. If 3 or fewer units are on the shelf, the shelf is refilled overnight. The demand distribution is as follows:

x	0	1	2	3	4	5	6
P{D=x}	.1	.15	.25	.25	.15	.05	.05

(Assume that the demand never exceeds 6.)

The system is modeled as a Markov chain, with the state defined as the number of units on the shelf at the end of each day. The probability transition matrix is:

Transition Probabilities

from \ to	1	2	3	4	5	6	7	8	9
1	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
2	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
3	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
4	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
5	0.25	0.25	0.25	0.15	0.1	0	0	0	0
6	0.1	0.15	0.25	0.25	0.15	0.1	0	0	0
7	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0	0
8	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0
9	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1

The mean first passage matrix is:

Mean First Passage Times

from \ to	1	2	3	4	5	6	7	8	9
1	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
2	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
3	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
4	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
5	12.2711	10.4926	6.64702	7.25983	6.12634	6.35574	7.50281	13.7757	23.1867
6	14.3163	11.5197	6.47734	6.42023	5.85772	5.88683	7.68799	13.9609	23.3719
7	14.632	12.4406	7.01794	6.24734	5.2518	5.79028	7.22475	14.3004	23.7114
8	15.2275	12.1857	7.62978	6.77218	5.19576	5.29848	7.10625	13.2454	24.1281
9	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756

The steady-state distribution of the above Markov chain is:

Steady State Distribution

i	π_i
1	0.06471513457
2	0.07698357218
3	0.1304613771
4	0.1355295351
5	0.16322964
6	0.1698706746
7	0.1384131423
8	0.0754980776
9	0.04529884656

Name ___________________________

___ 6. If the shelf is restocked Sunday p.m., the probability that exactly six items are counted on the shelf Monday p.m. is **(choose nearest answer!)**

a. 5%	b. 10%	c. 15%
d. 20%	e. 25%	f. 30%

___ 7. If the shelf is restocked Sunday p.m., the expected number of days until exactly six items are counted on the shelf is **(choose nearest answer!)**

a. 3 or less	b. 4	c. 5
d. 6	e. 7	f. 8
g. 9	h. 10	i. 11 or more

___ 8. In a 30-day month, the expected number of stockouts (i.e., nothing on shelf) is **(choose nearest answer!)**

a. 1 or less	b. 2	c. 2.5
d. 3	e. 3.5	f. 4
g. 5	h. 6	i. 7 or more

___ 9. What is the expected number of days between stockouts? **(choose nearest answer!)**

a. 5 or less	b. 6	c. 7
d. 8	e. 9	f. 10
g. 11	h. 12	i. 13 or more

___ 10. The expected number of items on the shelf at the end of a day is **(choose nearest answer!)**

a. 1 or less	b. 2	c. 2.5
d. 3	e. 3.5	f. 4
g. 5	h. 6	i. 7 or more

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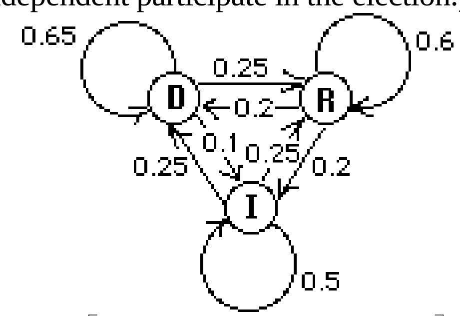
57:022 Principles of Design II

Quiz #7 Solutions

March 12, 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Part One: In the Iowa primary elections, held every two years (in even-numbered years), voters can change parties every election year regardless of their past affiliation. At each primary election in Outaluck County, suppose that 25% of the Democrats switch to the Republican party and 10% declare themselves Independent, while 20% of the Republicans switch to the Democratic party and 20% declare themselves Independent. At each primary election, half of the Independents join a party, and they are as likely to join the Democrats as the Republicans. (All eligible voters who are not Independent participate in the election.)



$$P = \begin{bmatrix} 0.65 & 0.25 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.25 & 0.25 & 0.5 \end{bmatrix},$$

$$P^2 = \begin{bmatrix} 0.4975 & 0.3375 & 0.165 \\ 0.3 & 0.46 & 0.24 \\ 0.3375 & 0.3375 & 0.325 \end{bmatrix},$$

$$P^3 = \begin{bmatrix} 0.4321 & 0.3681 & 0.1998 \\ 0.3470 & 0.411 & 0.242 \\ 0.3681 & 0.3681 & 0.2638 \end{bmatrix}, \text{ and } \lim_{n\to\infty} P^n = \begin{bmatrix} 0.3846 & 0.3846 & 0.2308 \\ 0.3846 & 0.3846 & 0.2308 \\ 0.3846 & 0.3846 & 0.2308 \end{bmatrix}$$

_f_ 1. If Mrs. Smith voted in the Republican primary in 1996, what is the probability that she will vote in the Republican primary in the following election year (1998)?

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60% = p_{22}
g. 70% h. 80% i. 90%

_c_ 2. If Mrs. Smith voted in the Republican primary in 1996, what is the probability that she will vote in the Democratic primary 2 elections hence, i.e., in 2000?

a. 10% b. 20% c. 30% = $p_{21}^{(2)}$
d. 40% e. 50% f. 60%
g. 70% h. 80% i. 90%

_h_ 3. If Mr. Jones voted in the Democratic primary in 1996, what is the probability that he will vote in the primary 3 elections hence, i.e., in 2002? **(choose nearest answer!)**

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60%
g. 70% h. 80% $\approx p_{11}^{(3)} + p_{12}^{(3)}$ i. 90%

_d_ 4. The fraction of the voters in this county who belong to the Republican party is expected to be **(choose nearest answer!)**

a. 10% b. 20% c. 30%
d. 40% (π_2 = 38.46%) e. 50% f. 60%
g. 70% h. 80% i. 90%

_c_ 5. Which of the following equations must be satisfied by the steadystate probability distribution?

a. $0.65\pi_1 + 0.25\pi_2 + 0.1\pi_3 = 0$ b. $0.2\pi_1 + 0.6\pi_2 + 0.2\pi_3 = 0$
c. $\pi_2 = 0.25\pi_1 + 0.6\pi_2 + 0.25\pi_3$ d. $\pi_1 = 0.65\pi_1 + 0.25\pi_2 + 0.1\pi_3$
e. $0.1\pi_1 + 0.2\pi_2 + 0.5\pi_3 = 0$ f. *None of the above*

Part Two: Consider an (s,S) inventory system in which the number of items on the shelf is checked at the end of each day. The maximum number on the shelf is 8. If 3 or fewer units are on the shelf, the shelf is refilled overnight. The demand distribution is as follows:

x	0	1	2	3	4	5	6
P{D=x}	.1	.15	.25	.25	.15	.05	.05

(Assume that the demand never exceeds 6.)

The system is modeled as a Markov chain, with the state defined as the number of units on the shelf at the end of each day. The probability transition matrix is:

Transition Probabilities

from \ to	1	2	3	4	5	6	7	8	9
1	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
2	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
3	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
4	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
5	0.25	0.25	0.25	0.15	0.1	0	0	0	0
6	0.1	0.15	0.25	0.25	0.15	0.1	0	0	0
7	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0	0
8	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0
9	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1

The mean first passage matrix is:

Mean First Passage Times

from \ to	1	2	3	4	5	6	7	8	9
1	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
2	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
3	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
4	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756
5	12.2711	10.4926	6.64702	7.25983	6.12634	6.35574	7.50281	13.7757	23.1867
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9	15.4523	12.9898	7.6651	7.37847	5.69593	5.24463	6.39169	12.6646	22.0756

The steady-state distribution of the above Markov chain is:

Steady State Distribution

i	π_i
1	0.06471513457
2	0.07698357218
3	0.1304613771
4	0.1355295351
5	0.16322964
6	0.1698706746
7	0.1384131423
8	0.0754980776
9	0.04529884656

_e_ 6. If the shelf is restocked Sunday p.m., the probability that exactly six items are counted on the shelf Monday p.m. is **(choose nearest answer!)**

a. 5% b. 10% c. 15%
d. 20% e. 25% = p_{97} f. 30%

_d_ 7. If the shelf is restocked Sunday p.m., the expected number of days until exactly six items are counted on the shelf is **(choose nearest answer!)**

a. 3 or less b. 4 c. 5
d. 6 $\left(m_{97} \approx 6.392\right)$ e. 7 f. 8
g. 9 h. 10 i. 11 or more

_b_ 8. In a 30-day month, the expected number of stockouts (i.e., nothing on shelf) is **(choose nearest answer!)**

a. 1 or less b. 2 c. 2.5
d. 3 e. 3.5 f. 4
g. 5 h. 6 i. 7 or more

Note: $30\pi_1 = 30(0.0647) \approx 1.94145$

_i_ 9. What is the expected number of days between stockouts? **(choose nearest answer!)**

a. 5 or less b. 6 c. 7
d. 8 e. 9 f. 10
g. 11 h. 12 i. 13 or more

Note: $m_{11} = 15.4523$

_f_ 10. The expected number of items on the shelf at the end of a day is **(choose nearest answer!)**

a. 1 or less b. 2 c. 2.5
d. 3 e. 3.5 f. 4
g. 5 h. 6 i. 7 or more

Note: $0\pi_1 + 1\pi_2 + 2\pi_3 + 3\pi_4 + 4\pi_5 + 5\pi_6 + 6\pi_7 + 7\pi_8 + 8\pi_9 = 3.968$

Name ________________________

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57:022 Principles of Design II

Quiz #8

Wednesday, April 9, 1997
«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Part One: Indicate whether true (= +) or false (=o)

___ 1. In a Markov chain model of a system, the probability of a transition from state i to state j must be independent of the initial state of the system.

___ 2. A transient state cannot be, at the same time, an absorbing state.

___ 3. The quantity $f_{ij}^{(n)}$ represents the number of stages required to reach state j from state i.

___ 4. m_{ij} represents the expected value of the probability distribution

$P\{N_{ij}=n\} = f_{ij}^{(n)}$, n=0,1,2,3, · · ·

___ 5. The random variable a_{ij} represents the number of stages required to reach absorbing state j from transient state i.

___ 6. N_{ij} is a random variable with probability distribution $P\{N_{ij}=n\} = f_{ij}^{(n)}$, n=0,1,2,3, · · ·

___ 7. e_{ij} represents the expected value of the probability distribution

$P\{X_n=j \mid X_0 = i\} = p_{ij}^{(n)}$, n=0,1,2,3, · · ·

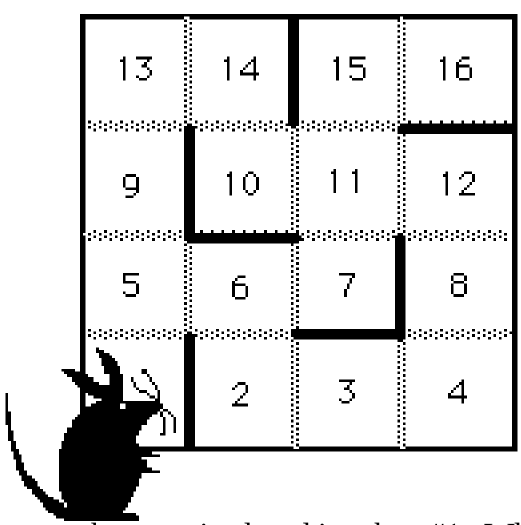
___ 8. e_{ij} is defined only when i and j are both transient states.

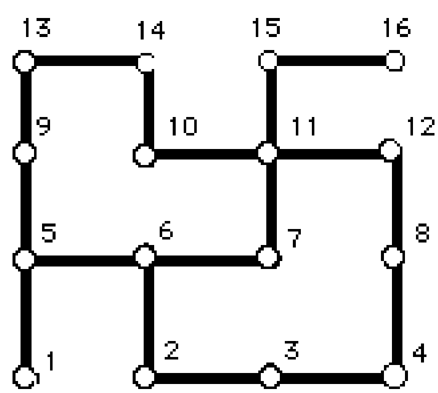
___ 9. The random variable N_{ij} represents the # of stages required to reach state j from state i.

___ 10. m_{ij} represents the expected value of the probability distribution

$P\{X_n=j \mid X_0 = i\} = p_{ij}^{(n)}$, n=0,1,2,3, · · ·

Part Two: We wish to model the passage of a rat through a maze, shown below:





We will assume that a rat is placed into box #1. While in any box, the rat is assumed to be equally likely to choose each of the doors leaving the box (including the one by which he entered the box). A reward has been placed in box #16.

___ 11. In the Markov chain model of this maze, $p_{11,12}$ =

a. 1	b. 1/2	c. 1/3
d. 1/4	e. 1/5	f. None of the above

___ 12. The box which is occupied most frequently, over a long time period, is

a. 1	b. 5	c. 6
d. 11	e. 16	f. None of the above

Name ________________________

___ 13. The probability that, starting in box #1, the rat follows a direct path to box #16, is (choose *interval*):

a. [0, 0.1%]	b. [0.1%, 1%]	c. [1%, 2%]
d. [2%, 5%]	e. [5%, 10%]	f. 10% or greater

___ 14. If we perform a large number of experiments in which the rat is placed in box #1, the average number of moves that he requires to first reach box #16, is (choose *nearest* value):

a. 10	b. 20	c. 30
d. 40	e. 50	f. 60
g. 70	h. 80	i. 85 or more

___ 15. If, instead, the rat begins in box #11, the average number of moves that he requires to first reach box #16, is (choose *nearest* value):

a. 10	b. 20	c. 30
d. 40	e. 50	f. 60
g. 70	h. 80	i. 85 or more

Steady State Distribution

i	P{i}
1	0.02941
2	0.05882
3	0.05882
4	0.05882
5	0.08824
6	0.08824
7	0.05882
8	0.05882
9	0.05882
10	0.05882
11	0.1176
12	0.05882
13	0.05882
14	0.05882
15	0.05882
16	0.02941

First Passage Probabilities

n	$f_{1,16}^{(n)}$
1	0
	⋮
5	0
6	0.006944
7	0
8	0.01264
9	0
10	0.01649
11	0
12	0.0189
13	0
14	0.0203
15	0
16	0.02102

Mean First Passage Times:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
34	31	43.6	49.1	1	11.3	23.1	47.6	19.6	33	23.3	39	31.1	35.6	54.3	87.3
59.6	17	18.2	29.4	26.6	8.6	20.4	33.6	39.6	36	20.6	30.6	45.4	44.2	51.6	84.6
65.2	11.2	17	15.7	32.2	15.3	23.7	24.4	44	37	20.5	26	48.7	46.4	51.5	84.5
68.7	20.4	13.7	17	35.7	20	25	13.2	46.4	36	18.4	19.3	50	46.5	49.4	82.4
33	30	42.6	48.1	11.3	10.3	22.1	46.6	18.6	32	22.3	38	30.1	34.6	53.3	86.3
52.1	20.8	34.5	41.2	19.1	11.3	15.2	40.8	33.2	33	18.8	33.3	40.2	40.1	49.8	82.8
60.7	29.4	39.7	43	27.7	12	17	39.2	38.4	28	10.4	28.3	42	38.5	41.4	74.4
70.2	27.6	25.4	16.2	37.2	22.6	24.2	17	46.8	33	14.2	10.6	49.2	44.6	45.2	78.2
43.8	35.2	46.6	51.0	10.8	16.6	25.0	48.4	17	27	21.8	38.6	16.0	25.0	52.8	85.8
64.4	38.8	46.8	47.8	31.4	23.6	21.8	41.8	34.2	17	8.4	28.6	29.8	18.4	39.4	72.4
67.3	36	42.9	42.8	34.3	22	16.8	35.6	41.6	21	8.5	21.3	41.8	34.9	31	64
69.8	32.8	35.2	30.5	36.8	23.3	21.5	18.8	45.2	28	8.1	17	46.5	40.8	39.1	72.1
52.7	38.4	48.7	52	19.7	21	26	48.2	13.4	20	19.4	37.3	17	13.5	50.4	83.4
59.6	39.6	48.8	50.9	26.6	23.3	24.9	46	24.8	11	14.9	34	15.9	17	45.9	78.9
70.3	39	45.9	45.8	37.3	25	19.8	38.6	44.6	24	3	24.3	44.8	37.9	17	33
71.3	40	46.9	46.8	38.3	26	20.8	39.6	45.6	25	4	25.3	45.8	38.9	1	34

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57:022 Principles of Design II

Quiz #8 Solution

April 9, 1997

«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»«»

Part One: Indicate whether true (= +) or false (=o)

\+ 1. In a Markov chain model of a system, the probability of a transition from state i to state j must be independent of the initial state of the system.

\+ 2. A transient state cannot be, at the same time, an absorbing state.

o 3. The quantity $f_{ij}^{(n)}$ represents the number of stages required to reach state j from state i.

Note: $f_{ij}^{(n)}$ is a **probability**!

\+ 4. m_{ij} represents the expected value of the probability distribution

$P\{N_{ij}=n\} = f_{ij}^{(n)}$, n=0,1,2,3, ···

o 5. The random variable a_{ij} represents the number of stages required to reach absorbing state j from transient state i. *Note:* a_{ij} is a **probability**!

\+ 6. N_{ij} is a random variable with probability distribution $P\{N_{ij}=n\} = f_{ij}^{(n)}$, n=0,1,2,3, ···

o 7. e_{ij} represents the expected value of the probability distribution

$P\{X_n=j \mid X_0 = i\} = p_{ij}^{(n)}$, n=0,1,2,3, ··· Note: If states i & j are both transient, then

$$e_{ij} = \sum_{n=0}^{\infty} p_{ij}^{(n)}$$ is the expected number of visits to state j, starting in state i.

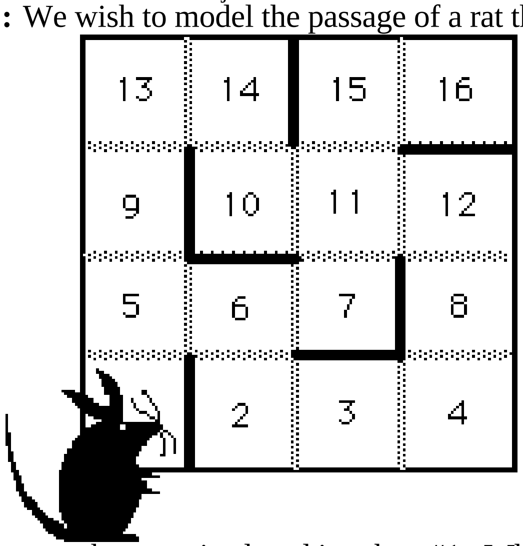
\+ 8. e_{ij} is defined only when i and j are both transient states.

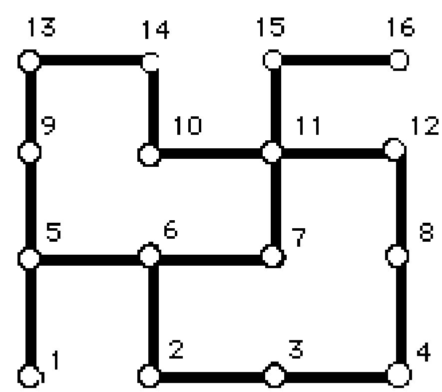
\+ 9. The random variable N_{ij} represents the # of stages required to reach state j from state i.

o 10. m_{ij} represents the expected value of the probability distribution

$P\{X_n=j \mid X_0 = i\} = p_{ij}^{(n)}$, n=0,1,2,3, ··· ***Note:*** see question (4) above.

Part Two: We wish to model the passage of a rat through a maze, shown below:





We will assume that a rat is placed into box #1. While in any box, the rat is assumed to be equally likely to choose each of the doors leaving the box (including the one by which he entered the box). A reward has been placed in box #16.

_d_ 11. In the Markov chain model of this maze, $p_{11,12}$ =

a. 1	b. 1/2	c. 1/3
d. 1/4	e. 1/5	f. None of the above

_d_ 12. The box which is occupied most frequently, over a long time period, is

a. 1 b. 5 c. 6
d. 11 **(π_{11} = 11.76%)** e. 16 f. None of the above

_b_ 13. The probability that, starting in box #1, the rat follows a direct path to box #16, is (choose *interval*):

a. [0, 0.1%] b. [0.1%, 1%] **(0.6944%)** c. [1%, 2%]
d. [2%, 5%] e. [5%, 10%] f. 10% or greater

_i_ 14. If we perform a large number of experiments in which the rat is placed in box #1, the average number of moves that he requires to first reach box #16, is (choose *nearest* value):

a. 10 b. 20 c. 30
d. 40 e. 50 f. 60
g. 70 h. 80 i. 85 or more **$m_{1,16}$ = 87.3)**

_f_ 15. If, instead, the rat begins in box #11, the average number of moves that he requires to first reach box #16, is (choose *nearest* value):

a. 10 b. 20 c. 30
d. 40 e. 50 f. 60 **($m_{11,16}$ = 64)**
g. 70 h. 80 i. 85 or more

Steady State Distribution

i	P{i}
1	0.02941
2	0.05882
3	0.05882
4	0.05882
5	0.08824
6	0.08824
7	0.05882
8	0.05882
9	0.05882
10	0.05882
11	0.1176
12	0.05882
13	0.05882
14	0.05882
15	0.05882
16	0.02941

First Passage Probabilities

n	$f_{1,16}^{(n)}$
1	0
⋮	⋮
5	0
6	0.006944
7	0
8	0.01264
9	0
10	0.01649
11	0
12	0.0189
13	0
14	0.0203
15	0
16	0.02102

Mean First Passage Times:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
34	31	43.6	49.1	1	11.3	23.1	47.6	19.6	33	23.3	39	31.1	35.6	54.3	87.3
59.6	17	18.2	29.4	26.6	8.6	20.4	33.6	39.6	36	20.6	30.6	45.4	44.2	51.6	84.6
65.2	11.2	17	15.7	32.2	15.3	23.7	24.4	44	37	20.5	26	48.7	46.4	51.5	84.5
68.7	20.4	13.7	17	35.7	20	25	13.2	46.4	36	18.4	19.3	50	46.5	49.4	82.4
33	30	42.6	48.1	11.3	10.3	22.1	46.6	18.6	32	22.3	38	30.1	34.6	53.3	86.3
52.1	20.8	34.5	41.2	19.1	11.3	15.2	40.8	33.2	33	18.8	33.3	40.2	40.1	49.8	82.8
60.7	29.4	39.7	43	27.7	12	17	39.2	38.4	28	10.4	28.3	42	38.5	41.4	74.4
70.2	27.6	25.4	16.2	37.2	22.6	24.2	17	46.8	33	14.2	10.6	49.2	44.6	45.2	78.2
43.8	35.2	46.6	51.0	10.8	16.6	25.0	48.4	17	27	21.8	38.6	16.0	25.0	52.8	85.8
64.4	38.8	46.8	47.8	31.4	23.6	21.8	41.8	34.2	17	8.4	28.6	29.8	18.4	39.4	72.4
67.3	36	42.9	42.8	34.3	22	16.8	35.6	41.6	21	8.5	21.3	41.8	34.9	31	64
69.8	32.8	35.2	30.5	36.8	23.3	21.5	18.8	45.2	28	8.1	17	46.5	40.8	39.1	72.1
52.7	38.4	48.7	52	19.7	21	26	48.2	13.4	20	19.4	37.3	17	13.5	50.4	83.4
59.6	39.6	48.8	50.9	26.6	23.3	24.9	46	24.8	11	14.9	34	15.9	17	45.9	78.9
70.3	39	45.9	45.8	37.3	25	19.8	38.6	44.6	24	3	24.3	44.8	37.9	17	33
71.3	40	46.9	46.8	38.3	26	20.8	39.6	45.6	25	4	25.3	45.8	38.9	1	34

Name ______________________________

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57:022 Principles of Design II

Quiz #9

Friday, April 18, 1997

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For each of the following statements about SLAM, **indicate "+" if True and "O" if False.**

___a. If the SERVER module is used in the ARENA model of an M/M/2 queueing system, a SERVER module is required for each of the two servers.

___b. In the ARENA model of an M/M/1 queueing system with arrival rate λ=15/hour, the "time between arrivals" box in the dialog box for the ARRIVE module should be EXPO(15).

___c. Little's Law says that the average length of a queue is equal to the average time spent in the queue times the average arrival rate of the queueing system.

___d. The notation **L** in queueing analysis represents the average length of the queue.

___e. The notation $\mathbf{W_q}$ in queueing analysis represents the average time spent in the queue.

___f. Little's Law applies only to queues which can be modeled as a birth-death process.

___g. The notation μ in a birth/death process is referred to as the *death rate.*

___h. A queueing system in which the arrival process is Poisson and the service time has a Normal distribution can be modeled as a continuous-time Markov chain.

___i. A birth-death process is a special case of a continuous-time Markov chain.

___j. You must attach the ARENA "Common" panel in order to use the ARRIVE and SERVER modules in your model.

___k. In order to obtain a report showing the average time spent in a queue, you should use a STATISTICS module in your ARENA model.

___l. If the arrival rate for an M/M/1 queueing system is 4 per hour and the service rate is 5 per hour, then one can expect the server to be idle 20% of the time.

___m. The utilization of the server in an M/M/1 system is 1 - π_0, where π_0 is the probability that no "customers" are in the system.

___n. By combining the two M/M/1 queues (with λ=4/hour and μ=5/hour) into a single M/M/2 queue as in this homework assignment, the steady-state average time in the system spent by a customer was reduced to less than one-half its original value.

___o. The "resource capacity" to be specified in the SERVER module of ARENA is the maximum amount of time which the entity may spend in that station.

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57:022 Principles of Design II

Quiz #9 Solutions

Spring 1997

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For each of the following statements about SLAM, **indicate "+" if True and "O" if False.**

_O_a. If the SERVER module is used in the ARENA model of an M/M/2 queueing system, a SERVER module is required for each of the two servers. ***Note: The capacity of the server resource may be specified to be 2.***

_O_b. In the ARENA model of an M/M/1 queueing system with arrival rate λ=15/hour, the "time between arrivals" box in the dialog box for the ARRIVE module should be EXPO(15). ***Note: The mean of the exponential distribution will be 4 minutes = $^1/_{15}$ hour.***

_+_c. Little's Law says that the average length of a queue is equal to the average time spent in the queue times the average arrival rate of the queueing system.

_O_d. The notation **L** in queueing analysis represents the average length of the queue. ***Note: L denotes the average number of customers in the system, which includes the one currently being served.***

_+_e. The notation $\mathbf{W_q}$ in queueing analysis represents the average time spent in the queue.

_+_f. Little's Law applies only to queues which can be modeled as a birth-death process.

_+_g. The notation μ in a birth/death process is referred to as the *death rate.*

_O_h. A queueing system in which the arrival process is Poisson and the service time has a Normal distribution can be modeled as a continuous-time Markov chain. ***Note: If the service time has Normal distribution, it is not "memoryless", and cannot be modeled as a Markov chain.***

_+_i. A birth-death process is a special case of a continuous-time Markov chain.

_+_j. You must attach the ARENA "Common" panel in order to use the ARRIVE and SERVER modules in your model.

_O_k. In order to obtain a report showing the average time spent in a queue, you should use a STATISTICS module in your ARENA model. ***Note: you may specify that you wish a report of the average time spent in a server's queue in the dialog box of the SERVER module.***

_+_l. If the arrival rate for an M/M/1 queueing system is 4 per hour and the service rate is 5 per hour, then one can expect the server to be idle 20% of the time.

_+_m. The utilization of the server in an M/M/1 system is 1 - π_0, where π_0 is the probability that no "customers" are in the system.

_+_n. By combining the two M/M/1 queues (with λ=4/hour and μ=5/hour) into a single M/M/2 queue as in this homework assignment, the steady-state average time in the system spent by a customer was reduced to less than one-half its original value.

_O_o. The "resource capacity" to be specified in the SERVER module of ARENA is the maximum amount of time which the entity may spend in that station. ***Note: "resource capacity" refers to the number of servers!***

Name ________________________________

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57:022 Principles of Design II

Quiz #10 -- Friday, May 2, 1997

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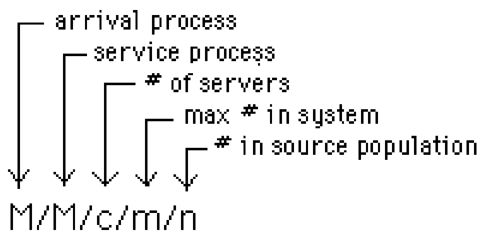
Queueing Models

Note: Kendall's notation:



True (+) or False (**o**)?

___1. In an M/M/1 queueing system, with arrival rate 2/minute and service time averaging 20 seconds, we would expect the server to be busy more than 75% of the time.

___2. In an M/M/1/N queueing system, N represents the capacity of the waiting line.

___3. Consider an M/M/1 queueing system, with an average of 15 seconds between arrivals and an average of 2 customers in the system. According to Little's Law, we would expect the average time spent by a customer in the system to be at least 1 minute.

___4. In an M/M/1/4/4 queueing system, the arrival rate varies by state, while the service rate is the same for every state (except state 0).

___5. In an M/M/2 queueing system, the arrival rate varies by state, while the service rate is the same for every state (except state 0).

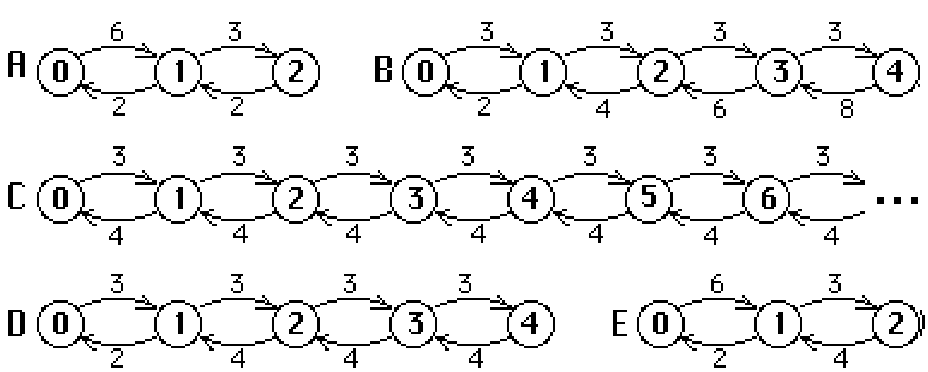
6. Indicate standard notation:

_____	Probability system is empty	_____	Service rate
_____	Average length of queue	_____	Average time in queue
_____	Arrival rate	_____	Average time in system

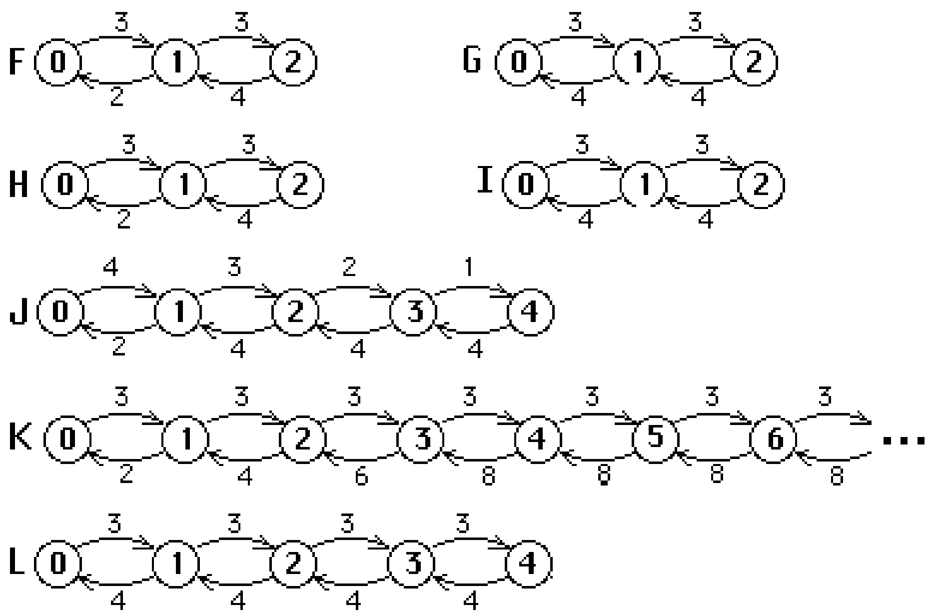
(a.) L	(e.) L_q	(i.) λ
(b.) M	(f.) $1/\mu$	(j.) W
(c.) W_q	(g.) ρ	(k.) $1/\lambda$
(d.) N	(h.) π_0	(l.) μ

7. For each queue classification below, indicate a birth/death (B/D) model below (**A** through **L**) which is consistent for some birth & death rates. If "none", indicate "**X**".

Queue	B/D Model	Queue	B/D Model
M/M/1	____	M/M/4/4	____
M/M/4	____	M/M/1/2/2	____
M/M/2/4	____	M/M/1/4	____

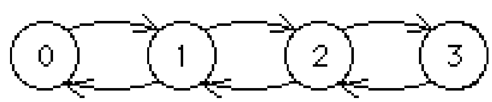


Name ______________________________



A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading jobs) has exponential distribution with mean 10 minutes. The machine will then process its job without human attendance for an average of 30 minutes (but with actual time having exponential distribution) before it requires the operator's attention again. *(Note that the arrival & service rates differ from those in the HW assignment!)*

8. Label the transition rates on the birth/death diagram below:



____9. Which equation below is used to compute the steady-state probability π_0? *(Note: The arithmetic is correct!)*

a. $\frac{1}{\pi_0} = 1 + \frac{6}{6} + \frac{24}{36} + \frac{48}{216} = \frac{1}{0.3553}$

b. $\frac{1}{\pi_0} = 1 + \frac{2}{6} + \frac{4}{36} + \frac{8}{216} = \frac{1}{0.729}$

c. $\frac{1}{\pi_0} = 1 + \frac{6}{2} + \frac{36}{4} + \frac{216}{8} = \frac{1}{0.001316}$

d. $\frac{1}{\pi_0} = 1 + \frac{6}{6} + \frac{6}{24} + \frac{6}{48} = \frac{1}{0.4384}$

e. $\frac{1}{\pi_0} = 1 + \frac{6}{6} + \frac{6}{4} + \frac{6}{2} = \frac{1}{0.125}$

f. *None of the above*

____ 10. The average utilization of the machines is computed by the formula

a. λ/μ

b. $\sum_{i=0}^{3} (3-i)\pi_i \Big/ 3$

c. $\sum_{i=0}^{3} \pi_i \Big/ 3$

d. $3\lambda/\mu$

e. $\sum_{i=0}^{3} i\pi_i \Big/ 3$

f. $\sum_{i=0}^{3} i\pi_{3-i}$

g. *None of the above*

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57:022 Principles of Design II

Quiz #10 -- Friday, May 2, 1997

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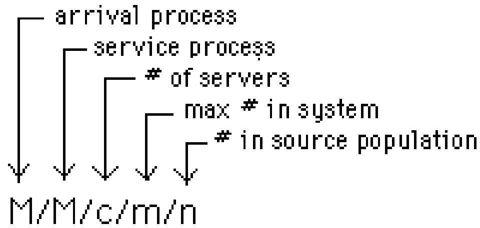
Queueing Models

Note: Kendall's notation:



True (+) or False (**o**)?

_**o**_1. In an M/M/1 queueing system, with arrival rate 2/minute and service time averaging 20 seconds, we would expect the server to be busy more than 75% of the time.

Note: λ = 2/minute, μ = 3/minute, and so the utilization $\rho=\lambda/\mu = 2/3$.

_**o**_2. In an M/M/1/N queueing system, N represents the capacity of the waiting line.

Note: N represents the capacity of the **system**, which is 1 greater than the capacity of the queue.

_**o**_3. Consider an M/M/1 queueing system, with an average of 15 seconds between arrivals and an average of 2 customers in the system. According to Little's Law, we would expect the average time spent by a customer in the system to be at least 1 minute.

Note: λ =4/minute and L=2, and so by Little's Law ($L=\lambda W$),

$$W = \frac{L}{\lambda} = \frac{2}{4/\text{minute}} = 0.5 \text{ minute}$$

_+_4. In an M/M/1/4/4 queueing system, the arrival rate varies by state, while the service rate is the same for every state (except state 0).

_**o**_5. In an M/M/2 queueing system, the arrival rate varies by state, while the service rate is the same for every state (except state 0).

Note: The service rate in state 1 is the rate at which an individual server works, but in states 2,3,4,..., the service rate will be twice that.

6. Indicate standard notation:

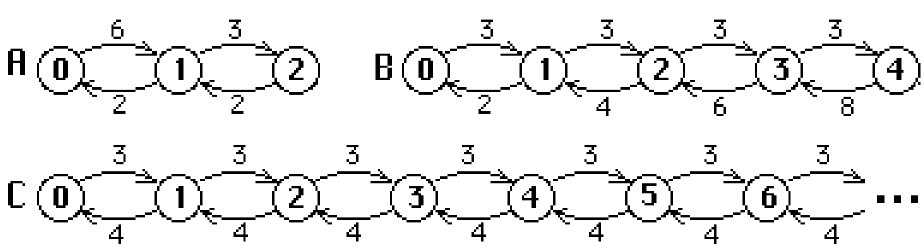
(h.) π_0 = Probability system is empty (l.) μ = Service rate

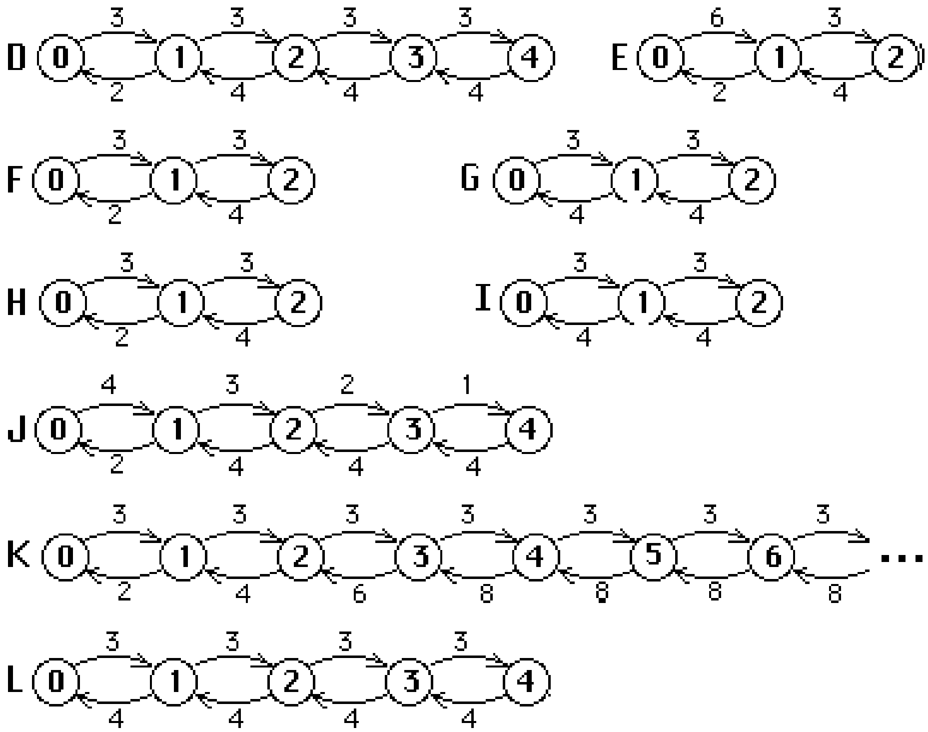
(e.) L_q = Average length of queue (c.) W_q = Average time in queue

(i.) λ = Arrival rate (j.) W = Average time in system

7. For each queue classification below, indicate a birth/death (B/D) model below (**A** through **L**) which is consistent for some birth & death rates. If "none", indicate "**X**".

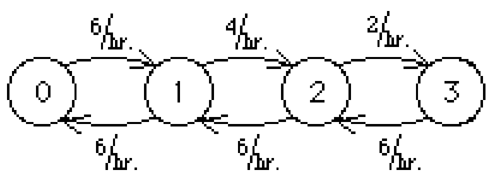
Queue	B/D Model	Queue	B/D Model
M/M/1	C	M/M/4/4	B
M/M/4	K	M/M/1/2/2	A
M/M/2/4	D	M/M/1/4	L





A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading jobs) has exponential distribution with mean 10 minutes. The machine will then process its job without human attendance for an average of 30 minutes (but with actual time having exponential distribution) before it requires the operator's attention again. *(Note that the arrival & service rates differ from those in the HW assignment!)*

8. Label the transition rates on the birth/death diagram below:



Note that the service rate μ is $^6/_{hr.}$, the rate at which the machine operator is able to unload/reload the machines, while the arrival rate for each of the three machines is $\lambda = {}^2/_{hr.}$ In state #0, for example, the machine operator is idle and three machines are processing jobs, so that the arrival rate is $3\lambda = {}^6/_{hr.}$

_a__9. Which equation below is used to compute the steady-state probability π_0? *(Note: The arithmetic is correct!)*

a. $\dfrac{1}{\pi_0} = 1 + \dfrac{6}{6} + \dfrac{24}{36} + \dfrac{48}{216} = \dfrac{1}{0.3553}$

b. $\dfrac{1}{\pi_0} = 1 + \dfrac{2}{6} + \dfrac{4}{36} + \dfrac{8}{216} = \dfrac{1}{0.729}$

c. $\dfrac{1}{\pi_0} = 1 + \dfrac{6}{2} + \dfrac{36}{4} + \dfrac{216}{8} = \dfrac{1}{0.001316}$

d. $\dfrac{1}{\pi_0} = 1 + \dfrac{6}{6} + \dfrac{6}{24} + \dfrac{6}{48} = \dfrac{1}{0.4384}$

e. $\dfrac{1}{\pi_0} = 1 + \dfrac{6}{6} + \dfrac{6}{4} + \dfrac{6}{2} = \dfrac{1}{0.125}$

f. *None of the above*

__b_ 10. The average utilization of the machines is computed by the formula

a. λ/μ b. $\sum_{i=0}^{3} (3-i)\pi_i \Big/ 3$ c. $\sum_{i=0}^{3} \pi_i \Big/ 3$

d. $3\lambda/\mu$ e. $\sum_{i=0}^{3} i\pi_i / 3$ f. $\sum_{i=0}^{3} i\pi_{3-i}$

g. *None of the above*

The average (i.e. expected) number of machines processing jobs at any time in steady state is

$\sum_{i=0}^{3} (3-i)\pi_i$, since in state #i, i machines are in the "system", i.e. the machine operator's queue plus service facility, and therefore 3-i are processing jobs.