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57:022 Principles of Design II
Quizzes Spring '95

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<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #1** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Vehicles arrive at a toll booth on the freeway at the average rate of 5/minute in a completely random fashion. The vehicles are counted and arrival times are recorded, beginning at 12:00 noon. Eighty percent of the vehicles are cars, while the remainder are trucks, buses, etc.

Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!

___ 1. the vehicle# of the second vehicle which is *not* a car.
___ 2. an indicator X_n, which is 1 if vehicle #n is a car, 0 otherwise.
___ 3. vehicle# of the first vehicle which is *not* a car.
___ 4. time of arrival of first vehicle
___ 5. time of arrival of vehicle #2
___ 6. time between arrival of vehicle #1 and vehicle #2
___ 7. number of vehicles arriving during the first 5 minutes
___ 8. the number of cars among the first 10 vehicles to arrive

Probability distributions:

A. Binomial
B. Exponential
C. Pascal
D. Poisson
E. Erlang
F. Geometric
G. Bernouilli
H. Normal

Write the alphabetic letter corresponding to the numerical value of the following quantities.

___ 9. P{*exactly* 5 vehicles arrive during the first minute}
___ 10. P{the first non-car is vehicle #5}
___ 11. P{the second non-car is vehicle #8}
___ 12. P{two of the first ten vehicles are cars}
___ 13. P{the first vehicle arrives during the first $^1/_5$ minute}
___ 14. P{the fifth vehicle arrives during the first minute}
___ 15. P{vehicle #5 is not a car}

Numerical values:

J. $\binom{10}{2}(0.2)^8(0.8)^2$

K. $1 - \sum_{x=0}^{4} \frac{5^x}{x!} e^{-5}$

L. $(0.8)^4(0.2)$

M. $5\,e^{-5}$

N. $\binom{9}{1}(0.8^8)(0.2^2)$

O. $\frac{5^5}{5!} e^{-5}$

P. $1 - e^{-1}$

Q. 0.2

R. *None of the above*

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #2** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

___ 1. The "Cumulative Distribution Function" (CDF) of a random variable X is
- a. $F(x) = P\{X=x\}$
- b. $f(x) = P\{x\}$
- c. $F(x) = P\{X \le x\}$
- d. $f(x) = P\{X|x\}$
- e. $F(x) = P\{X \ge x\}$
- f. $f(x) = P\{x \mid X\}$

___ 2. The arrival of parts to be processed by a machine is a Poisson process, with the rate 4/hour. The actual number of parts which arrive during the first hour has the
- a. Normal distribution
- b. Binomial distribution
- c. Exponential distribution
- d. Poisson distribution
- e. Uniform distribution
- f. *None of the above*

___ 3. The time between arrivals of parts in the preceding question has the
- a. Normal distribution
- b. Binomial distribution
- c. Exponential distribution
- d. Poisson distribution
- e. Uniform distribution
- f. *None of the above*

___ 4. The CDF of the distribution in (3) above, i.e., the inter-arrival times, is
- a. $1 - e^{-4t}$
- b. e^{-4t}
- c. $4e^{-4t}$
- d. $1 - 4e^{-4t}$
- e. $4 - e^{-4t}$
- f. *None of the above*

___ 5. An inter-arrival time T can be randomly generated by using a uniformly-generated random variable X and computing
- a. $T = -\frac{\ln X}{4}$
- b. $T = e^{-4X}$
- c. $T = -\frac{\ln(1-X)}{4}$
- d. $T = 1 - e^{-4X}$
- e. Either (a) or (c)
- f. Either (b) or (d)

The time between arrivals of fifty cars are measured. It is assumed that these observations will have an exponential distribution with mean of 4 minutes, although the actual average value of the observations was 3.68 minutes. We wish to decide whether the discrepancy between the assumed arrival rate (1 every 4 minutes) and the observed arrival rate (1 every 3.68 minutes) is so large as to disqualify our assumption. The number of observations O_i falling within each of several intervals is shown in the table below. We wish to test the "goodness of fit" of the exponential distribution with mean 4 minutes, and have prepared the table below:

i	Interval	O_i	p_i	$E_i = 50p_i$	$\frac{(E_i - O_i)^2}{E_i}$
1	0-1	12	0.221199	11.06	0.0798984
2	1-2	11	0.17227	8.61351	0.661212
3	2-3	6	0.134164	6.70821	0.0747674
4	3-5	6	0.185862	9.29309	1.16693
5	5-9	10	0.181106	9.05528	0.0985611
6	9-∞	5	0.105399	5.26996	0.0138291

total = 2.0952

A portion of a table of the chi-square distribution is given below:

deg.of	Chi-square Dist'n $P\{D \geq \chi^2\}$					
freedom	99%	95%	90%	10%		5%
	1%					
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

Indicate whether true or false, using "**+**" for true, "**o**" for false.

____ 6. The probability p_i that a car arrives in interval #3, i.e., [2,3], is F(2) - F(3), where F(t) is the CDF of the interarrival times.
____ 7. The quantity D=2.0952 is assumed to have the chi-square distribution.
____ 8. If the assumption is correct, the arrivals of the cars forms a Poisson process.
____ 9. The chi-square distribution for this test will have 6 "degrees of freedom".
____ 10. The CDF of the inter-arrival time distribution is $F(t) = P\{T \leq t\}$
____ 11. The parameter of the exponential distribution was assumed to be λ = 1/4 min. = 0.25/minute.
____ 12. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D exceeds 2.0952 is more than 10%.
____ 13. The quantity $(E_i - O_i)^2/E_i$ is assumed to have a "chi-square" distribution.
____ 14. Based upon these observations, the exponential distribution with mean 4 minutes should <u>not</u> be rejected as a model for the interarrival times of the vehicles.
____ 15. The chi-square distribution for this test will have 6 "degrees of freedom".
____ 16. The number of observations O_i in interval #i is a random variable with approximately binomial distribution with n=50 and probability of "success" $p=p_i$.
____ 17. The quantity E_i is a random variable with approximately a Poisson distribution.
____ 18. The smaller the value of D, the better the fit for the distribution being tested.
____ 19. The quantity E_i is the expected number of observations in interval #i, if the assumption is true.
____ 20. The quantity D is assumed to have approximately a Normal distribution.
____ 21. The degrees of freedom is reduced by 1 because the total number of observations is fixed at 50.
____ 22. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D is less than 2.0952 is more than 10%.

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #3** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

___ 1. The "Cumulative Distribution Function" (CDF) of a random variable X is
a. $f(x) = P\{x \mid X\}$
b. $f(x) = P\{x\}$
c. $F(x) = P\{X=x\}$
d. $F(x) = P\{X \geq x\}$
e. $f(x) = P\{X|x\}$
f. $F(x) = P\{X \leq x\}$

___ 2. Suppose that a steel chain is made up of many links. The strength of the chain is, of course, the strength of its weakest link, since the chain fails whenever any link fails. A reasonable assumption for the probability distribution for the strength of the chain is
a. Normal distribution
b. Exponential distribution
c. Uniform distribution
d. Weibull distribution
e. Gumbel distribution
f. *None of the above*

___ 4. The CDF, i.e., F(x), of the Gumbel distribution with parameters α and u is

a. $e^{\left(-e^{-\alpha(x-u)}\right)}$

b. $1 - e^{-\alpha(x-u)}$

c. $1 - e^{-(x/u)^{\alpha}}$

d. $u - \frac{\ln(-\ln x)}{\alpha}$

e. $1 - \frac{u \ln(-\ln x)}{\alpha}$

f. *None of the above*

___ 5. The CDF, i.e., F(x), of the Weibull distribution with parameters k and u is

a. $e^{\left(-e^{-k(x-u)}\right)}$

b. $1 - e^{-k(x-u)}$

c. $1 - e^{-(x/u)^{k}}$

d. $u - \frac{\ln(-\ln x)}{k}$

e. $1 - \frac{u \ln(-\ln x)}{k}$

f. *None of the above*

___ 6. The "coefficient of variation" of a probability distribution with mean μ and variance σ^2, is

a. $\sqrt{\mu^2 + \sigma^2}$

b. μ^2/σ^2

c. σ^2/μ^2

d. σ/μ

e. μ/σ

f. none of the above

___ 7. The "Gamma" function is related to the factorial function for integers by

a. $\Gamma(1-k) = k!$

b. $\Gamma(1+k) = k!$

c. $\Gamma(k) = k!$

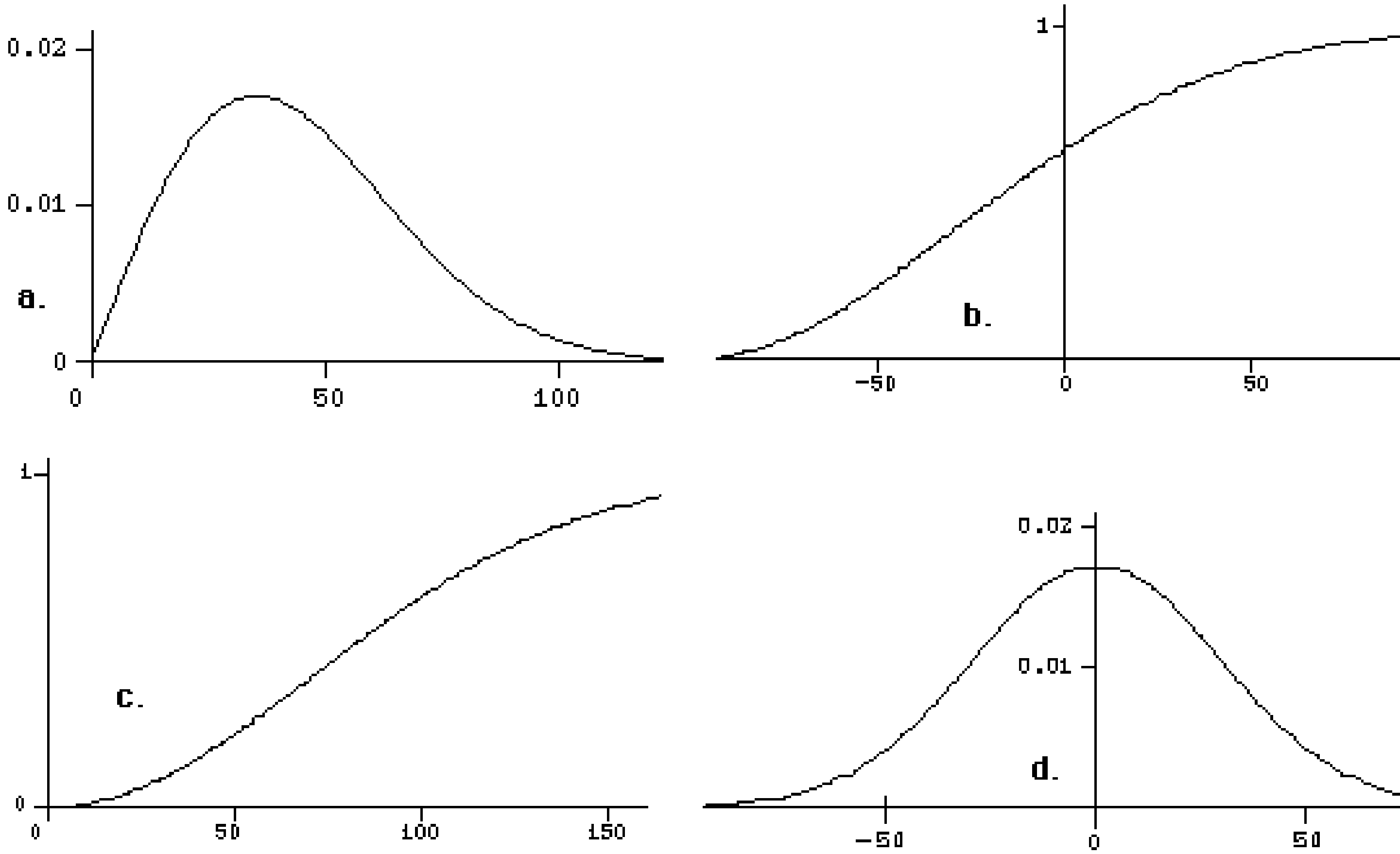
d. $\Gamma(k) = (k+1)!$

e. $\Gamma(1+1/k) = k!$

f. *None of the above*

___ 8. To generate a single random number x having a Weibull distribution F(x), you must first
- a. obtain a single random number y uniformly-distributed in the interval [0,1].
- b. obtain two random numbers (x,y) having uniform distribution in [0,1].
- c. Plot a randomly-generated point (x,y), and accept x if $y \leq f(x)$, otherwise try again.
- d. Derive the inverse of the function F, and compute $F^{-1}(y)$ where y has uniform distribution in [0,1].
- e. Both (a) and (d) are true.
- f. Both (b) and (c) are true.
- g. *None of the above.*

___ 9. Which of the following figures could possibly represent the cumulative distribution function F(x) of a Weibull distribution?



___ 10. Given a set of data points (x_i,y_i), i=1,2,...n, "linear regression" is a method for determining a relationship y = f(x) which
- a. minimizes the sum of the errors $\sum_i y_i - f(x_i)$
- b. minimizes the sum of the absolution values of the errors: $\sum_i |y_i - f(x_i)|$
- c. minimizes the sum of the squares of the errors: $\sum_i (y_i - f(x_i))^2$
- d. minimizes the maximum error max $\{y_i - f(x_i)\}$
- e. *None of the above*

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #4** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Statements below refer to today's homework assignment (HW#4).
Indicate "+" for true, "o" for false:

___ 1. We assumed in this HW that the number of motors which have failed at time t , $N_f(t)$, has a Weibull distribution.

___ 2. If the assumption of Weibull distribution were correct, a plot of $N_f(t)$ vs. t should be approximately on a straight line.

___ 3. If you use the Cricket Graph program to fit a line, it will choose the straight line which minimizes the sum of the squares of the errors, i.e., the sum of the squares of the vertical distances between each data point and the line.

___ 4. The quantity F(t) is, in theory, the fraction of the motors which have failed at time t (or earlier).

___ 5. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has survived (not failed) until time t.

___ 6. If the assumption of Weibull distribution were correct, a plot of $\ln \ln {}^1/_{R(t)}$ vs. ln t should be approximately on a straight line, where R(t) is the fraction of motors surviving at time t.

___ 7. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

___ 8. A value of k greater than 1.0 indicates an increasing failure rate, and k less than 1.0 indicates a decreasing failure rate.

___ 9. The method used in this homework to estimate the Weibull parameters u & k does <u>not</u> require that the motors be operated until all have failed.

___ 10. Given a coefficient of variation for the Weibull distribution (the ratio σ/μ), the parameter k can be determined.

___ 11. If ten motors are installed in a manufacturing facility, the number still functioning after 200 days has a binomial distribution.

___ 12. If the failure rate is decreasing, it may be more appropriate to use the Gumbel distribution than the Weibull.

___ 13. The expected number of machines E_i which fail in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$

___ 14. In the chi-square goodness-of-fit test, the number of degrees of freedom is equal to the number of "cells" of the histogram (in this case, 12).

___ 15. According to the results of this homework exercise, the failure rate of the motors is decreasing rather than increasing over time.

Select the letter below which indicates each correct answer:

When plotting the points to fit a straight line,

___ 16. The vertical axis should represent ...

___ 17. The horizontal axis should represent ...

___ 18. The slope of the line should be approximately ...

___ 19. The vertical intercept (y-intercept) of the line should be approximately ...

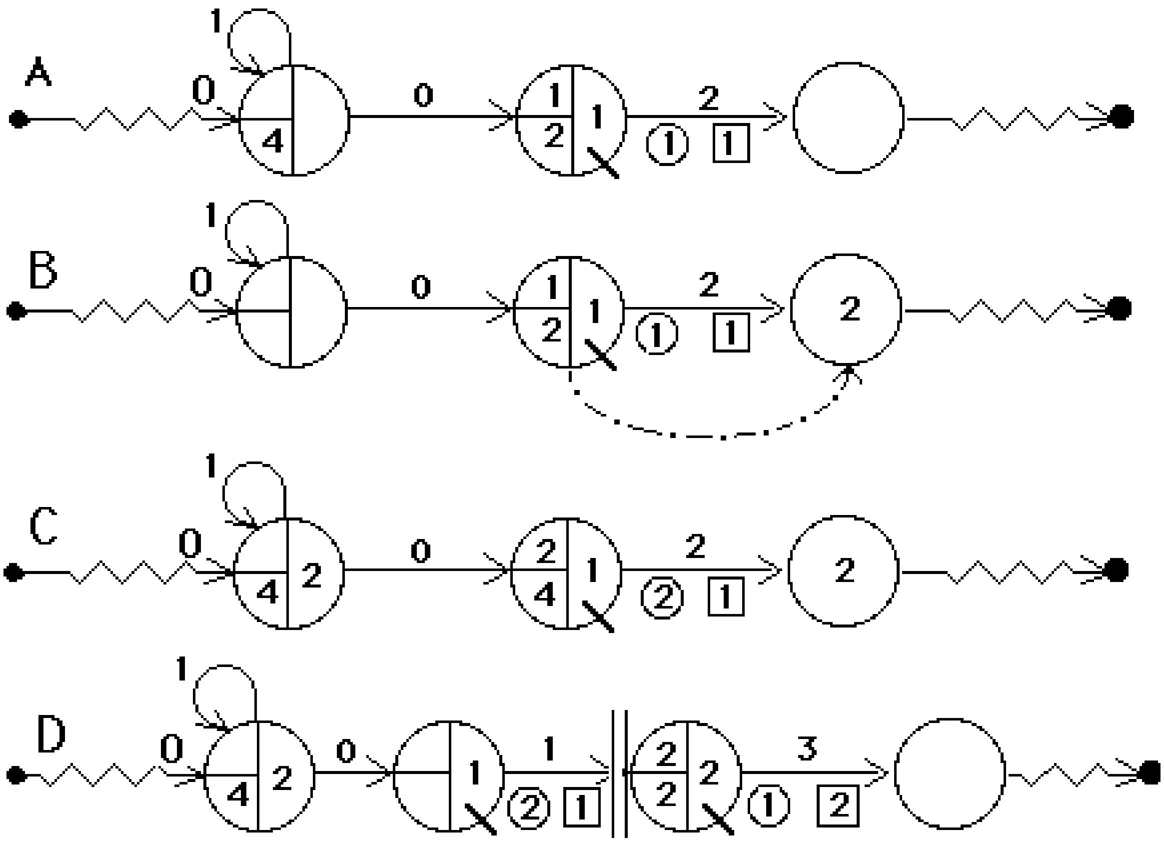
___ 20. Given Weibull parameters u and k, the quantity $u\Gamma(1+{}^1/_k)$ should be...

a. t	g. R_t	m. shape parameter k
b. ln t	h. ln R_t	n. scale parameter u
c. $\ln {}^1/_t$	i. $\ln {}^1/_{R_t}$	o. mean value μ
d. ln ln t	j. ln ln R_t	p. standard deviation σ
e. $\ln \ln {}^1/_t$	k. $\ln \ln {}^1/_{R_t}$	q. ln u
f. k ln u	l. u ln k	r. ln k

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #5** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Note that

- all activity durations in the SLAM network below are *constants* , and none are random!
- first entity is created at time=0



___ 1. In Network **A**, *before* the first created entity, there are how many entities already in the network?

a. none b. one c. two d. three
e. four f. five g. can't be determined h. *NOTA*

___ 2. In Network **A**, the first entity to *leave the system* (& is terminated) leaves at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 3. In Network **A**, the first created entity *enters the queue* at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 4. In Network **A**, the first created entity *begins being served* at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 5. In Network **B**, the first entity which *cannot enter the queue* will arrive at the queue at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 6. In Network **C**, the total number of servers is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 7. In Network **C**, the total number of entities which will *leave* the system is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 8. In Network **C**, the simulation will terminate at time=

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 9. In Network **D**, the total number of servers is
a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

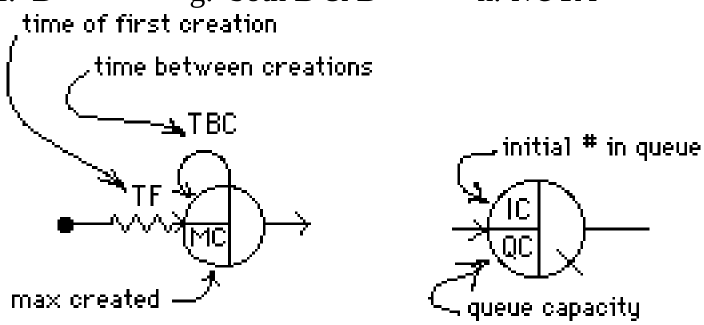
___ 10. In Network **D**, the total number of queues is
a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 11. In Network **D**, the first entity to leave the system will leave at time =
a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 12. In Network **D**, the total number of entities which will leave the system is
a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 13. Of the four SLAM networks, the network in which "blocking" may occur is
a. A b. B c. both C & D d. both A & B
e. C f. D g. both B & D h. *NOTA*

___ 14. Of the four SLAM networks, the network in which "balking" may occur is
a. A b. B c. both C & D d. both A & B
e. C f. D g. both B & D h. *NOTA*



<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #6** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Markov Chain Model of a Reservoir: A city's water supply comes from a reservoir. Careful study of this reservoir over the past twenty years has shown that, if the reservoir was full at the beginning of one summer, then the probability that it would be full at the beginning of the next summer is 80%; however, if the reservoir was not full at the beginning of one summer, the probability that it would be full at the beginning of the next summer is only 40%.

Powers of P

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.72 & 0.28 \\ 0.56 & 0.44 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.688 & 0.312 \\ 0.624 & 0.376 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.6752 & 0.3248 \\ 0.6496 & 0.3504 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.67008 & 0.32992 \\ 0.65984 & 0.34016 \end{bmatrix}$$

Steady State Distribution

i	Pi
1	0.66666667
2	0.33333333

Mean First Passage Times

from \ to	1	2
1	1.5	5
2	2.5	3

Expected no. of visits during first 5 stages

from \ to	1	2
1	3.55328	1.44672
2	2.89344	2.10656

First Passage Probabilities

stage 1: $\begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$

stage 2: $\begin{bmatrix} 0.08 & 0.16 \\ 0.24 & 0.08 \end{bmatrix}$

stage 3: $\begin{bmatrix} 0.048 & 0.128 \\ 0.144 & 0.064 \end{bmatrix}$

stage 4: $\begin{bmatrix} 0.0288 & 0.1024 \\ 0.0864 & 0.0512 \end{bmatrix}$

stage 5: $\begin{bmatrix} 0.01728 & 0.08192 \\ 0.05184 & 0.04096 \end{bmatrix}$

___ 1. Over a 100-year period, how many summers can the reservoir be expected to be *not* full?
a. between 30 and 40 b. between 40 and 50 c. between 50 and 60
d. between 60 and 70 e. between 70 and 80 f. *NOTA*

___ 2. If the reservoir was *not* full at the beginning of summer '94, what is the expected number of summers during the next five years ('95-'99) that the reservoir will not be full?
a. less than 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. *NOTA*

___ 3. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be full at the beginning of summer '95 is
a. between 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75
d. between 0.75 and 1.0 e. *NOTA*

___ 4. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be full at the beginning of summer '96 is
a. 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75
d. between 0.75 and 1.0 e. *NOTA*

___ 5. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be not be full again until sometime *after* '96 is
a. 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75

d. between 0.75 and 1.0 e. *NOTA*

___ 6. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be not be full again until sometime *after* '98 is

a. 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75
d. between 0.75 and 1.0 e. *NOTA*

___ 7. If the reservoir was *not* full at the beginning of summer '94, the expected number of years until it is full again is

a. less than 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. *NOTA*

In response to pressure from the Board of Regents to increase the number of students who complete their degrees within four years, the Engineering College admissions office has modeled the academic career of a student as a Markov chain:

Each student's state is observed at the beginning of each *fall* semester. For example, if a student is a junior at the beginning of the current fall semester, there is an 80% chance that he/she will be a senior at the beginning of the next fall semester, a 15% chance that he/she will still be a junior, and a 5% chance that he/she will have quit. (For simplicity we will assume that once a student quits, he/she never re-enrolls.)

State	Description
1	Freshman
2	Sophomore
3	Junior
4	Senior
5	Drop-out
6	Graduate

___ 8. The number of recurrent states in this model is

a. zero b. one c. two d. three
e. four f. five g. six h. *NOTA*

___ 9. The number of transient states in this model is

a. zero b. one c. two d. three
e. four f. five g. six h. *NOTA*

___ 10. The number of absorbing states in this model is

a. zero b. one c. two d. three
e. four f. five g. six h. *NOTA*

___ 11. If a student enters the college as a freshman, how many years can he or she expect to spend as a student in the college?

a. between 0 and 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. between 5 and 6 g. *NOTA*

___ 12. If a student enters the college as a freshman, what is the probability that, in the fall semester of the 4th year (i.e., 3 years later), he/she is a senior?

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

Transition probabilities (P)

from \ to	1	2	3	4	5	6
1	0.1	0.8	0	0	0.1	0
2	0	0.1	0.85	0	0.05	0
3	0	0	0.15	0.8	0.05	0
4	0	0	0	0.1	0.05	0.85
5	0	0	0	0	1	0
6	0	0	0	0	0	1

E = Expected No. Visits to Transient States

from \ to	1	2	3	4
1	1.1111111	0.98765432	0.98765432	0.87791495
2	0	1.1111111	1.1111111	0.98765432
3	0	0	1.1764706	1.0457516
4	0	0	0	1.1111111

P^2

from \ to	1	2	3	4	5	6
1	0.01	0.16	0.68	0	0.15	0
2	0	0.01	0.2125	0.68	0.0975	0
3	0	0	0.0225	0.2	0.0975	0.68
4	0	0	0	0.01	0.055	0.935
5	0	0	0	0	1	0
6	0	0	0	0	0	1

P^3

from \ to	1	2	3	4	5	6
1	0.001	0.024	0.238	0.544	0.193	0
2	0	0.001	0.040375	0.238	0.142625	0.578
3	0	0	0.003375	0.038	0.108625	0.85
4	0	0	0	0.001	0.0555	0.9435
5	0	0	0	0	1	0
6	0	0	0	0	0	1

A = Absorption Probabilities

from \ to	5	6
1	0.25377229	0.74622771
2	0.16049383	0.83950617
3	0.11111111	0.88888889
4	0.055555556	0.94444444

P^4

from \ to	1	2	3	4	5	6
1	0.0001	0.0032	0.0561	0.2448	0.2334	0.4624
2	0	0.0001	0.00690625	0.0561	0.15659375	0.7803
3	0	0	0.00050625	0.0065	0.11069375	0.8823
4	0	0	0	0.0001	0.05555	0.94435
5	0	0	0	0	1	0
6	0	0	0	0	0	1

___ 13. If a student enters the college as a freshman, what is the probability that he/she will drop out before graduating?

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

___ 14. If a student transfers into the college as a junior, how many years can he or she expect to spend as a student in the college (not counting years before transfer)?

a. between 0 and 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. between 5 and 6 g. *NOTA*

___ 15. What fraction of the students who transfer in as beginning juniors will eventually graduate?

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

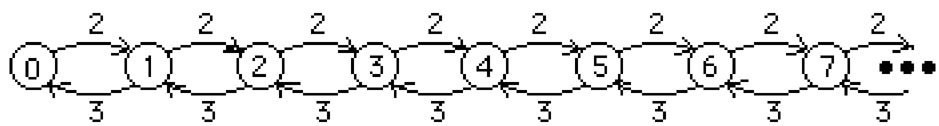
___ 16. What fraction of the students who begin as freshman will have graduated in four years?

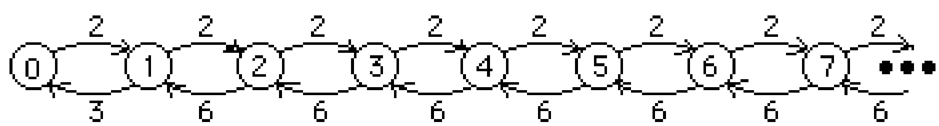
a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

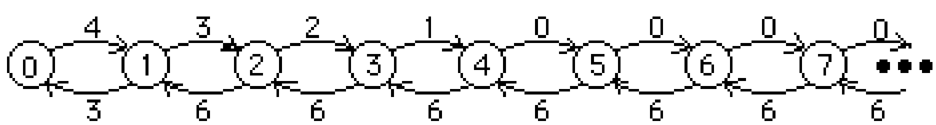
<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #7** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

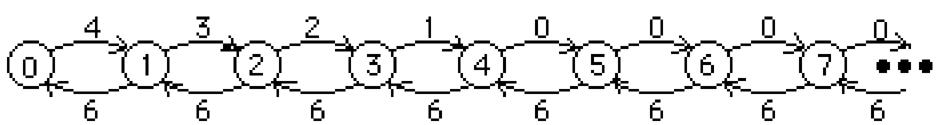
Birth-Death Processes For each birth-death model of a queue in diagrams (1) through (8) below, indicate the correct Kendall's classification from among the following choices. (Note that some classifications might not be matched with any birth-death diagram, while others might be matched to more than one!)

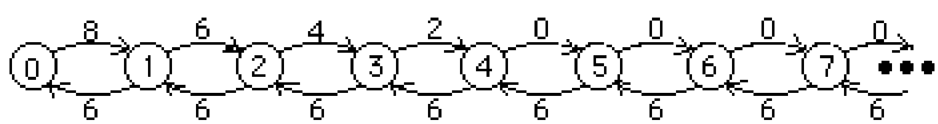
a. M/M/1 d. M/M/2 g. M/M/1/4 j. M/M/2/3
b. M/M/4 e. M/M/2/4 h. M/M/2/4/4 k. M/M/2/2/4
c. M/M/1/2/4 f. M/M/4/2 i. M/M/4/4 l. *None of Above*

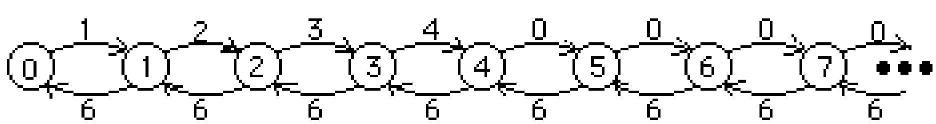
___ 1


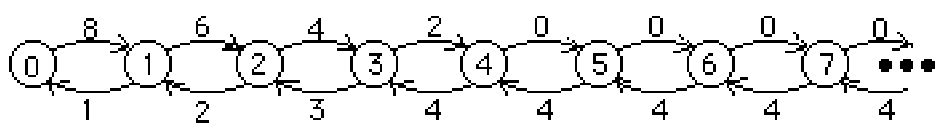
___ 2


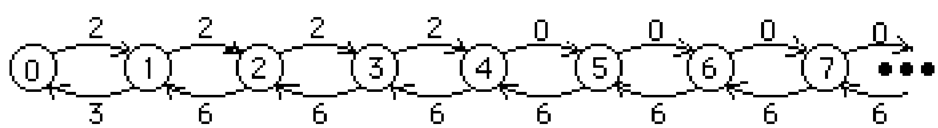
___ 3


___ 4


___ 5


___ 6


___ 7


___ 8


An average of 1 car per minute arrive (forming a Poisson process) arrive at the MacBurger's drive-in window. However, if four or more cars are in line (including the car at the window), an arriving car will not enter the line (i.e., "balk"). It takes an average of 1.5 minutes (exponentially distributed) to serve a car. *(Note that the arrival & service rates differ from those in the HW assignment!)*

9. Label the transition rates on the birth/death diagram below:

____**10.** Which equation is used to compute the steady-state probability π_0? *(Note: The arithmetic is correct!)*

a. $\frac{1}{\pi_0} = 1 + \frac{1}{3/2} + \frac{1}{3/2} + \frac{1}{3/2} + \frac{1}{3/2} = \frac{1}{0.2727}$

b. $\frac{1}{\pi_0} = 1 + \frac{1}{2/3} + \frac{1}{2/3} + \frac{1}{2/3} + \frac{1}{2/3} = \frac{1}{0.1428}$

c. $\frac{1}{\pi_0} = 1 + \frac{1}{3/2} + \frac{1}{9/4} + \frac{1}{27/8} + \frac{1}{81/16} = \frac{1}{0.3838}$

d. $\frac{1}{\pi_0} = 1 + \frac{1}{2/3} + \frac{1}{4/9} + \frac{1}{8/27} + \frac{1}{16/81} = \frac{1}{0.0758}$

e. *None of the above*

____**11.** What is the average arrival rate $\underline{\lambda}$ (in cars/minue)?

a. $\sum_{i=1}^{4} \pi_i = 1- \pi_0$

b. $\sum_{i=0}^{3} \pi_i =1-\pi_4$

c. $\sum_{i=0}^{4} \pi_i = 1$

d. $\frac{\lambda}{\mu}$

e. *None of the above*

____ 12. The average number of cars waiting for the drive-in window (not including a car at the window) is 1.83. What is the average time which each car waits?

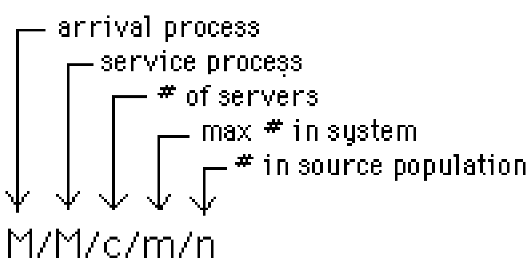
a. $1.83\underline{\lambda}$

b. $1.83/\underline{\lambda}$

c. $\underline{\lambda}/1.83$

d. None of the above

Note: Kendall's notation:



<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Solutions** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Quiz #1:

_Pascal__ 1. the vehicle# of the second vehicle which is *not* a car.
Bernouilli 2. an indicator X_n, which is 1 if vehicle #n is a car, 0 otherwise.
Geometric 3. vehicle# of the first vehicle which is *not* a car.

Exponential 4. time of arrival of first vehicle
Erlang 5. time of arrival of vehicle #2
Exponential 6. time between arrival of vehicle #1 and vehicle #2
Poission 7. number of vehicles arriving during the first 5 minutes
Binomial 8. the number of cars among the first 10 vehicles to arrive

_**O**_ 9. P{*exactly* 5 vehicles arrive during the first minute}
_**L**_ 10. P{the first non-car is vehicle #5}
_**R**_ 11. P{the second non-car is vehicle #8}
_**J**_ 12. P{two of the first ten vehicles are cars}
_**P**_ 13. P{the first vehicle arrives during the first $^1/_5$ minute}
_**K**_ 14. P{the fifth vehicle arrives during the first minute}
_**Q**_ 15. P{vehicle #5 is not a car}

Quiz #2:

_**c**_ 1. The "Cumulative Distribution Function" (CDF) of a random variable X is
a. F(x) = P{X=x} b. f(x) = P{x}
c. F(x) = P{X≤x} d. f(x) = P{X|x}
e. F(x) = P{X≥x} f. f(x) = P{x | X}

_**d**_ 2. The arrival of parts to be processed by a machine is a Poisson process, with the rate 4/hour. The actual number of parts which arrive during the first hour has the
a. Normal distribution b. Binomial distribution
c. Exponential distribution d. Poisson distribution
e. Uniform distribution f. *None of the above*

_**c**_ 3. The time between arrivals of parts in the preceding question has the
a. Normal distribution b. Binomial distribution
c. Exponential distribution d. Poisson distribution
e. Uniform distribution f. *None of the above*

_**a**_ 4. The CDF of the distribution in (3) above, i.e., the inter-arrival times, is
a. $1 - e^{-4t}$ b. e^{-4t}
c. $4e^{-4t}$ d. $1 - 4e^{-4t}$
e. $4 - e^{-4t}$ f. *None of the above*

_**e**_ 5. An inter-arrival time T can be randomly generated by using a uniformly-generated random variable X and computing
a. $T = -\frac{\ln X}{4}$ b. $T = e^{-4X}$
c. $T = -\frac{\ln (1-X)}{4}$ d. $T = 1 - e^{-4X}$
e. Either (a) or (c) f. Either (b) or (d)

_**o**_ 6. The probability p_i that a car arrives in interval #3, i.e., [2,3], is F(2) - F(3), where F(t) is the CDF of the interarrival times.
Should be F(3)-F(2)

_+_ 7. The quantity D=2.0952 is assumed to have the chi-square distribution.
_+_ 8. If the assumption is correct, the arrivals of the cars forms a Poisson process.
_**o**_ 9. The chi-square distribution for this test will have 6 "degrees of freedom".
_+_ 10. The CDF of the inter-arrival time distribution is F(t) = P{T ≤ t}
_+_ 11. The parameter of the exponential distribution was assumed to be λ = 1/4 min. = 0.25/minute.
_+_ 12. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D exceeds 2.0952 is more than 10%.
Since P{D≥7.779}=10%, we have P{D≥2.0952}>10%
_+_ 13. The quantity $(E_i-O_i)^2/E_i$ is assumed to have a "chi-square" distribution.

_+_ 14. Based upon these observations, the exponential distribution with mean 4 minutes should not be rejected as a model for the interarrival times of the vehicles.

_o_ 15. The chi-square distribution for this test will have 6 "degrees of freedom".

_+_ 16. The number of observations O_i in interval #i is a random variable with approximately binomial distribution with n=50 and probability of "success" $p=p_i$.

_o_ 17. The quantity E_i is a random variable with approximately a Poisson distribution.
Ei is not a random variable.

_+_ 18. The smaller the value of D, the better the fit for the distribution being tested.

_+_ 19. The quantity E_i is the expected number of observations in interval #i, if the assumption is true.

_o_ 20. The quantity D is assumed to have approximately a Normal distribution.
Chi-Square distribution.

_o_ 21. The degrees of freedom is reduced by 1 because the total number of observations is fixed at 50.

_+_ 22. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D is less than 2.0952 is more than 10%.
Since P{D≥1.064}=90%, we have P{D≤1.064}=10% or P{D≤2.0952}>10%.

Quiz #3:

_f_ 1. The "Cumulative Distribution Function" (CDF) of a random variable X is

a. f(x) = P{x | X}
b. f(x) = P{x}
c. F(x) = P{X=x}
d. F(x) = P{X≥x}
e. f(x) = P{X|x}
f. F(x) = P{X≤x}

_d_ 2. Suppose that a steel chain is made up of many links. The strength of the chain is, of course, the strength of its weakest link, since the chain fails whenever any link fails. A reasonable assumption for the probability distribution for the strength of the chain is

a. Normal distribution
b. Exponential distribution
c. Uniform distribution
d. Weibull distribution
e. Gumbel distribution
f. *None of the above*

_a_ 4. The CDF, i.e., F(x), of the Gumbel distribution with parameters α and u is

a. $e^{\left(-e^{-\alpha(x-u)}\right)}$
b. $1 - e^{-\alpha(x-u)}$
c. $1 - e^{-(x/u)^{\alpha}}$
d. $u - \frac{\ln(-\ln x)}{\alpha}$
e. $1 - \frac{u \ln(-\ln x)}{\alpha}$
f. *None of the above*

_c_ 5. The CDF, i.e., F(x), of the Weibull distribution with parameters k and u is

a. $e^{\left(-e^{-k(x-u)}\right)}$
b. $1 - e^{-k(x-u)}$
c. $1 - e^{-(x/u)^{k}}$
d. $u - \frac{\ln(-\ln x)}{k}$
e. $1 - \frac{u \ln(-\ln x)}{k}$
f. *None of the above*

_d_ 6. The "coefficient of variation" of a probability distribution with mean μ and variance σ^2, is

a. $\sqrt{\mu^2 + \sigma^2}$
b. μ^2/σ^2
c. σ^2/μ^2
d. σ/μ

e. μ/σ

f. none of the above

_b_ 7. The "Gamma" function is related to the factorial function for integers by

a. $\Gamma(1\text{-}k)$ = k!

b. $\Gamma(1+k)$ = k!

c. $\Gamma(k)$ = k!

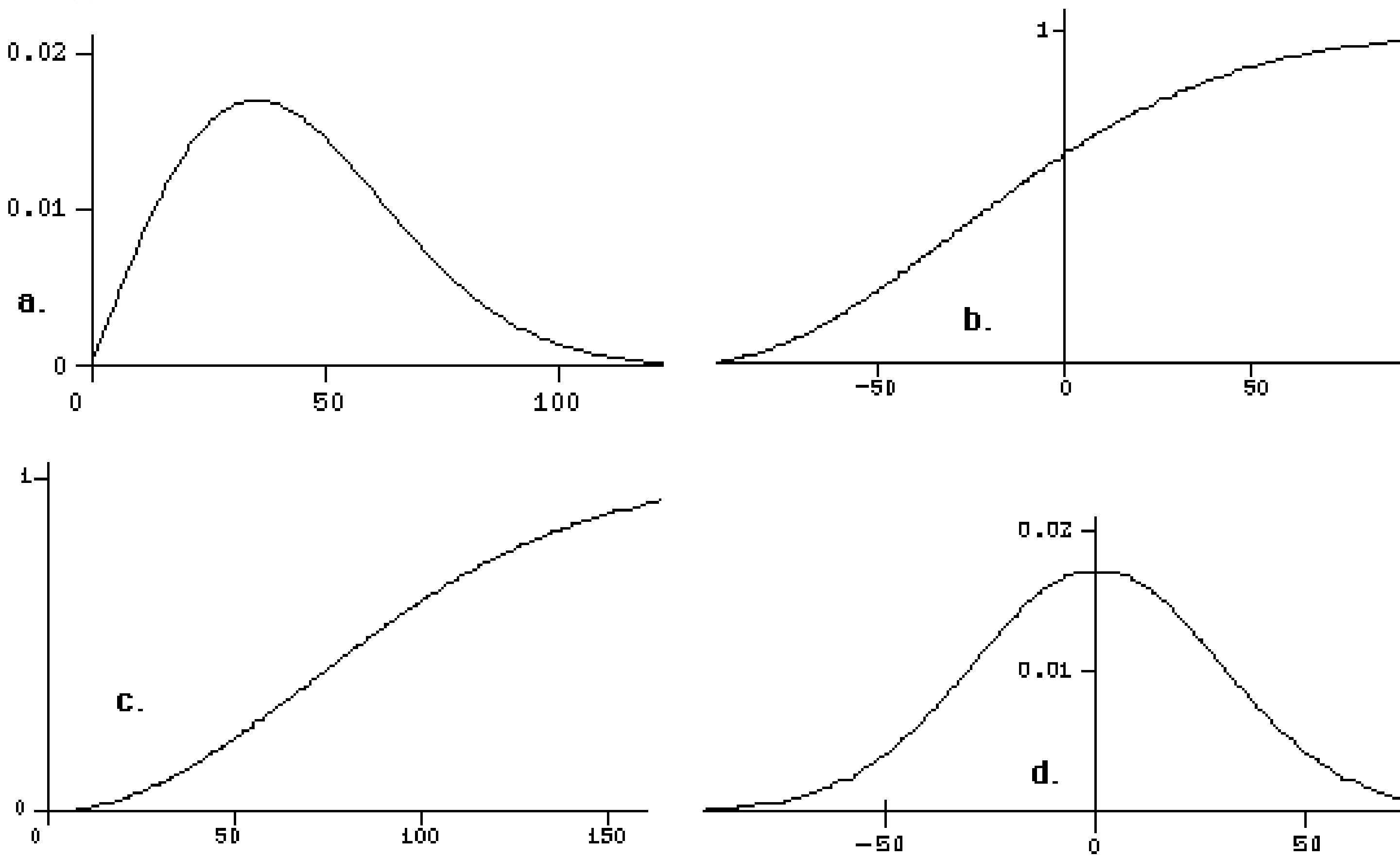
d. $\Gamma(k)$ = (k+1)!

e. $\Gamma\left(1+1/k\right)$ = k!

f. *None of the above*

_**e**_ 8. To generate a single random number x having a Weibull distribution F(x), you must first
a. obtain a single random number y uniformly-distributed in the interval [0,1].
b. obtain two random numbers (x,y) having uniform distribution in [0,1].
c. Plot a randomly-generated point (x,y), and accept x if $y \leq f(x)$, otherwise try again.
d. Derive the inverse of the function F, and compute $F^{-1}(y)$ where y has uniform distribution in [0,1].
e. Both (a) and (d) are true.
f. Both (b) and (c) are true.
g. *None of the above.*

_**c**_ 9. Which of the following figures could possibly represent the cumulative distribution function F(x) of a Weibull distribution?



_**c**_ 10. Given a set of data points (x_i, y_i), i=1,2,...n, "linear regression" is a method for determining a relationship y = f(x) which
a. minimizes the sum of the errors $\sum_i y_i - f(x_i)$
b. minimizes the sum of the absolution values of the errors: $\sum_i |y_i - f(x_i)|$
c. minimizes the sum of the squares of the errors: $\sum_i (y_i - f(x_i))^2$
d. minimizes the maximum error max $\{y_i - f(x_i)\}$
e. *None of the above*

Quiz #4:

_**o**_ 1. We assumed in this HW that the number of motors which have failed at time t , $N_f(t)$, has a Weibull distribution.

_**o**_ 2. If the assumption of Weibull distribution were correct, a plot of $N_f(t)$ vs. t should be approximately on a straight line.

_+_ 3. If you use the Cricket Graph program to fit a line, it will choose the straight line which minimizes the sum of the squares of the errors, i.e., the sum of the squares of the vertical distances between each data point and the line.

_+_ 4. The quantity F(t) is, in theory, the fraction of the motors which have failed at time t (or earlier).

_**o**_ 5. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has survived (not failed) until time t.

_+_ 6. If the assumption of Weibull distribution were correct, a plot of $\ln \ln {}^1/_{R(t)}$ vs. ln t should be approximately on a straight line, where R(t) is the fraction of motors surviving at time t.

_+_ 7. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

_+_ 8. A value of k greater than 1.0 indicates an increasing failure rate, and k less than 1.0 indicates a decreasing failure rate.

_+_ 9. The method used in this homework to estimate the Weibull parameters u & k does <u>not</u> require that the motors be operated until all have failed.

_+_ 10. Given a coefficient of variation for the Weibull distribution (the ratio ${}^\sigma/_\mu$), the parameter k can be determined.

_+_ 11. If ten motors are installed in a manufacturing facility, the number still functioning after 200 days has a binomial distribution.

_**o**_ 12. If the failure rate is decreasing, it may be more appropriate to use the Gumbel distribution than the Weibull.

_+_ 13. The expected number of machines E_i which fail in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$

_**o**_ 14. In the chi-square goodness-of-fit test, the number of degrees of freedom is equal to the number of "cells" of the histogram (in this case, 12).

_+_ 15. According to the results of this homework exercise, the failure rate of the motors is decreasing rather than increasing over time.

Select the letter below which indicates each correct answer:

When plotting the points to fit a straight line,

_**k**_ 16. The vertical axis should represent ...

_**b**_ 17. The horizontal axis should represent ...

_**m**_ 18. The slope of the line should be approximately ...

_**f**_ 19. The vertical intercept (y-intercept) of the line should be approximately ...

_**o**_ 20. Given Weibull parameters u and k, the quantity $u\Gamma(1+{}^1/_k)$ should be...

<u>*Quiz #5:*</u>

_**c**_ 1. In Network **A**, *before* the first created entity, there are how many entities already in the network?

a. none	b. one	c. two	d. three	
e. four	f. five	g. can't be determined		h. *NOTA*

_**c**_ 2. In Network **A**, the first entity to *leave the system* (& is terminated) leaves at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. *NOTA*

_**a**_ 3. In Network **A**, the first created entity *enters the queue* at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. *NOTA*

_**e**_ 4. In Network **A**, the first created entity *begins being served* at time =

a. 0	b. 1	c. 2	d. 3	e. 4

f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**b**_ 5. In Network **B**, the first entity which *cannot enter the queue* will arrive at the queue at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**c**_ 6. In Network **C**, the total number of servers is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**c**_ 7. In Network **C**, the total number of entities which will *leave* the system is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**c**_ 8. In Network **C**, the simulation will terminate at time=

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**d**_ 9. In Network **D**, the total number of servers is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**c**_ 10. In Network **D**, the total number of queues is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**d**_ 11. In Network **D**, the first entity to leave the system will leave at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**h**_ 12. In Network **D**, the total number of entities which will leave the system is

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

_**f**_ 13. Of the four SLAM networks, the network in which "blocking" may occur is

a. A b. B c. both C & D d. both A & B
e. C f. D g. both B & D h. *NOTA*

_**b**_ 14. Of the four SLAM networks, the network in which "balking" may occur is

a. A b. B c. both C & D d. both A & B
e. C f. D g. both B & D h. *NOTA*

Quiz #6:

_**a**_ 1. Over a 100-year period, how many summers can the reservoir be expected to be *not* full?

a. between 30 and 40 b. between 40 and 50 c. between 50 and 60
d. between 60 and 70 e. between 70 and 80 f. *NOTA*

_**c**_ 2. If the reservoir was *not* full at the beginning of summer '94, what is the expected number of summers during the next five years ('95-'99) that the reservoir will not be full?

a. less than 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. *NOTA*

_**b**_ 3. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be full at the beginning of summer '95 is

a. between 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75
d. between 0.75 and 1.0 e. *NOTA*

_**c**_ 4. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be full at the beginning of summer '96 is

a. 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75
d. between 0.75 and 1.0 e. *NOTA*

_**b**_ 5. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be not be full again until sometime *after* '96 is $1 - \left(f_{21}^{1} + f_{21}^{2}\right) = 1 - 0.4 - 0.24 = 0.36$

a. 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75

d. between 0.75 and 1.0 e. *NOTA*

_**a**_ 6. If the reservoir was *not* full at the beginning of summer '94, the probability that it will be not be full again until sometime *after* '98 is

$1 - \left(f^1_{21} + f^2_{21} + f^3_{21} + f^4_{21}\right) = 1 - 0.4 - 0.24 - 0.144 - 0.0864 \approx 0.13$

a. 0 and 0.25 b. between 0.25 and 0.50 c. between 0.50 and 0.75
d. between 0.75 and 1.0 e. *NOTA*

_**c**_ 7. If the reservoir was *not* full at the beginning of summer '94, the expected number of years until it is full again is $m_{21} = 2.5$

a. less than 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. *NOTA*

_**c**_ 8. The number of recurrent states in this model is

a. zero b. one c. two d. three
e. four f. five g. six h. *NOTA*

_**e**_ 9. The number of transient states in this model is

a. zero b. one c. two d. three
e. four f. five g. six h. *NOTA*

_**c**_ 10. The number of absorbing states in this model is

a. zero b. one c. two d. three
e. four f. five g. six h. *NOTA*

_**d**_ 11. If a student enters the college as a freshman, how many years can he or she expect to spend as a student in the college?

$e_{11} + e_{12} + e_{13} + e_{14} = 1.111 + 0.9876 + 0.9876 + 0.8779 \approx 3.96$

a. between 0 and 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. between 5 and 6 g. *NOTA*

_**c**_ 12. If a student enters the college as a freshman, what is the probability that, in the fall semester of the 4th year (i.e., 3 years later), he/she is a senior? $p^3_{14} = 0.544$

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

_**b**_ 13. If a student enters the college as a freshman, what is the probability that he/she will drop out before graduating? $a_{15} = 0.2538$

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

_**c**_ 14. If a student transfers into the college as a junior, how many years can he or she expect to spend as a student in the college (not counting years before transfer)?

$e_{33} + e_{34} = 1.1765 + 1.0457 = 2.222$

a. between 0 and 1 b. between 1 and 2 c. between 2 and 3
d. between 3 and 4 e. between 4 and 5 f. between 5 and 6 g. *NOTA*

_**e**_ 15. What fraction of the students who transfer in as beginning juniors will eventually graduate?

$a_{36} = 0.8888$

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

_**c**_ 16. What fraction of the students who begin as freshman will have graduated in four years?

$p^4_{16} = 0.4624$

a. between 0 and 0.2 b. between 0.2 and 0.4 c. between 0.4 and 0.6
d. between 0.6 and 0.8 e. between 0.8 and 1.0 f. *NOTA*

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>