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 57:022 Principles of Design II
 Quizzes Spring '93
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◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊ **Quiz #1** ◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊

Vehicles arrive at a toll booth on the freeway at the average rate of 6/minute in a completely random fashion. The vehicles are counted and arrival times are recorded, beginning at 12:00 noon. Ninety percent of the vehicles are cars, while the remainder are trucks, buses, etc.

Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!

- ___ 1. time of arrival of first vehicle
- ___ 2. time of arrival of vehicle #2
- ___ 3. time between arrival of vehicle #1 and vehicle #2
- ___ 4. number of vehicles arriving during the first 5 minutes
- ___ 5. vehicle# of the first vehicle which is *not* a car.
- ___ 6. the number of cars among the first 10 vehicles to arrive
- ___ 7. the vehicle# of the second vehicle which is *not* a car.
- ___ 8. an indicator for vehicle #n which is 1 if a car, 0 otherwise.

Probability distributions:

- | | |
|--------------|----------------|
| A. Bernoulli | E. Geometric |
| B. Erlang | F. Exponential |
| C. Poisson | G. Pascal |
| D. Binomial | H. Normal |

Write the alphabetic letter corresponding to the numerical value of the following quantities.

- ___ 9. $P\{\text{exactly 6 vehicles arrive during the first minute}\}$
- ___ 10. $P\{\text{two of the first 6 vehicles are cars}\}$
- ___ 11. $P\{\text{the first vehicle arrives during the first } 1/6 \text{ minute}\}$
- ___ 12. $P\{\text{the sixth vehicle arrives during the first minute}\}$
- ___ 13. $P\{\text{the first non-car is vehicle #6}\}$
- ___ 14. $P\{\text{the second non-car is vehicle #6}\}$
- ___ 15. $P\{\text{vehicle #6 is not a car}\}$

Numerical values:

- | | |
|---|----------------------------------|
| J. $6e^{-6}$ | K. 0.1 |
| L. $1 - \sum_{x=0}^5 \frac{6^x}{x!} e^{-6}$ | M. $\frac{6^6}{6!} e^{-6}$ |
| N. $(0.9^5)(0.1)$ | O. $1 - e^{-1}$ |
| P. $\binom{6}{2} 0.1^4 0.9^2$ | Q. $\binom{5}{1} (0.9^4)(0.1^2)$ |
- R. None of the above

Solutions:

- | | |
|-------------|---|
| Exponential | 1. time of arrival of first vehicle |
| Erlang | 2. time of arrival of vehicle #2 |
| Exponential | 3. time between arrival of vehicle #1 and vehicle #2 |
| Poisson | 4. number of vehicles arriving during the first 5 minutes |

$$\begin{aligned}
 (1234)^2 &= 01522756 & (0152)^2 &= 00032104 & (5227)^2 &= 27321529 \\
 0.1479 + 0.0776 &= 0.2255 & (1479)^2 &= 02187441 & 0.1297 + 0.2269 &= 0.3566 \\
 \frac{-\ln(1-0.3215)}{5} &= 0.0776 & \frac{-\ln(1-0.5227)}{5} &= 0.1479 & \frac{-\ln(0.1523)}{5} &= 0.3764 \\
 \frac{-\ln(0.3215)}{5} &= 0.2269 & \frac{-\ln(0.5227)}{5} &= 0.1297 & f(t) &= e^{-t} \\
 \frac{-\ln(1-0.2756)}{5} &= 0.0645 & \frac{-\ln(0.2756)}{5} &= 0.2578 & F(t) &= 1 - e^{-t}
 \end{aligned}$$

Solutions:

- | | |
|-------------|---|
| Exponential | 1. probability dist'n of the time of arrival of first vehicle |
| Poisson | 2. probability dist'n of the number of vehicles arriving during the first minute |
| Exponential | 3. probability dist'n of the time between arrival of first and second vehicles |
| Geometric | 4. probability dist'n of the vehicle# of the first vehicle which is <i>not</i> a car. |
| Binomial | 5. probability dist'n of the number of cars among the first 10 vehicles to arrive |

- | | |
|---------------------------|--|
| <u>0.5227</u> | 6. The first uniformly distributed random number R_1 is? |
| <u>0.1479</u> (or 0.1297) | 7. The first simulated vehicle arrives at time? |
| <u>1-0.5227=0.4773</u> | 8. If F is the cumulative dist'n function of the interarrival time, $F(0.1479) = \dots?$ |
| <u>0.3215</u> | 9. The second uniformly distributed random number R_2 is? |
| <u>0.2255</u> (or 0.3566) | 10. The second simulated vehicle arrives how many minutes after the simulation begins? |

Note 1: The inverse transformation is found by solving $F(t)=R$ for t:

$$F(t) = 1 - e^{-t} = R \quad t = -\frac{\ln(1-R)}{5}$$

If R is uniformly distributed between 0 and 1, then 1-R is also. Substituting the uniformly distributed random number for either R or 1-R yields two possible values for t, hence the alternate answers in (8.) and (10.) above.

Note 2: To obtain the arrival time of the second car, add the two interarrival times: either 0.1479+0.0776 or 0.1297+0.2269, depending upon whether you substitute for R or 1-R in the inverse transformation.

Quiz #3

The time between arrivals of forty vehicles are measured. The number of observations O_i falling within each half-minute interval is shown in the table below. The average is computed by weighting the midpoint of each interval by its number of observations: $0.25 \times 9 + 0.75 \times 4 + 1.25 \times 5 + \dots = 2.225$ minutes. We wish to test the "goodness of fit" of the exponential distribution with mean 2.225 minutes.

i	Interval	O_i	p_i	E_i	$(E_i - O_i)^2 / E_i$
1	0.0-0.5	9	0.2015	8.0594	0.1098
2	0.5-1.0	4	0.1609	6.4355	0.9217
3	1.0-1.5	5	0.1285	5.1389	0.0038
4	1.5-2.0	3	0.1026	4.1035	0.2967
5	2.0-2.5	7	0.0819	3.2767	4.2308
6	2.5-3.0	3	0.0654	2.6165	0.0562
7	> 3.0	9	0.2592	10.3696	0.1809

The sum of the values in the last column is $D = 5.8$.

Indicate whether true or false, using "+" for true, "o" for false.

- ___ 1. The probability p_i that a car arrives in an interval $\#i$, $[t_1, t_2]$, is $F(t_2) - F(t_1)$
- ___ 2. The CDF of the distribution is $F(t) = P\{T \leq t\}$
- ___ 3. The quantity D is assumed to have the chi-square distribution.
- ___ 4. The chi-square distribution for this test will have 7 "degrees of freedom".
- ___ 5. The number of observations O_i in interval $\#i$ is a random variable with approximately Poisson distribution.
- ___ 6. The arrivals of the vehicles is assumed to form a Poisson process.
- ___ 7. The square of a $N(0,1)$ random variable has chi-square distribution.
- ___ 8. The chi-square distribution for this test will have 5 "degrees of freedom".
- ___ 9. The parameter of the exponential distribution is assumed to be $\lambda = 1/2.225 = 0.45$
- ___ 10. Given T has the exponential distribution with mean 2.225 minutes, the probability that D exceeds 5.8 is more than 10%.
- ___ 11. The quantity $(E_i - O_i)^2/E_i$ is assumed to have the normal $N(0,1)$ distribution.
- ___ 12. The exponential distribution with mean 2.225 minutes should not be rejected as a model for the interarrival times of the vehicles.
- ___ 13. The CDF of the distribution is assumed to be $F(t) = 1 - e^{-\lambda t}$
- ___ 14. The number of observations, O_i , in an interval should have the binomial distribution.
- ___ 15. The quantity E_i is a random variable with approximately Poisson distribution.
- ___ 16. The smaller the value of D , the worse the fit for the distribution being tested.
- ___ 17. The quantity E_i is the expected number of observations in interval $\#i$
- ___ 18. When running SLAM, you will see the "\$" (Aegis) prompt, rather than "%" (Unix).
- ___ 19. The quantity D is assumed to have approximately a Normal distribution.
- ___ 20. The degrees of freedom is reduced by 2 because (i) the total number of observations is fixed, and (ii) the data was used to estimate one parameter for the distribution being tested.

deg.of freedom	Chi-square Dist'n $P\{D \leq \chi^2\}$					
	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

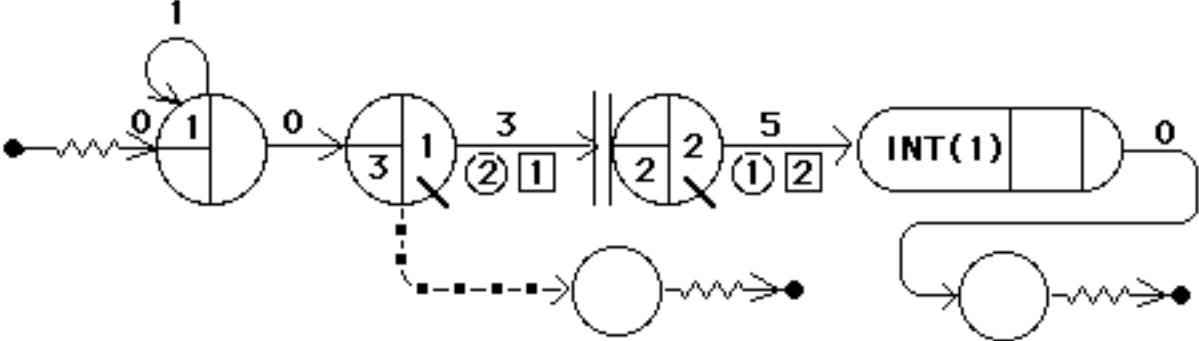
Solutions:

- True 1. The probability p_i that a car arrives in an interval $\#i$, $[t_1, t_2]$, is $F(t_2) - F(t_1)$
- False 2. The CDF of the distribution is $F(t) = P\{T \leq t\}$
The CDF is $F(t) = P\{T \leq t\}$
- True 3. The quantity D is assumed to have the chi-square distribution.
- False 4. The chi-square distribution for this test will have 7 "degrees of freedom".
It will have 5 degrees of freedom.
- False 5. The number of observations O_i in interval $\#i$ is a random variable with approximately Poisson distribution. **It has binomial dist'n, approximately Normal dist'n by Central Limit Theorem.**
- True 6. The arrivals of the vehicles is assumed to form a Poisson process.
- False 7. The square of a $N(0,1)$ random variable has chi-square distribution.
- True 8. The chi-square distribution for this test will have 5 "degrees of freedom".
- True 9. The parameter of the exponential distribution is assumed to be $\lambda = 1/2.225 = 0.45$
- True 10. Given T has the exponential distribution with mean 2.225 minutes, the probability that D exceeds 5.8 is more than 10%.
- False 11. The quantity $(E_i - O_i)^2/E_i$ is assumed to have the normal $N(0,1)$ distribution.

- True 12. The exponential distribution with mean 2.225 minutes should not be rejected as a model for the interarrival times of the vehicles.
- False 13. The CDF of the distribution is assumed to be $F(t) = 1 - e^{-t}$ **F(t) = 1 - e^{-t}**
- True 14. The number of observations, O_i, in an interval should have the binomial distribution.
- False 15. The quantity E_i is a random variable with approximately Poisson distribution. **E_i is not a random variable, but the expected value of a random variable.**
- False 16. The smaller the value of D, the worse the fit for the distribution being tested.
- True 17. The quantity E_i is the expected number of observations in interval #i
- True 18. When running SLAM, you will see the "\$" (Aegis) prompt, rather than "%" (Unix).
- False 19. The quantity D is assumed to have approximately a Normal distribution. **D has approximately a Chi-square distribution.**
- True 20. The degrees of freedom is reduced by 2 because (i) the total number of observations is fixed, and (ii) the data was used to estimate one parameter for the distribution being tested.

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #4** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Note that in the SLAM network below, all activity durations are constants (in minutes), and none random! The first entity is created at time=0.



- Before each statement, write the number indicated by (?).*
- 1. The maximum number of entities in the second queue is (?).
 - 2. The first entity to be created will depart the system after (?) minutes.
 - 3. The second entity to be created will be served by server #1 beginning at time=(?).
 - 4. At time=1 minute, the first queue will contain (?) entities.
 - 5. At time=2 minutes, the first queue will contain (?) entities.
 - 6. At time=3 minutes, the first queue will contain (?) entities.
 - 7. The first entity begins service #2 at time = (?).
 - 8. The second entity to be created will depart the system after (?) minutes.
- Before each statement, write "+" if true, "o" if false.*
- 9. When an entity arrives at the second queue and finds that it is full, the entity will be lost and a warning message will be displayed.
 - 10. The node following the second queue node will collect statistics on the time between arrivals (INT(1)).
 - 11. In this system, two entities can be receiving service at the same time in activity #1.
 - 12. When an entity arrives at the first queue and finds that it is full, the simulation will terminate.

Solutions:

- 1. The maximum number of entities in the second queue is 2.
- 2. The first entity to be created will depart the system after 8 minutes. *It will spend 3 minutes in activity 1 and 5 minutes in activity 2, with zero waiting time.*
- 3. The second entity to be created will be served by server #1 beginning at time= 1. *Since the circled 2 means that there are two servers, entity 1 does not need to wait, but begins service when it arrives.*

4. At time=1 minute, the first queue will contain 0 entities.
5. At time=2 minutes, the first queue will contain 1 entities. *The third entity to be created arrives at time=2, and must wait since service of the first entity is not yet complete.*
6. At time=3 minutes, the first queue will contain 1 entities. *At this time, service of the first entity is complete, and so entity #3 begins service and leaves the queue, just as the fourth entity arrives to take its place.*
7. The first entity begins service #2 at time = 3.
8. The second entity to be created will depart the system after 13 minutes. *The first entity finishes activity 2 at time=8, at which time the second entity can begin. The duration is 5, so it will be complete at time=13.*

◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷-◁•▷ **Quiz #5** ▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷•◁-▷

Statements below refer to today's homework assignment (HW#5).

Indicate "+" for true, "o" for false:

- 1. We assumed in this HW that the number of failures at time t , $N_f(t)$, has a Weibull distribution.
- 2. If the assumption of Weibull distribution were correct, a plot of the data on the graph should be approximately on a straight line.
- 3. The Cricket Graph program fits a line which minimizes the sum of the errors, i.e., the vertical distance between each data point and the line.
- 4. The quantity R_t is the fraction of the bulbs which have failed at time t (or earlier).
- 5. The Weibull CDF, i.e., $F(t)$, gives, for each bulb, the probability that it has failed at or before time t .
- 6. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.
- 7. According to the results of this homework exercise, the failure rate of the light bulbs is decreasing rather than increasing.
- 8. A positive value of k indicates an increasing failure rate, and negative k indicates a decreasing failure rate.
- 9. The method used in the HW#5 to estimate the Weibull parameters u & k does not require that the bulbs be tested until all have failed.
- 10. Given a coefficient of variation for the Weibull distribution (the ratio $\frac{\sigma}{\mu}$), the parameter k can be determined.
- 11. If 10 bulbs are installed in an office's light fixtures, the number still functioning after 100 days has a Weibull distribution.
- 12. If the failure rate is increasing, it may be more appropriate to use the Gumbel distribution than the Weibull.

Select the letter below which indicates each correct answer:

- 13. The label on the vertical axis should be ...
- 14. The label on the horizontal axis should be ...
- 15. The slope of the line fit by Cricket Graph should be approximately ...
- 16. The vertical intercept of the line fit by Cricker Graph should be approximately ...

- | | | |
|------------------|--------------------|------------------------|
| a. t | g. R_t | m. shape parameter k |
| b. $\ln t$ | h. $\ln R_t$ | n. scale parameter u |
| c. $\ln 1/t$ | i. $\ln 1/R_t$ | o. mean value μ |
| d. $\ln \ln t$ | j. $\ln \ln R_t$ | p. standard deviation |
| e. $\ln \ln 1/t$ | k. $\ln \ln 1/R_t$ | q. $\ln u$ |
| f. $k \ln u$ | l. $u \ln k$ | r. $\ln k$ |

Solutions:

1. We assumed in this HW that the number of failures at time t , $N_f(t)$, has a Weibull distribution. *The failure time of each bulb is assumed to have Weibull distribution.*
2. If the assumption of Weibull distribution were correct, a plot of the data on the graph should be approximately on a straight line.
3. The Cricket Graph program fits a line which minimizes the sum of the errors, i.e., the vertical distance between each data point and the line. *The sum of the squares of the errors is minimized!*
4. The quantity R_t is the fraction of the bulbs which have failed at time t (or earlier). *It is the fraction which have survived.*
5. The Weibull CDF, i.e., $F(t)$, gives, for each bulb, the probability that it has failed at or before time t .
6. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.
7. According to the results of this homework exercise, the failure rate of the light bulbs is decreasing rather than increasing.
8. A positive value of k indicates an increasing failure rate, and negative k indicates a decreasing failure rate. *Parameter k is always >0 . If $k < 1$, the failure rate is decreasing.*
9. The method used in the HW#5 to estimate the Weibull parameters u & k does not require that the bulbs be tested until all have failed.
10. Given a coefficient of variation for the Weibull distribution (the ratio $\frac{\sigma}{\mu}$), the parameter k can be determined.
11. If 10 bulbs are installed in an office's light fixtures, the number still functioning after 100 days has a Weibull distribution. *It should have binomial distribution, with $n=10$ and $p=1-F(100)=R(100)$.*
12. If the failure rate is increasing, it may be more appropriate to use the Gumbel distribution than the Weibull.

Select the letter below which indicates each correct answer:

- k 13. The label on the vertical axis should be ...
- b 14. The label on the horizontal axis should be ...
- m 15. The slope of the line fit by Cricket Graph should be approximately ...
- f 16. The vertical intercept of the line fit by Cricker Graph should be approximately ...

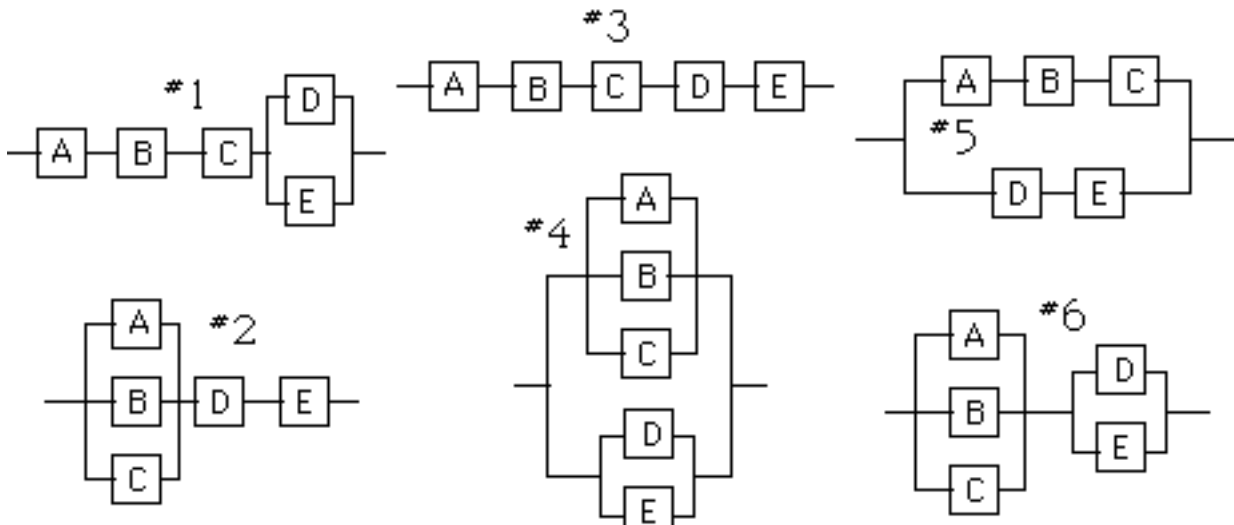
<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #6** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Part One: A system consists of five components (A,B,C,D, &E). The probability that each component fails during the first year of operation is 10% for A, B, and C, and 20% for D and E. For each alternative of (a) through (e), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)

Diagram Reliability

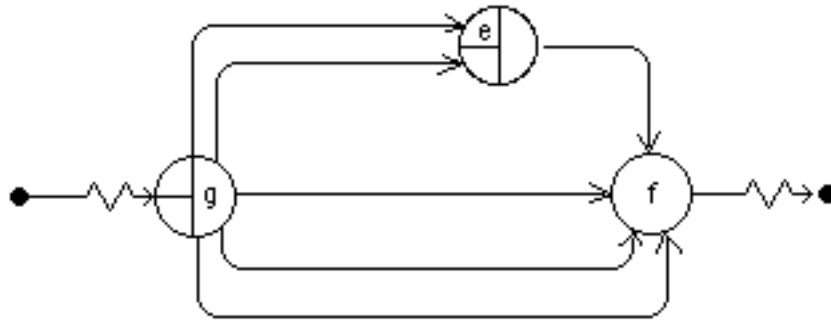
- ___ ___ a. The system can function if A, B, and C all function or if both D and E function.
- ___ ___ b. The system requires that D & E both function, and at least one of A, B, & C.
- ___ ___ c. The system requires at least one of A, B, & C, and at least one of D & E.
- ___ ___ d. The system requires all of A, B, & C, and at least one of D & E.



Reliabilities:

- | | | |
|-----------------------------|-------------------------|---------------------------------|
| 1. $1 - (0.1)^3(0.2)^2$ | 2. $(0.9)^3[1-(0.2)^2]$ | 3. $[1-(0.1)^3](0.8)^2$ |
| 4. $[1-(0.1)^3][1-(0.2)^2]$ | 5. $(0.9)^3(0.8)^2$ | 6. $1 - [1-(0.9)^3][1-(0.2)^2]$ |
| 7. None of the above | | |

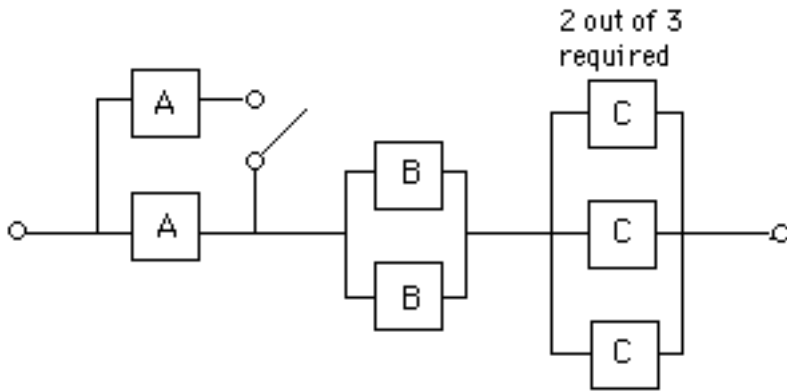
Part Two. For the SLAM network below, indicate the appropriate values for the parameters e, f, and g, in order to simulate the system (b) above, "The system requires all of A, B, & C, and at least one of D & E."



e. _____ f. _____ g. _____

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #7** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Part One: A system has 3 types of components (A, B, & C) which are subject to failure. As shown below, the system requires at least one of components A and B, and at least two of component C. One of component A is in stand-by. When the first component A has failed, the second is to be switched on. Assume that the sensor/switch has 99% reliability.



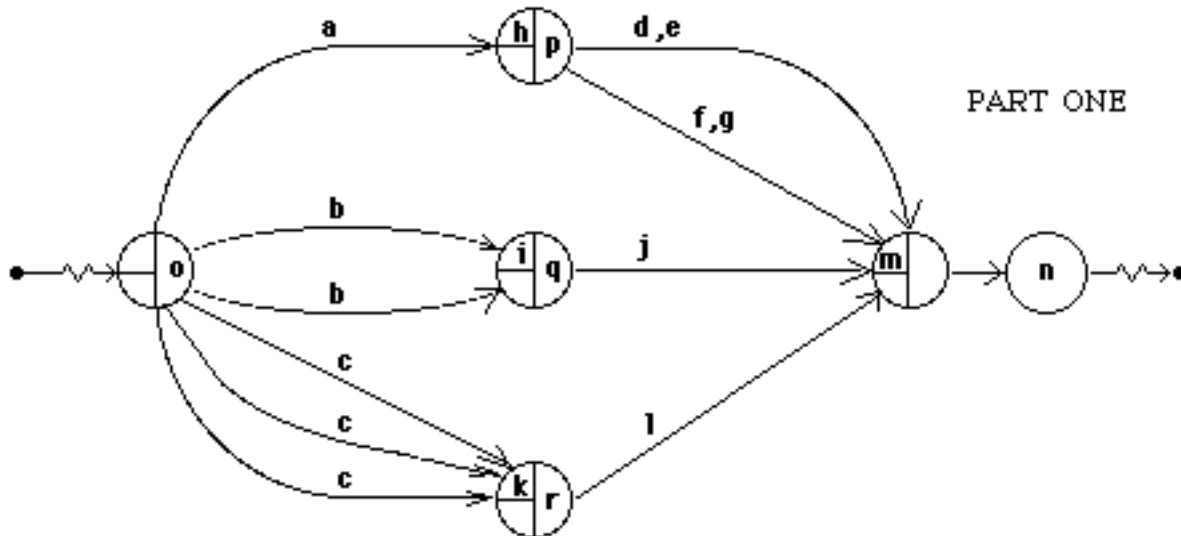
The lifetime distributions of the three component types are:

Component A: Normal with expected lifetime 100 days, and standard deviation 25 days.

Component B: Exponential, with expected lifetime 150 days.

Component C: Exponential, with expected lifetime 200 days.

Indicate the values (a through r) on the SLAM network below which will simulate this system.

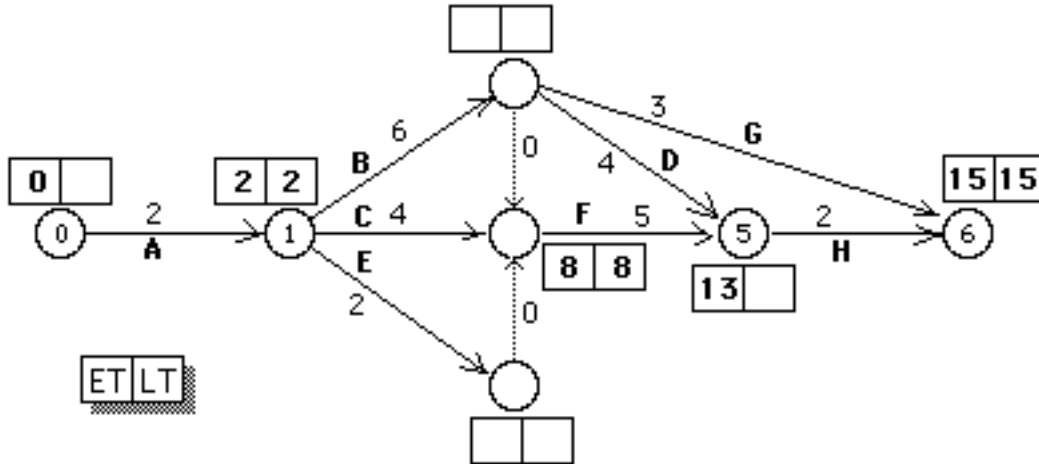


Part Two: Suppose that in the system above, the spare unit of component A is in "warm standby", i.e., it is possible that the spare can fail while in standby, while the primary unit of component A is in use. The lifetime of the spare has normal distribution with mean 200 days and standard deviation 40 days. Again, the sensor/switch, which senses that the primary unit of A has failed and switches on the spare, has 99% reliability.

Indicate the values (s through z, , , , and) on the SLAM network below which will simulate this system.

G	Paint & finish work on building	B	3	1
H	Facility test & checkout	D,F	2	1

1. Three nodes in the AOA network below are not labeled. Label them.



- Complete the computation of the earliest & latest expected times for the events (indicated in the boxes ABOVE). *There are six values to be computed!*
- If each duration is its expected value, indicate whether activities D & F are critical, and for activity G, compute:

ES = earliest start time LS = latest start time
 EF = earliest finish time LF = latest finish time
 TF = total float (slack)

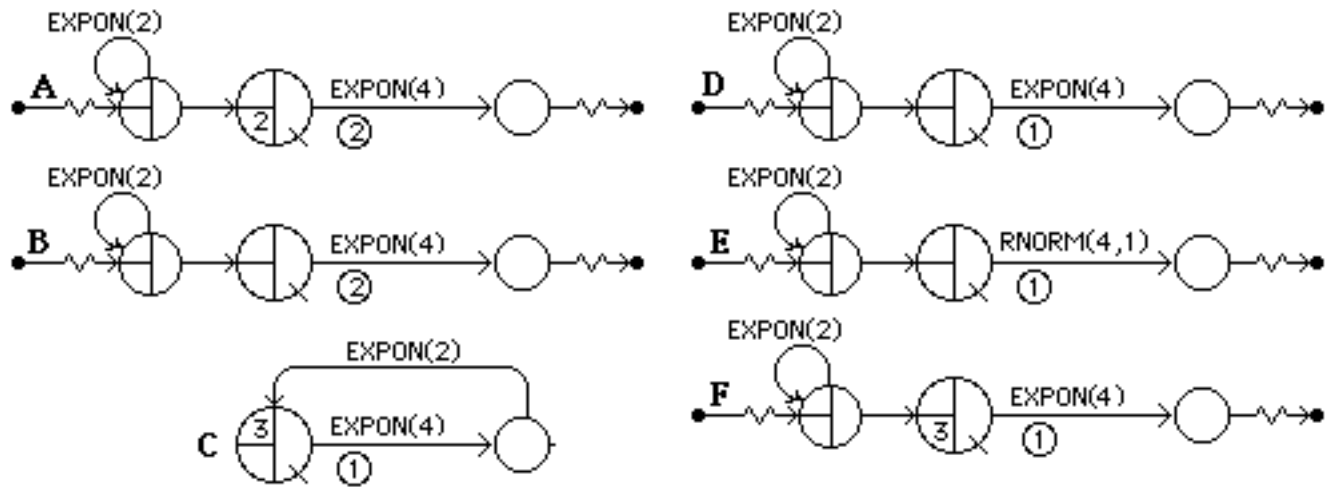
Activity	Duration	ES	LS	EF	LF	TF	Critical?
A	2	0	0	2	2	0	Yes
B	6	2	2	8	8	0	Yes
C	4	2	4	6	8	2	No
D	2	8	9	12	13	1	—
E	4	2	6	4	8	4	No
F	5	8	8	13	13	0	—
G	3	—	—	—	—	—	No
H	2	13	13	15	15	0	Yes

- What is the expected completion time for the project? _____
- Under the assumptions of PERT, what is...
 the standard deviation of the completion time? _____
 the probability distribution of the completion time?
 (circle one: Exponential Triangular Beta Normal Gamma Weibull)

◁•>-<•>-<•>-<•>-<•>-<•>-<•>-<•> **Quiz #10** <•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Match SLAM diagram (A through F) & Queue Classification:

- | | |
|---------------|-----------------|
| _____ M/M/1 | _____ M/M/1/4/4 |
| _____ M/M/1/4 | _____ M/M/2 |
| _____ M/G/1 | _____ M/M/2/4 |



Indicate standard notation:

- | | | | |
|-------|-----------------------------|-------|-----------------------|
| _____ | Probability system is empty | _____ | Utilization |
| _____ | Average length of queue | _____ | Expected service time |
| _____ | Arrival rate | _____ | Service rate |
| _____ | Average time in system | _____ | Average time in queue |

- | | | |
|------------|--------------|------------|
| (G.) L | (H.) L_q | (I.) |
| (J.) M | (K.) $1/\mu$ | (L.) W |
| (M.) W_q | (N.) | (O.) $1/$ |
| (P.) N | (Q.) 0 | (R.) μ |

