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57:022 Principles of Design II
Quizzes Spring '92
<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #1**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Write the number corresponding to the correct probability distribution in each blank below. Note that some distributions may apply in more than one case, while others not at all!

Cars arrive at a toll booth on the freeway at the average rate of 8/minute in a completely random fashion. The arrival times are recorded, beginning at 8:00 am. *What probability distribution does each of the following random variables have?*

___ a. time of arrival of first car
___ b. time of arrival of car #2
___ c. time between arrival of car #1 and car #2
___ d. number of cars arriving during the first minute (between 8:00 and 8:01)

Under normal circumstances, 5% of the items produced by a certain process are defective. All items are routinely inspected as soon as they are produced. *Which probability distributions would best be used to compute the probability that...*

___ e. exactly one of the first ten items is defective
___ f. the first and second item are both defective
___ g. the second defective item is item #10
___ h. two or fewer defective items are found among the first 20 inspected
___ i. the fifth item is the first to be found defective

Probability distributions:

1. Bernouilli
2. Binomial
3. Poisson
4. Erlang
5. Geometric
6. Exponential
7. Pascal
8. Normal

Solutions:

Exponential a. time of arrival of first car
Erlang b. time of arrival of car #2
Exponential c. time between arrival of car #1 and car #2
Poisson d. number of cars arriving during the first minute (between 8:00 and 8:01)
Binomial e. exactly one of the first ten items is defective
Binomial f. the first and second item are both defective
Pascal g. the second defective item is item #10
Binomial h. two or fewer defective items are found among the first 20 inspected
Geometric i. the fifth item is the first to be found defective

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #2**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

For each of the random variables below, write the number corresponding to the correct probability distribution . Note that some distributions may apply in more than one case, while others not at all! Assume that arrivals of vehicles form a Poisson process.

_____ a. the number of cars passing through an intersection during a 1-minute green light.
_____ b. the number of left-handed students in a class of 20.
_____ c. the strength of a 10-foot steel chain

_____ d. the time until the arrival of the third car at an intersection during a red light
_____ e. the total weight of a group of persons on an elevator, when loaded to its capacity of 18 persons
_____ f. the weight of the heaviest person on an elevator, when loaded to its capacity of 18 persons
_____ g. the time you must wait for a bus after arriving at the bus stop
_____ h. the lifetime of an electronic device with several dozen components which might fail (each necessary for the device to function)
_____ i. age of the oldest alumnus of the University of Iowa
_____ j. number of defective items found when testing a batch of 12.
_____ k. the result of tossing a single coin
_____ l. the number of items produced in order to obtain 5 acceptable items, if each is tested before producing the next
_____ m. the magnitude of the highest rate of flow into the Coralville Reservoir next year

<u>Probability distributions:</u>

1. Bernouilli	2. Geometric
3. Binomial	4. Exponential
5. Poisson	6. Pascal (negative binomial)
7. Erlang (Gamma) with k>1	8. Normal
9. Gumbel	10. Uniform
11. Weibull	12. Chi-square

Solutions:

Poisson:	the number of cars passing through an intersection during a 1-minute green light.
Binomial:	the number of left-handed students in a class of 20.
Weibull:	the strength of a 10-foot steel chain
Erlang:	the time until the arrival of the third car at an intersection during a red light
Normal:	the total weight of a group of persons on an elevator, when loaded to its capacity of 18 persons
Gumbel:	the weight of the heaviest person on an elevator, when loaded to its capacity of 18 persons
Exponential:	the time you must wait for a bus after arriving at the bus stop
Weibull:	the lifetime of an electronic device with several dozen components which might fail (each necessary for the device to function)
Gumbel:	age of the oldest alumnus of the University of Iowa
Binomial:	number of defective items found when testing a batch of 12.
Bernouilli:	the result of tossing a single coin
Pascal:	the number of items produced in order to obtain 5 acceptable items, if each is tested before producing the next
Gumbel:	the magnitude of the highest rate of flow into the Coralville Reservoir next year

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #3**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Fill each of the five blanks with a letter from the list at the bottom of the page.

Suppose that you have turned on simultaneously 100 light bulbs, and at the beginning of each day for the next five days (t=1,2,3,4,5) observed the number of bulbs still burning (N_1,N_2,N_3, N_4, N_5, respectively). You then calculate the "reliability", which is Rt= $N_t/_{100}$. If the lifetime of a bulb has a Weibull probability distribution, then the five data points should lie on a straight line if you plot them on "Weibull probability paper", which is equivalent to plotting, on normal graph paper:

on the vertical axis: y=______
on the horizontal axis: x=_____

When you do this, the slope of the line is ______ , the x-intercept is _______, and the y-intercept is ________.

a. t
b. ln t
c. ln $^1/_t$
d. ln ln t
e. ln ln $^1/_t$
f. k ln u

g. R_t
h. ln R_t
i. ln $^1/_{R_t}$
j. ln ln R_t
k. ln ln $^1/_{R_t}$
l. u ln k

m. shape parameter k
n. scale parameter u
o. mean value m
p. standard deviation s
q. ln u
r. ln k

Solution:

Suppose that you have turned on simultaneously 100 light bulbs, and at the beginning of each day for the next five days (t=1,2,3,4,5) observed the number of bulbs still burning (N_1,N_2,N_3, N_4, N_5, respectively). You then calculate the "reliability", which is Rt= $N_t/_{100}$. If the lifetime of a bulb has a Weibull probability distribution, then the five data points should lie on a straight line if you plot them on "Weibull probability paper", which is equivalent to plotting, on normal graph paper:

on the vertical axis: y= **ln ln $^1/_{R_t}$**

on the horizontal axis: x=**ln t**

When you do this, the slope of the line is **k** ,

the x-intercept is **u**,

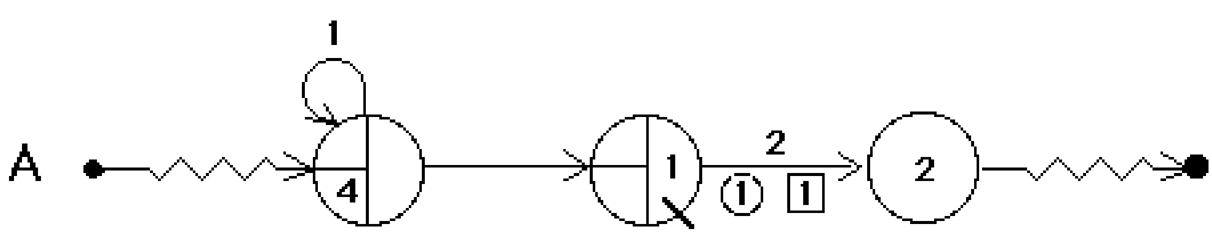
and the y-intercept is **k ln u**.

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #4**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

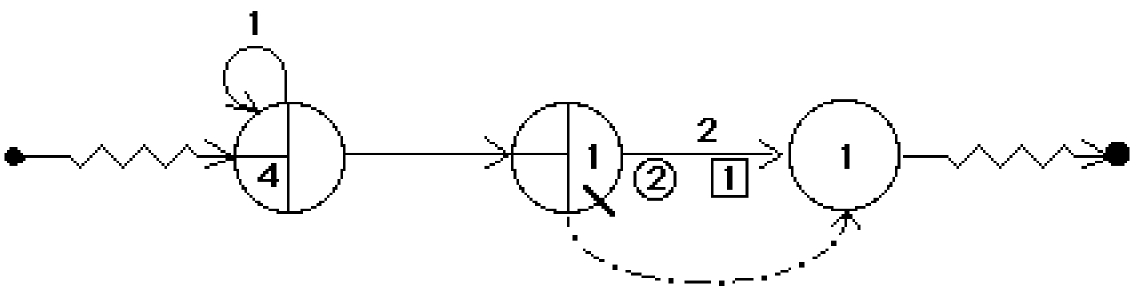
For each SLAM network below, state

- the time at which the first entity leaves the system,
- the time at which the simulation ends (or the last entity leaves the system, whichever is first), and
- the number of entities which have left the system when the simulation ends.

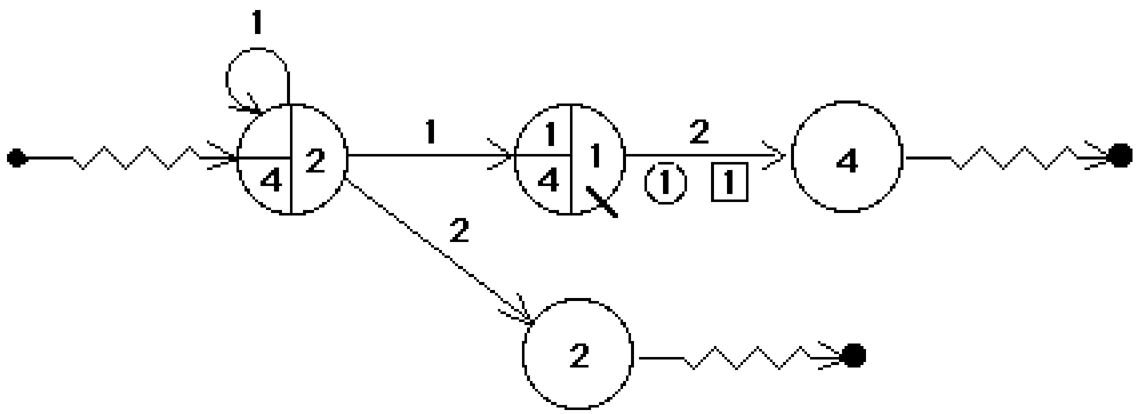
Note that all activity durations are constants, and none random!



B



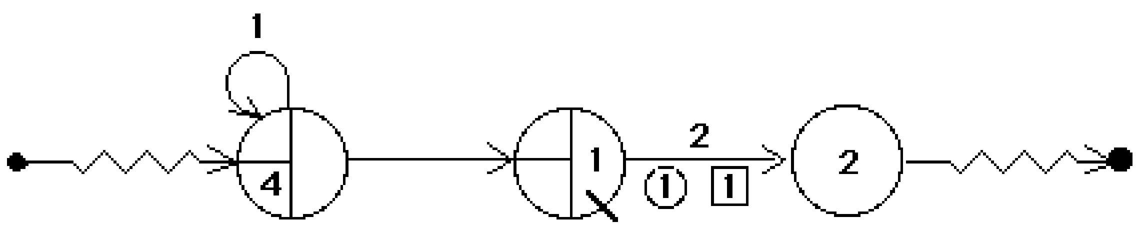
C



Network	Time first entity leaves	Time Simulation Ends	Number of entities which leave system
A	___________	___________	___________
B	___________	___________	___________
C	___________	___________	___________

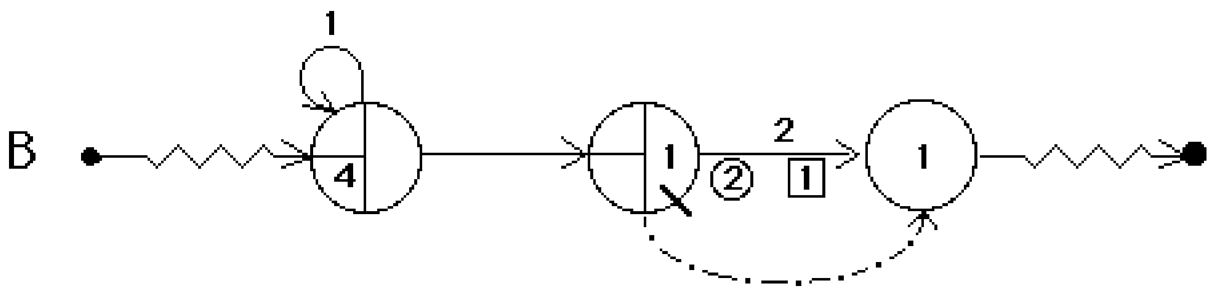
Solutions:

A



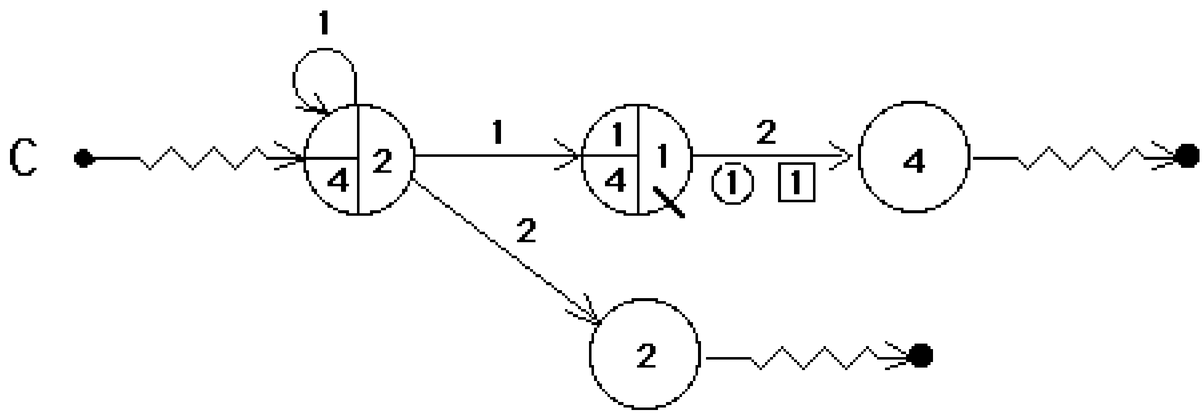
Sequence of events in network A:

Time	Event
0	Entity #1 created & enters queue
	Service begins on entity #1
1	Entity #2 created & enters queue
2	Entity #1 service completed, leaves system
	Service begins on entity #2
	Entity #3 created & enters queue
3	Entity #4 created & enters queue
4	Entity #2 service completed, leaves system
	Entity #5 created & enters queue
	Service begins on entity #3



Sequence of events in network B:

Time	Event
0	Entity #1 created & enters queue
	Service begins on entity #1
1	Entity #2 created & enters queue
2	Service begins on entity #2
	Entity #3 created & enters queue
	Entity #1 service completed, leaves system
	(which triggers termination of simulation)



Note that, since the queue initially contains one entity (#2), the server is busy with an entity (#1). Note also that when an entity is created, it leaves the create node by *both branches*.
Sequence of events in network C:

Time	Event
0	Entity #3 created
	(One copy begins each activity leaving create node)
	Service begins on entity #1
1	Entity #3 arrives in queue (length now 2)
	Entity #4 created
	(One copy begins each activity leaving create node)
2	Entity #1 service completed, leaves system
	Service begins on entity #2
	Entity #3 arrives at terminate node & leaves system
	Entity #4 arrives in queue (length still 2)
	Entity #5 created
	(One copy begins each activity leaving create node)
3	Entity #5 arrives in queue (length now 3)
	Entity #4 arrives at terminate node & leaves system
	(which triggers termination of simulation)

Network	Time first entity leaves	Time Simulation Ends	Number of entities which leave system
A	2	4	2
B	2	2	1
C	2	3	3

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #5**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Indicate whether true or false below:

___ 1. The earliest start time for an activity depends on the earliest finish time for the project.

___ 2. The critical path of a project is the longest path from beginning to end.

___ 3. In PERT, the project duration is assumed to be a random variable with a BETA distribution.

___ 4. In PERT, the duration of an activity is assumed to be a random variable with a BETA distribution.

___ 5. A SLAM network model of a project uses the activity-on-node representation.

___ 6. All activities on a critical path have their latest finish times equal to their early start times.

___ 7. In CPM (with the A-O-A network model) the earliest time for a node is the maximum of the early finish times of activities entering the node.

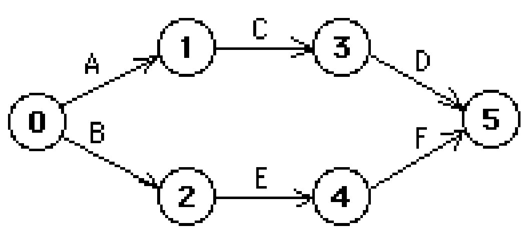
___ 8. In PERT, the calculation that the critical path will be completed by time T assumes that activity times are independent random variables.

___ 9. In PERT, the minimum completion time of the project is assumed to be a random variable with a Weibull distribution.

___ 10. Consider the project consisting of the following activities:

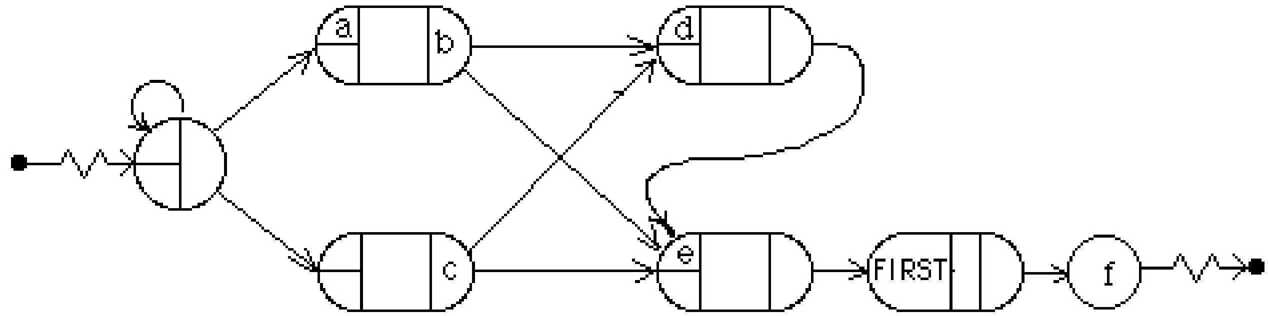
Activity	Immediate Predecessor(s)	Duration (days)
A	none	3
B	none	5
C	A	2
D	B,C	4
E	B	1
F	E	2

What arrow, if any, is missing from the network below?



a. From node 1 to node 4
b. From node 2 to node 3
c. From node 3 to node 4
d. None of the above

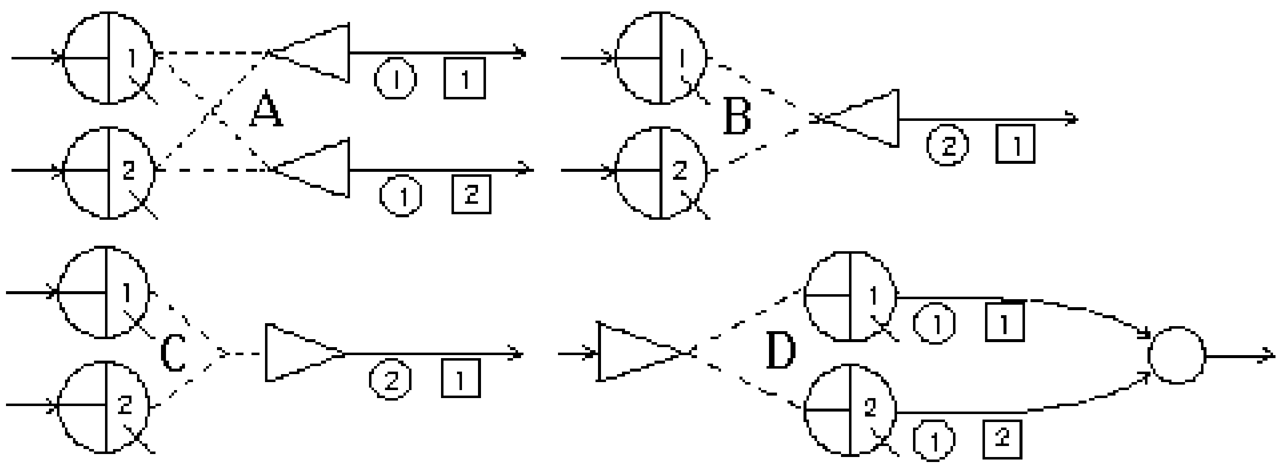
11. What are the correct numerical values (0,1,2,3,... ∞) of the parameters **a**, **b**, **c**, **d**, **e**, & **f** in the SLAM network model below? (Note that some other parameters are missing... do not assume that the default values are acceptable!)

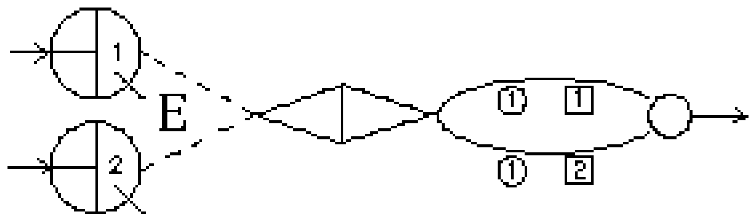


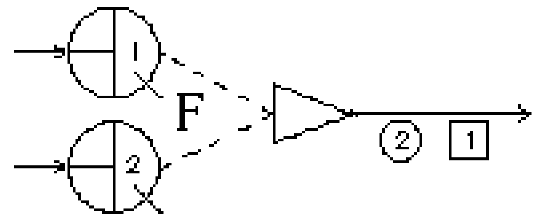
<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #6**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

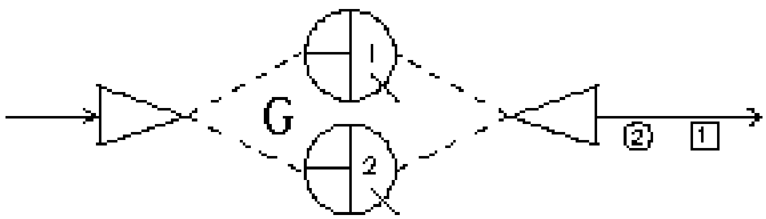
For each system described below, indicate the appropriate SLAM network setment (A through H) which might best model it. If no network segment shown could be used, indicate "N" (none).

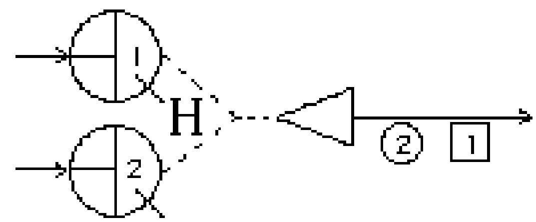
___ 1. Customers select the check-out lane at the grocery store which has the shortest queue.
___ 2. Two (identical) servers select their next job from the longer of two queues.
___ 3. Widgets and boxes arrive on two conveyors at the final station on an assembly line. Two (identical) workers have the task of selecting a widget and a box, packing it, and sealing the box.
___ 4. Two workers, who differ in the speed with which they work, select their next job from queue #1 if any wait there, and queue #2 otherwise.
___ 5. Arriving customers select the waiting area (of two available) with the lesser number of people already waiting. Two non-identical clerks serve the person who has waited the longest.
___ 6. Customers arrive at two queues to wait for service by either of two clerks. If both clerks are idle, customers prefer clerk #1. If a clerk finishes serving a customer and both queues have persons waiting, he selects the customer at the head of the longest queue.











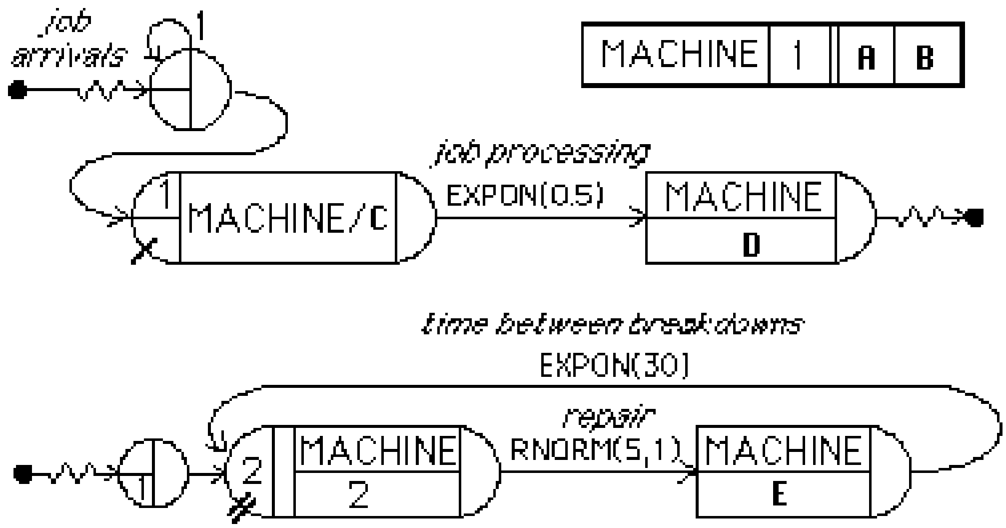
Solutions:

1. **D:** Customers select the check-out lane at the grocery store which has the shortest queue.
2. **B:** Two (identical) servers select their next job from the longer of two queues.
3. **H:** Widgets and boxes arrive on two conveyors at the final station on an assembly line. Two (identical) workers have the task of selecting a widget and a box, packing it, and sealing the box.
4. **A:** Two workers, who differ in the speed with which they work, select their next job from queue #1 if any wait there, and queue #2 otherwise.
5. **D** (or **N**, if the clerks may select from either waiting area) Arriving customers select the waiting area (of two available) with the lesser number of people already waiting. Two non-identical clerks serve the person who has waited the longest.
6. **E:** Customers arrive at two queues to wait for service by either of two clerks. If both clerks are idle, customers prefer clerk #1. If a clerk finishes serving a customer and both queues have persons waiting, he selects the customer at the head of the longest queue.

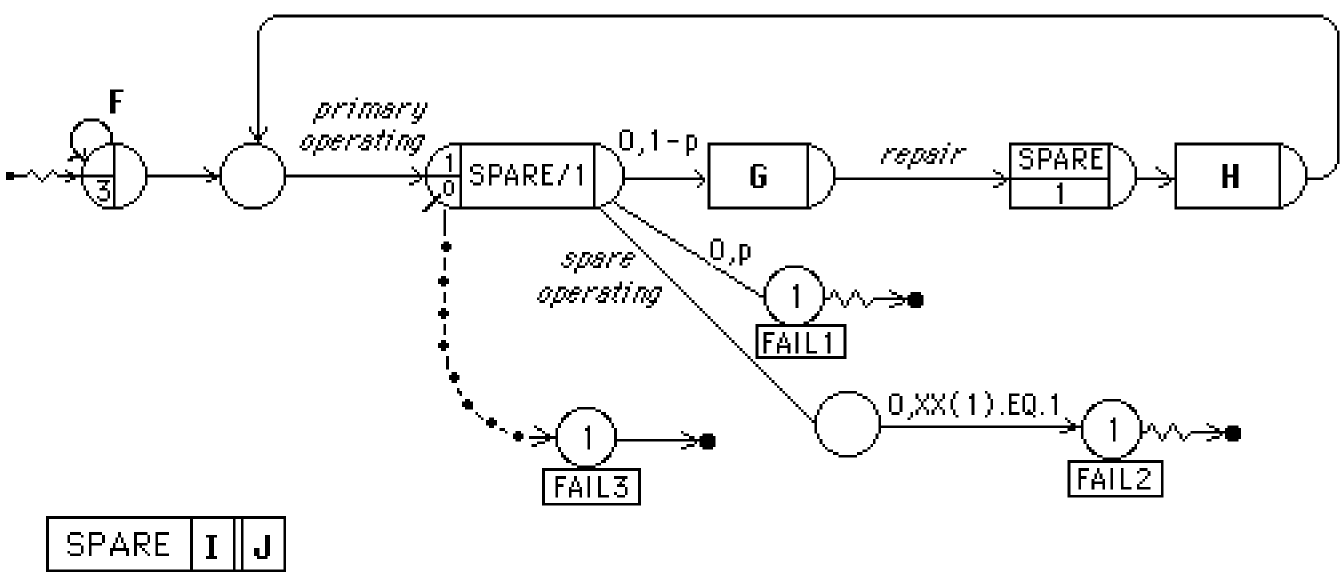
<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #7**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Below are SLAM network models for 3 examples which were described in the class (and on the HYPERCARD stacks). The values of some parameters are missing (designated by **A** through **U**, excluding letter **O**). Give appropriate numerical values of each parameter (0, 1, 2, 3, ...) on the attached answer sheet.

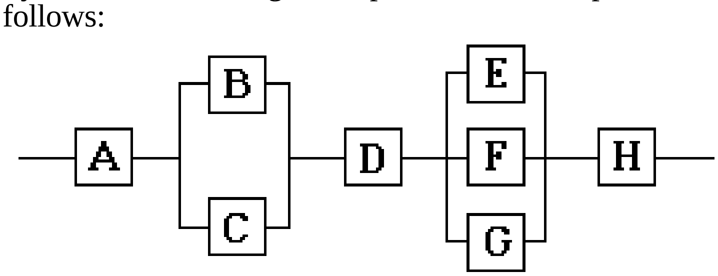
Network #1: Jobs arrive and are processed, one at a time, on a machine which is subject to breakdown. When a breakdown occurs, job processing is interrupted while the machine is repaired.



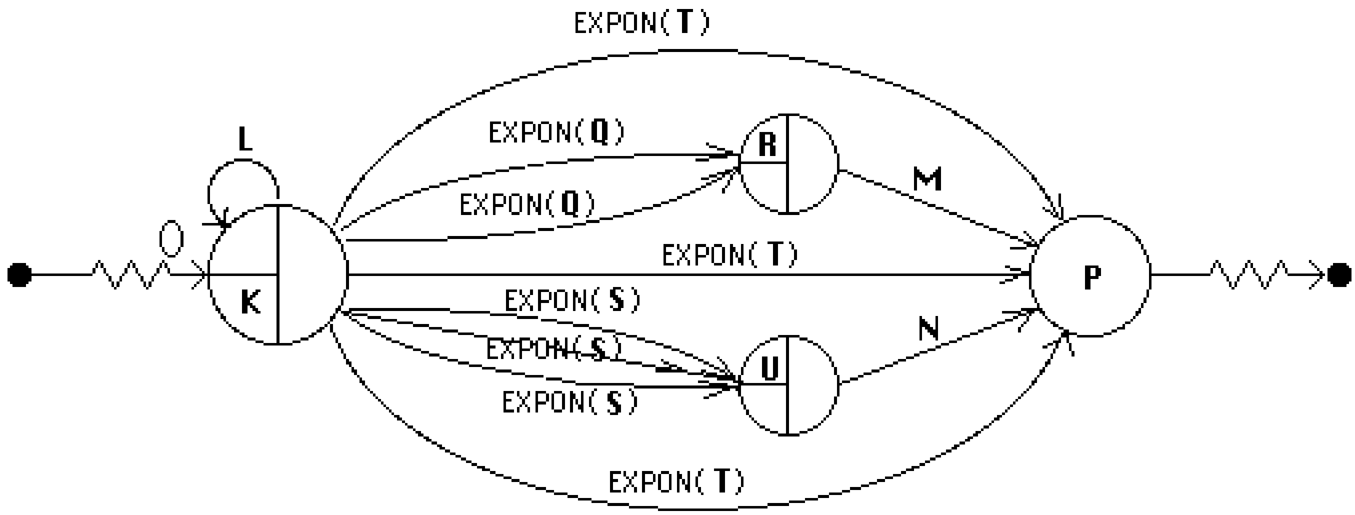
Network #2: A power plant has three primary generators. When one fails, a spare generator is switched on while the primary generator is being repaired. (The switch fails to function with probability p, in which case the system fails.) If either the spare generator or a second primary generator fails before the first is repaired, the system also fails.



Network #3: A system consists of eight components, in a complex series/parallel arrangement as follows:



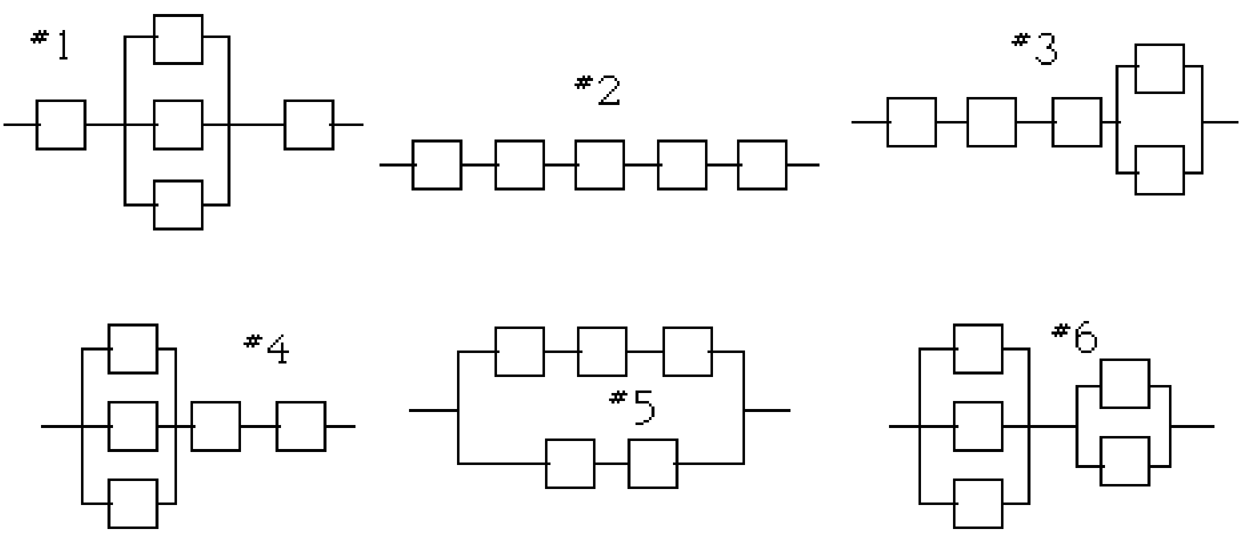
The components have lifetimes which are exponentially distributed with the following expected values: A, D, & H: 300 days; B&C: 200 days; E,F,&G: 100 days. The system is simulated in order to estimate its failure time.



<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #8**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Part One: A system consists of five components (A,B,C,D, &E). The system can function if either of components D or E (or both) are functioning, but fails if any of components A, B, or C fail. The probability that each component fails during the first year of operation is 10% for A, B, and C, and 20% for D and E.

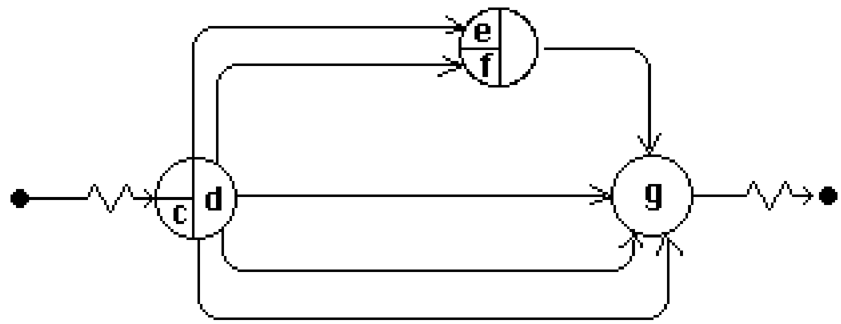
a. Which of the following diagrams represents this system?



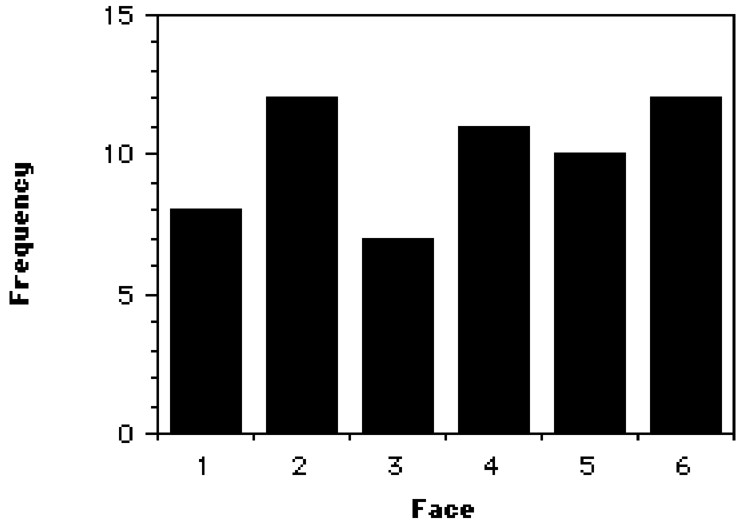
b. Which of the following expressions will give the reliability of the system?

1. $(0.9)^3(0.2)^2$ 2. $(0.9)^3[1-(0.2)^2]$ 3. $(1 - (0.1)^3(0.8)^2$
4. $(0.9)^3(0.8)^2$ 5. $[1-(0.9)^3](0.8)^2$ 6. $[1-(0.1)^3](0.8)^2$

In the SLAM network below, what are the appropriate values for the parameters c, d, e, f, and g, in order to simulate this system?


c
d
e
f
g

Part Two: An ordinary die (cube) is tossed sixty times, and the number of times that each face appears is recorded.



h. What is the name of the probability distribution of the number of observations (O_i) in cell #i?

1. Normal	2. Poisson	3. Binomial
4. Weibull	5. Chi-square	6. None of these

i. What is the expected value of the random variable O_i, rounded to the nearest integer?

j. What is the standard deviation of O_i, rounded to the nearest integer?

k. Complete the table below and write $D = \sum_{i=1}^{6} D_i$, rounded to the nearest integer, on the answer sheet.

i	Ei	Oi	Di
1	___	8	___
2	___	12	___
3	___	7	___
4	___	11	___
5	___	10	___
6	___	12	___

l. What is the name of the probability distribution of D?

1. Normal	2. Poisson	3. Binomial
4. Weibull	5. Chi-square	6. None of these

m. According to the table on the answer sheet, what is the probability that if the experiment were repeated, a value at least as large as this D would be observed?

n. Should this die be considered "loaded", i.e., unfair?

$P\{D > \chi^2\}$

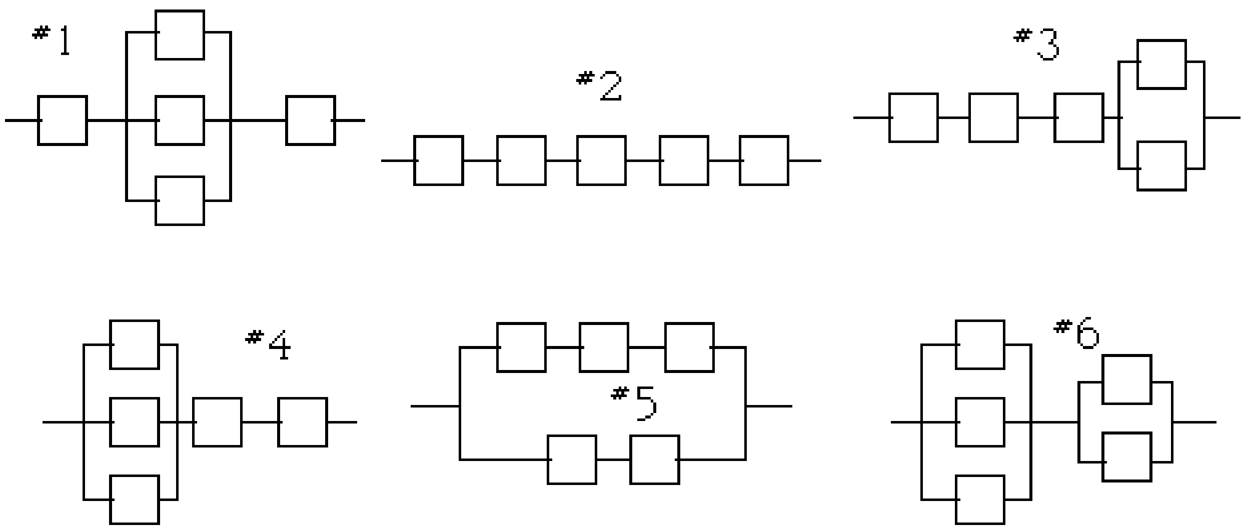
degrees of freedom	α=95%	α=90%	α=10%	α=5%	α=1%
1	.004	.0158	2.71	3.84	6.63
2	.103	.211	4.61	5.99	9.21

3	.352	.584	6.25	7.81	11.3
4	.711	1.06	7.78	9.49	13.3
5	1.15	1.61	9.24	11.1	15.1
6	1.64	2.20	10.6	12.6	16.8

Solutions:

Part One: A system consists of five components (A,B,C,D, &E). The system can function if either of components D or E (or both) are functioning, but fails if any of components A, B, or C fail. The probability that each component fails during the first year of operation is 10% for A, B, and C, and 20% for D and E.

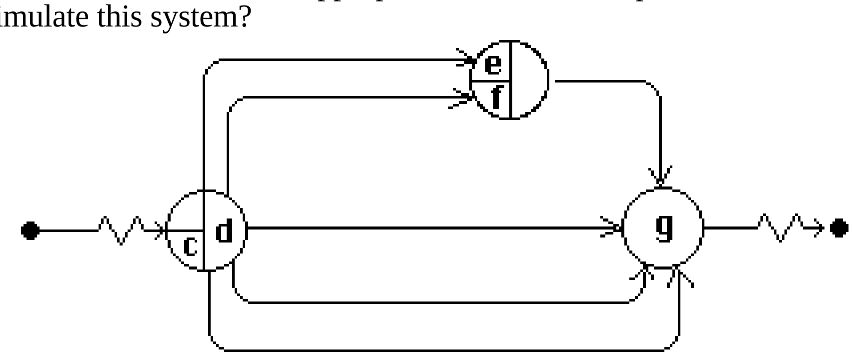
a. Which of the following diagrams represents this system? __**#3**__



b. Which of the following expressions will give the reliability of the system? __**#2**__

1. $(0.9)^3(0.2)^2$
2. $(0.9)^3[1-(0.2)^2]$
3. $(1 - (0.1)^3(0.8)^2$
4. $(0.9)^3(0.8)^2$
5. $[1-(0.9)^3](0.8)^2$
6. $[1-(0.1)^3](0.8)^2$

In the SLAM network below, what are the appropriate values for the parameters c, d, e, f, and g, in order to simulate this system?

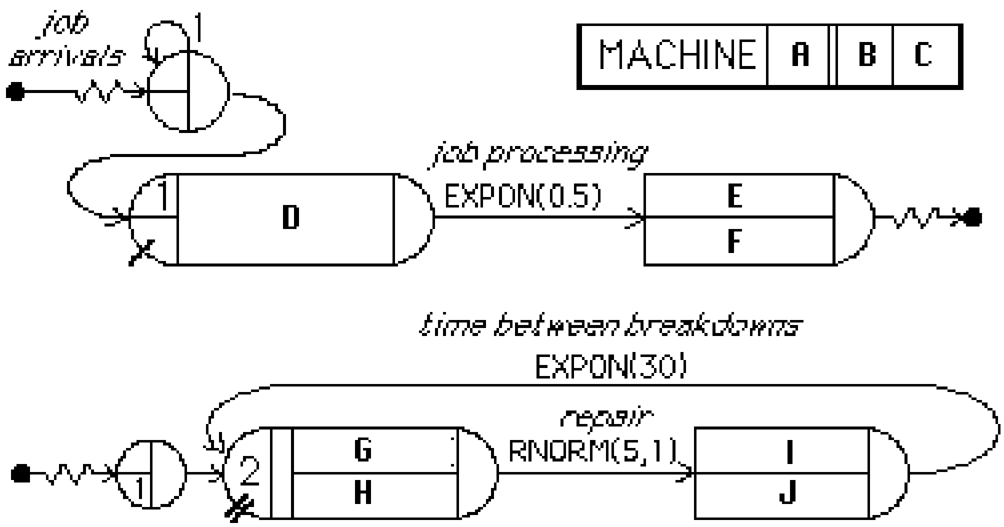


c=1, d=5, e=2, f is arbitrary (since no more than 2 entities can arrive at this node), g=1.

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #9**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Below are two examples which were discussed in class (& in the Hypercard Stacks). In each case, write the values of the parameters in the networks represented by **A** through **J.**

Part One: Machine Subject to Breakdowns
Jobs arrive at a machine, which is subject to breakdown. When breakdown occurs, the machine must be repaired before processing of the jobs may continue.



Part Two: Queue with Intermittent Service

Jobs arrive at a machine in a manufacturing facility, but an operator begins processing them only when 4 jobs have accumulated. When 4 jobs have accumulated, he begins operating the machine. After no more jobs are waiting, he then leaves the machine until 4 more jobs have accumulated.