Name ________________________

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57:022 Principles of Design II

Quiz #1, January 26, 2000

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Indicate with true (+) or false (o):

___ 1. A random number with *Pascal* distribution is the sum of random variables each having the *geometric* distribution.

___ 2. The *binomial* distribution is a special case of the Pascal distribution.

___ 3. In a Bernouilli process, the number of "successes" in n trials (N_n) has the *Poisson* distribution.

___ 4. If W_1 has the *geometric* distribution, then

$$P\{W_1=1\} \geq P\{W_1=2\} \geq P\{W_1=2\} \geq \ldots$$

••

For some of the questions which follow, you may refer to the table below. Only 3 significant digits are needed.

Binomial Cumulative Distribution Function (n= 10, p= 0.2)

x	P{x}	P{X ≤ x}	P{X > x}
0	0.10737418	0.10737418	0.89262582
1	0.26843546	0.37580964	0.62419036
2	0.30198989	0.67779953	0.32220047
3	0.20132659	0.87912612	0.12087388
4	0.08808038	0.96720650	0.03279350
5	0.02642412	0.99363062	0.00636938
6	0.00550502	0.99913564	0.00086436
7	0.00078643	0.99992207	0.00007793
8	0.00007373	0.99999580	0.00000420
9	0.00000410	0.99999990	0.00000010
10	0.00000010	1.00000000	0.00000000

••

The foreman of a casting section in a certain factory finds that, on the average, 1 in every 5 castings made is defective.

________5. If the section makes 10 castings a day, what is the probability that exactly 2 of these will be defective?

________6. What is the probability that 3 or more defective castings are made in one day?

________7. What is the probability that the first two castings are both defective (assuming independence)?

________8. What's the <u>name</u> of the probability distribution of the quality of casting #8 (either 1=defective or 0=OK)?

••

Advertising states that, for a certain lottery ticket, "every fifth ticket carries a prize". If you buy ten tickets, what is...

________9. the probability (numerical value) that you get *exactly* one winning ticket?

________10. the probability (numerical value) that you get *at least* one winning ticket?

If, instead of deciding in advance how many tickets to buy, you continue buying tickets until you have a winning ticket...

________11. What's the <u>name</u> of the probability distribution of the number of tickets you buy ?

If you continue buying tickets until you have *two* winning tickets...

________12. What's the <u>name</u> of the probability distribution of the number of tickets you buy ?

Some common probability distributions:

a. Bernouilli b. Random c. Binomial
d. Poisson e. Geometric f. Normal
g. Exponential h. Erlang i. Pascal

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57:022 Principles of Design II

Quiz #1, January 26, 2000

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Indicate with true (+) or false (o):

_+_ 1. A random number with *Pascal* distribution is the sum of random variables each having the *geometric* distribution.

_o_ 2. The *binomial* distribution is a special case of the Pascal distribution.

_o_ 3. In a Bernouilli process, the number of "successes" in n trials (N_n) has the *Poisson* distribution.

_+_ 4. If W_1 has the *geometric* distribution, then

$P\{W_1=1\} \geq P\{W_1=2\} \geq P\{W_1=2\} \geq \ldots$ (true, since $p \geq (1-p)p \geq (1-p)^2 p \geq \ldots$ for $0 \leq p \leq 1$)

••

For some of the questions which follow, you may refer to the table below. Only 3 significant digits are needed.

Binomial Cumulative Distribution Function (n= 10, p= 0.2)

x	P{x}	P{X ≤ x}	P{X > x}
0	0.10737418	0.10737418	0.89262582
1	0.26843546	0.37580964	0.62419036
2	0.30198989	0.67779953	0.32220047
3	0.20132659	0.87912612	0.12087388
4	0.08808038	0.96720650	0.03279350
5	0.02642412	0.99363062	0.00636938
6	0.00550502	0.99913564	0.00086436
7	0.00078643	0.99992207	0.00007793
8	0.00007373	0.99999580	0.00000420
9	0.00000410	0.99999990	0.00000010
10	0.00000010	1.00000000	0.00000000

••

The foreman of a casting section in a certain factory finds that, on the average, 1 in every 5 castings made is defective.

_0.302_ 5. If the section makes 10 castings a day, what is the probability that exactly 2 of these will be defective?

_0.322_ 6. What is the probability that 3 or more defective castings are made in one day?

_0.04_ 7. What is the probability that the first two castings are both defective (assuming independence)?

_Bernouilli_8. What's the name of the probability distribution of the quality of casting #8 (either 1=defective or 0=OK)?

••

Advertising states that, for a certain lottery ticket, "every fifth ticket carries a prize". If you buy ten tickets, what is...

_0.268_ 9. the probability (numerical value) that you get *exactly* one winning ticket?

_0.0892 10. the probability (numerical value) that you get *at least* one winning ticket?

If, instead of deciding in advance how many tickets to buy, you continue buying tickets until you have a winning ticket...

_Geometric_11. What's the name of the probability distribution of the number of tickets you buy ?

If you continue buying tickets until you have *two* winning tickets...

Pascal 12. What's the name of the probability distribution of the number of tickets you buy ?

Some common probability distributions:

a. Bernouilli	b. Random	c. Binomial
d. Poisson	e. Geometric	f. Normal
g. Exponential	h. Erlang	i. Pascal (= negative binomial)

Name ________________

57:022 Principles of Design II - Quiz #2
Wednesday, February 2, 2000

Part I. Along highway I-80 in Iowa, the probability that each passing car stops to pick up a hitchhiker is p=2%, i.e, an average of one in fifty drivers will stop; different drivers, of course, make their decisions whether to stop or not independently of each other.

___ 1. Consider a stochastic process in which X_n=1 if car n stops to pick up the hitchhiker, and X_n=0 otherwise. Then {X_n: n=1,2,3,...} is a

a. Binomial process b. Bernouilli process c. Poisson process
d. Markov process e. Exponential process f. None of the above

___ 2. P{ X_n =1} =

a. 0.50 b. 0.98 c. 0.025
d. 0.02 e. 0.2 f. None of the above

___ 3. If 25 cars pass the hitchhiker, the probability that *none* of them stop is

a. 25x(0.02) b. $(0.02)^{25}$ c. $(0.98)^{25}$
d. $(0.98)^{24}(0.02)$ e. $(0.02)^{24}(0.98)$ f. None of the above

___ 4. Given that a hitchhiker has counted 25 cars passing him without stopping, what is the probability that he will be picked up by the 30th car *or before?*

a. $(0.98)^{30}$ b. $1-(0.98)^{5}$ c. $1-(0.02)^{30}$
d. $1-(0.02)^{5}$ e. $(0.98)^{5}$ f. None of the above

Suppose that the arrivals of the cars form a Poisson process, at the average rate of 15 per minute. Define "success" for the hitchhiker to occur at time t provided that *both* an arrival occurs at t *and* that car stops to pick him up. Let T_1 be the time (in seconds) of the first "success", i.e., the time that he finally gets a ride, when he begins his wait at time T_1=0.

___ 5. The arrival rate of "successes" is

a. 1/minute b. 3/minute c. 2/minute
d. 0.3/minute e. 0.2/minute f. None of the above

___ 6. The random variable T_1 has what distribution?

a. Poisson b. Geometric c. Exponential
d. Pascal e. Erlang f. None of the above

___ 7. What is E(T_1), the expected (mean) value of T_1?

a. 10/3 minutes b. 3 minutes c. 4 minutes
d. 3/2 minutes e. 1/3 minute f. None of the above

___ 8. What's the probability that his waiting time is less than or equal to 5 min. (P{T_1 5}?

a. $1 - e^{-4.5}$ b. $1 - e^{-1.5}$ c. $e^{-1.5}$
d. $e^{-4.5}$ e. $1 - e^{1.5}$ f. None of the above

___ 9. What is the probability that he must wait *exactly* 5 minutes for a ride (P{T_1= 5}?

a. $1 - e^{-1.5}$ b. $e^{-1.5}$ c. $e^{4.5}$
d. $1 - e^{-4.5}$ e. 0.0 f. None of the above

___ 10. Suppose that after 3 minutes (during which 42 cars have passed by) he is still there waiting for a ride. What is the *conditional* expected value of T_1 (expected *total* waiting time, i.e., since time 0, given that he has already waited 3 minutes).

a. 10/3 minutes b. 3/10 minutes c. 15 minutes
d. 40/3 minutes e. 3/40 minutes f. None of the above

Name ________________

Vehicles arrive at a toll booth on the freeway at the average rate of 6/minute in a completely random fashion. The vehicles are counted and arrival times are recorded, beginning at 12:00 noon. Ninety percent of the vehicles are cars, while the remainder are trucks, buses, etc.

Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!

___ 1. time of arrival of the first vehicle
___ 2. time of arrival of vehicle #2
___ 3. time *between* arrivals of vehicle #1 and vehicle #2
___ 4. number of vehicles arriving during the first 5 minutes
___ 5. vehicle # of the first vehicle which is *not* a car.
___ 6. the number of cars among the first 10 vehicles to arrive
___ 7. the vehicle # of the second vehicle which is *not* a car.
___ 8. an indicator for vehicle #n which is 1 if a car, 0 otherwise.

Probability distributions:

A. Bernouilli
B. Erlang
C. Poisson
D. Binomial
E. Geometric
F. Exponential
G. Pascal
H. Normal

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57:022 Principles of Design II - Quiz #2 Solutions

Wednesday, February 2, 2000

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Part I. Along highway I-80 in Iowa, the probability that each passing car stops to pick up a hitchhiker is p=2%, i.e, an average of one in fifty drivers will stop; different drivers, of course, make their decisions whether to stop or not independently of each other.

_b_ 1. Consider a stochastic process in which $X_n=1$ if car n stops to pick up the hitchhiker, and $X_n=0$ otherwise. Then $\{X_n: n=1,2,3,...\}$ is a

a. Binomial process b. Bernouilli process c. Poisson process
d. Markov process e. Exponential process f. None of the above

_d_ 2. $P\{ X_n =1\}$ =

a. 0.50 b. 0.98 c. 0.025
d. 0.02 e. 0.2 f. None of the above

_c_ 3. If 25 cars pass the hitchhiker, the probability that *none* of them stop is

a. 25x(0.02) b. $(0.02)^{25}$ c. $(0.98)^{25}$
d. $(0.98)^{24}(0.02)$ e. $(0.02)^{24}(0.98)$ f. None of the above

_b_ 4. Given that a hitchhiker has counted 25 cars passing him without stopping, what is the probability that he will be picked up by the 30th car *or before?*

a. $(0.98)^{30}$ b. $1-(0.98)^5$ c. $1-(0.02)^{30}$
d. $1-(0.02)^5$ e. $(0.98)^5$ f. None of the above

Note: This is 1 minus the probability that 5 consecutive cars do not stop!

Suppose that the arrivals of the cars form a Poisson process, at the average rate of 15 per minute. Define "success" for the hitchhiker to occur at time t provided that *both* an arrival occurs at t *and* that car stops to pick him up. Let T_1 be the time (in seconds) of the first "success", i.e., the time that he finally gets a ride, when he begins his wait at time T_1=0.

_d_ 5. The arrival rate of "successes" is

a. 1/minute b. 3/minute c. 2/minute
d. 0.3/minute e. 0.2/minute f. None of the above

_c_ 6. The random variable T_1 has what distribution?

a. Poisson b. Geometric c. Exponential
d. Pascal e. Erlang f. None of the above

_a_ 7. What is $E(T_1)$, the expected (mean) value of T_1?

a. 10/3 minutes b. 3 minutes c. 4 minutes
d. 3/2 minutes e. 1/3 minute f. None of the above

_b_ 8. What's the probability that his waiting time is less than or equal to 5 min. $(P\{T_1 \leq 5\}$?

a. $1 - e^{-4.5}$ b. $1 - e^{-1.5}$ c. $e^{-1.5}$
d. $e^{-4.5}$ e. $1 - e^{1.5}$ f. None of the above

_e_ 9. What is the probability that he must wait *exactly* 5 minutes for a ride $(P\{T_1= 5\}$?

a. $1 - e^{-1.5}$ b. $e^{-1.5}$ c. $e^{4.5}$
d. $1 - e^{-4.5}$ e. 0.0 f. None of the above

_d_ 10. Suppose that after 3 minutes (during which 42 cars have passed by) he is still there waiting for a ride. What is the *conditional* expected value of T_1 (expected *total* waiting time, i.e., since time 0, given that he has already waited 3 minutes). Choose NEAREST value:

a. 3 minutes b. 4 minutes c. 5 minutes
d. 6 minutes *(6.3333)* e. 7 minutes f. More than 8 minutes

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Vehicles arrive at a toll booth on the freeway at the average rate of 6/minute in a completely random fashion. The vehicles are counted and arrival times are recorded, beginning at 12:00 noon. Ninety percent of the vehicles are cars, while the remainder are trucks, buses, etc.

Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!

_F_ 1. time of arrival of the first vehicle
_B_ 2. time of arrival of vehicle #2
_F_ 3. time *between* arrivals of vehicle #1 and vehicle #2
_C_ 4. number of vehicles arriving during the first 5 minutes
_E_ 5. vehicle # of the first vehicle which is *not* a car.
_D_ 6. the number of cars among the first 10 vehicles to arrive
_G_ 7. the vehicle # of the second vehicle which is *not* a car.
_A_ 8. an indicator for vehicle #n which is 1 if a car, 0 otherwise.

Probability distributions:

A. Bernouilli
B. Erlang
C. Poisson
D. Binomial
E. Geometric
F. Exponential
G. Pascal
H. Normal

Name ________________________________

57:022 Principles of Design II - Quiz #3
Wednesday, February 9, 2000

True (+) or False (o)?

__ 1. The rejection method to generate a random number can be used to simulate a random variable having a Normal distribution.

__ 2. The density function evaluated at the "mode" of a probability distribution is 50%.

__ 3. The maximum value of the cumulative distribution function is 1.

__ 4. The maximum value of the density function for a random variable is 1.

__ 5. The inverse transformation method to generate a random number can be used to simulate a random variable having a triangular distribution.

__ 6. The inverse transformation method requires as input a single random number in the interval [0,1].

__ 7. The rejection method requires as input a single random number in the interval [0,1].

__ 8. If we wanted to simulate a random variable with 2-Erlang distribution, we might generate two random numbers having the exponential distribution and sum them.

__ 9. The rejection method to generate a random number can be used to simulate a random variable having an exponential distribution.

___ 10. The "Cumulative Distribution Function" (CDF) of a random variable X is

a. $f(x) = P\{x \mid X\}$ b. $F(x) = P\{X \geq x\}$ c. $f(x) = P\{x\}$
d. $F(x) = P\{X \leq x\}$ e. $F(x) = P\{X = x\}$ f. $f(x) = P\{X \mid x\}$

We wish to generate some random numbers having an exponential distribution as the inter-arrival times (where the average is 5 minutes.) Suppose that a procedure for generating uniformly-distributed random numbers has yielded the value R=0.794. We want to generate a random value for T_1, i.e., the time at which the *first* car arrives.

___ 11. Using the Inverse Transformation method, then according to the table below the *nearest* value of T_1 should be

a. 1minute e. 5 minutes i. 9 minutes
b. 2 minutes f. 6 minutes j. 10 minutes
c. 3 minutes g. 7 minutes k. 11 minutes
d. 4 minutes h. 8 minutes l. *greater than 12 min.*

___ 12. Suppose that the next uniformly-generated random number is 0.218. Then corresponding arrival time T_2 of the *second* car is (choose *nearest* value):

a. 1minute e. 5 minutes i. 9 minutes
b. 2 minutes f. 6 minutes j. 10 minutes
c. 3 minutes g. 7 minutes k. 11 minutes
d. 4 minutes h. 8 minutes l. *greater than 12 min.*

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x	P{T≤x}	Δp	P{T>x}
0	0.00000000	0.00000000	1.00000000
1	0.18126925	0.18126925	0.81873075
2	0.32967995	0.14841071	0.67032005
3	0.45118836	0.12150841	0.54881164
4	0.55067104	0.09948267	0.44932896
5	0.63212056	0.08144952	0.36787944
6	0.69880579	0.06668523	0.30119421
7	0.75340304	0.05459725	0.24659696
8	0.79810348	0.04470045	0.20189652
9	0.83470111	0.03659763	0.16529889
10	0.86466472	0.02996360	0.13533528
11	0.88919684	0.02453212	0.11080316
12	0.90928205	0.02008521	0.09071795
13	0.92572642	0.01644438	0.07427358
14	0.93918994	0.01346352	0.06081006
15	0.95021293	0.01102299	0.04978707

Name ________________________________

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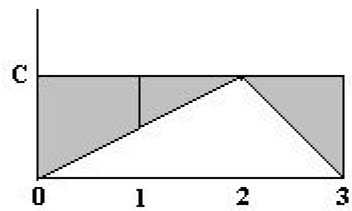
___ 13. We want to generate random numbers X between 0 and 3, having the triangular distribution whose density function is shown below. What is the value of C? *(Choose nearest value.)*

a. 0.2 d. 0.5 g. 0.9
b. 0.3 e. 0.6 h. 1.0
c. 0.4 f. 0.7 i. *greater than 1.0*

___ 14. Suppose that we generate two uniformly-distributed random numbers in the interval [0,1], namely R_1=0.713 and R_2=0.224, and apply the *rejection* method. What random number is generated from this pair of numbers? *(Choose nearest value.)*

a. 0.5 b. 1.0 c. 1.5
d. 2.0 e. 2.5 f. 3.0 g. *None of the above*



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57:022 Principles of Design II - Quiz #3 Solutions

Spring 2000

✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠

True (+) or False (o)?

_o_ 1. The rejection method to generate a random number can be used to simulate a random variable having a Normal distribution.

_o_ 2. The density function evaluated at the "mode" of a probability distribution is 50%.

_+_ 3. The maximum value of the cumulative distribution function is 1.

_o_ 4. The maximum value of the density function for a random variable is 1.

_+_ 5. The inverse transformation method to generate a random number can be used to simulate a random variable having a triangular distribution.

_+_ 6. The inverse transformation method requires as input a single random number in the interval [0,1].

_o_ 7. The rejection method requires as input a single random number in the interval [0,1].

_+_ 8. If we wanted to simulate a random variable with 2-Erlang distribution, we might generate two random numbers having the exponential distribution and sum them.

_o_ 9. The rejection method to generate a random number can be used to simulate a random variable having an exponential distribution.

_d_ 10. The "Cumulative Distribution Function" (CDF) of a random variable X is

a. f(x) = P{x | X}	b. F(x) = P{X ≥ x}	c. f(x) = P{x}
d. F(x) = P{X ≤ x}	e. F(x) = P{X = x}	f. f(x) = P{X | x}

We wish to generate some random numbers having an exponential distribution as the inter-arrival times (where the average is 5 minutes.) Suppose that a procedure for generating uniformly-distributed random numbers has yielded the value R=0.794. We want to generate a random value for T_1, i.e., the time at which the *first* car arrives.

_h_ 11. Using the Inverse Transformation method, then according to the table below the *nearest* value of T_1 should be

a. 1minute	e. 5 minutes	i. 9 minutes
b. 2 minutes	f. 6 minutes	j. 10 minutes
c. 3 minutes	g. 7 minutes	k. 11 minutes
d. 4 minutes	h. 8 minutes	l. *greater than 12 min.*

_i_ 12. Suppose that the next uniformly-generated random number is 0.218. Then the corresponding arrival time T_2 of the *second* car is (choose *nearest* value):

a. 1minute	e. 5 minutes	i. 9 minutes *(=8 minutes + 1 minute)*
b. 2 minutes	f. 6 minutes	j. 10 minutes
c. 3 minutes	g. 7 minutes	k. 11 minutes
d. 4 minutes	h. 8 minutes	l. *greater than 12 min.*

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x	P{T≤x}	Δp	P{T>x}
0	0.00000000	0.00000000	1.00000000
1	0.18126925	0.18126925	0.81873075
2	0.32967995	0.14841071	0.67032005
3	0.45118836	0.12150841	0.54881164
4	0.55067104	0.09948267	0.44932896
5	0.63212056	0.08144952	0.36787944
6	0.69880579	0.06668523	0.30119421
7	0.75340304	0.05459725	0.24659696
8	0.79810348	0.04470045	0.20189652
9	0.83470111	0.03659763	0.16529889
10	0.86466472	0.02996360	0.13533528
11	0.88919684	0.02453212	0.11080316
12	0.90928205	0.02008521	0.09071795
13	0.92572642	0.01644438	0.07427358
14	0.93918994	0.01346352	0.06081006
15	0.95021293	0.01102299	0.04978707

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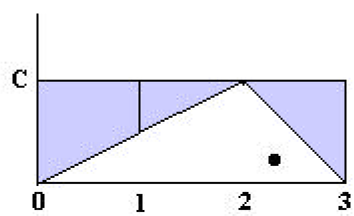
_b_ 13. We want to generate random numbers X between 0 and 3, having the triangular distribution whose density function is shown below. What is the value of C? *(Choose nearest value.)*

a. 0.2	d. 0.5	g. 0.9
b. 0.3 *(1/3)*	e. 0.6	h. 1.0
c. 0.4	f. 0.7	i. *greater than 1.0*

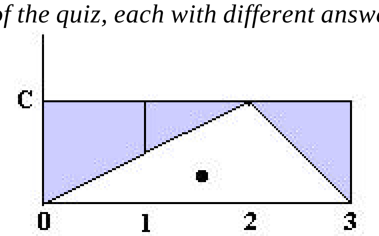
___ 14. Suppose that we generate two uniformly-distributed random numbers in the interval [0,1], namely R_1=???? and R_2=????, and apply the *rejection* method. What random number is generated from this pair of numbers? *(Choose nearest value.)*

a. 0.5	b. 1.0	c. 1.5	
d. 2.0	e. 2.5	f. 3.0	g. *None of the above*

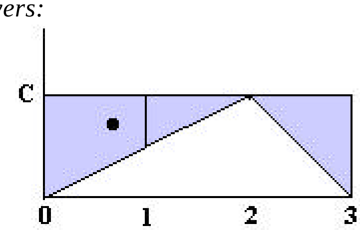
Note: There were three versions of the quiz, each with different answers:



R_1 = 0.713, R_2 = 0.224
Accepts X = 3×0.713 ~ 2



R_1 = 0.513, R_2 = 0.224
Accepts X = 3×0.513 ~ 1.5



R_1 = 0.224, R_2 = 0.713
Rejects value *(None of above)*

Name ______________________________

✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠

57:022 Principles of Design II - Quiz #4

Wednesday, February 16, 2000

✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠✠

The following statements refer to today's homework assignment in which you simulated the movement of dirt by trucks. (I assume that the entities of your model represent the trucks.)

True (+) or False (o)?

__ 1. The ARRIVE module is located on the SUPPORT template.
__ 2. SUPPORT and COMMON are names of templates.
__ 3. An ARRIVE module simulates each time a truck arrives at the loading area.
__ 4. The "capacity" of a SERVER module is the maximum number of entities which can wait at that server.
__ 5. Only one truck at a time may be loaded.
__ 6. The length of time to be simulated is specified in the SIMULATE module.
__ 7. The travel times to & from the loading area are assumed to be negligible and are ignored.
__ 8. The number of entities in the system is specified in the ARRIVE module.
__ 9. The entities depart the system at a DEPART module.
__ 10. Only one truck at a time may unload dirt.
__ 11. In this model, you specified how often observations are made of the system, and this number of observations appears in the output report.
__ 12. The number of replications (specified in the SIMULATE module) is the number of truckloads.
__ 13. The SERVER and ARRIVE modules appear on the same template.
__ 14. The SERVER and SIMULATE modules appear on different templates.
__ 15. The modules used to build the model are found on "templates".
__ 16. In order to start the simulation, you must enter the command "run".
__ 17. The number of servers in your model is equal to the number of trucks that are used.
__ 18. The ARENA simulation software is available on the Windows NT computers in the ICAEN labs.

Name ________________________

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57:022 Principles of Design II

Quiz #4 Solutions

Wednesday, February 16, 2000

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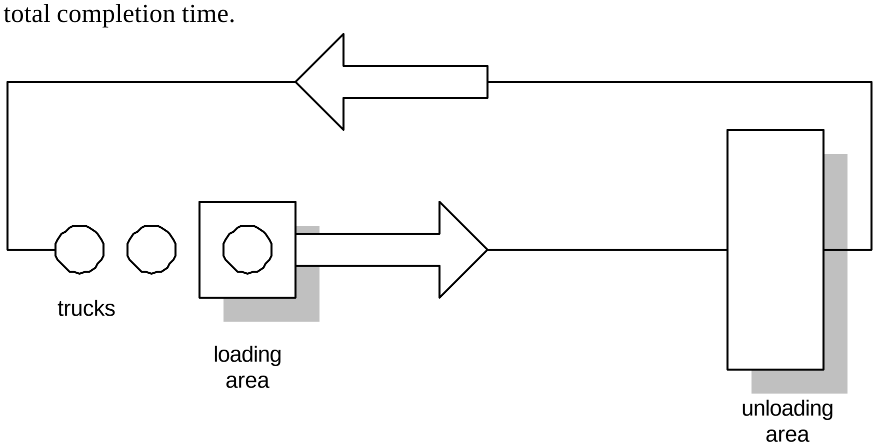
Restatement of homework problem: Bectol, Inc. is building a dam. A total of 1,000,000 cu ft of dirt is needed to construct the dam. A loader is used to collect dirt for the dam. Then the dirt is moved via dump trucks to the dam site. Only one loader is available, and it rents for $100 per hour. Bectol can rent, at $40 per hour, as many dump trucks as desired. Each dump truck can hold 1000 cu ft of dirt. Triangular distributions are assumed to describe the following various random quantities (primarily because the parameters are easily understood and estimated by the work crews):

Random variable	Best case (minimum time)	Most Likely	Worst case (maximum time)
Loading truck	8 minutes	12 minutes	18 minutes
Travel to unloading area	2 minutes	3 minutes	5 minutes
Unloading truck	1 minute	2 minutes	4 minutes
Return to loader	2 minutes	3 minutes	4 minutes

Simulate an 8-hour day to estimate the number of loads which can be moved per hour, so that you can estimate the total completion time.



The following statements refer to today's homework assignment in which you simulated the movement of dirt by trucks. (I assume that the entities of your model represent the trucks.)

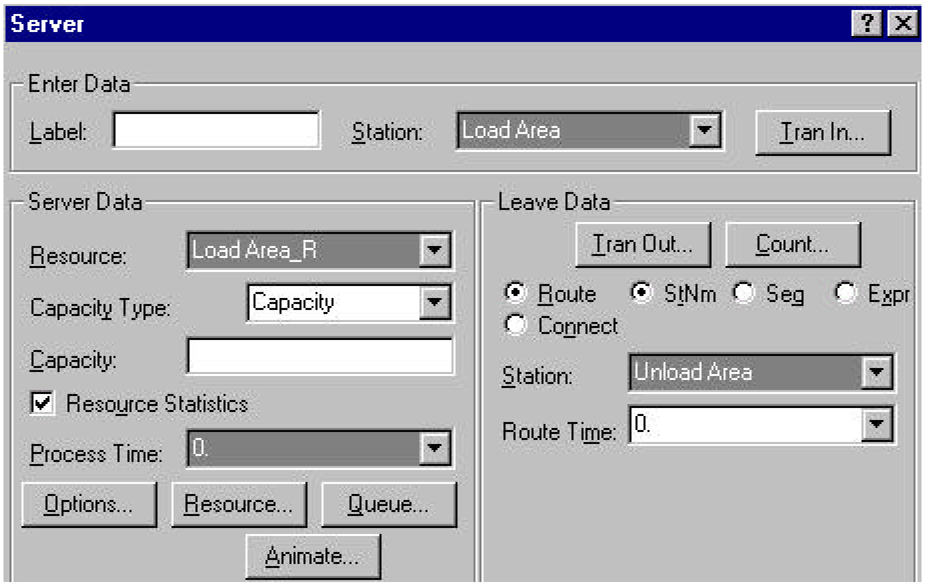
True (+) or False (o)?

o 1. The ARRIVE module is located on the SUPPORT template.
o 2. An ARRIVE module simulates each time a truck arrives at the loading area.
o 3. The "capacity" of a SERVER module is the maximum number of entities which can wait at that server.
\+ 4. The length of time to be simulated is specified in the SIMULATE module.
o 5. The travel times to & from the loading area are assumed to be negligible and are ignored.
\+ 6. The number of trucks in the system is specified in the ARRIVE module.
o 7. The entities depart the system at a DEPART module..
o 8. The number of replications (specified in the SIMULATE module) is the number of truckloads.
\+ 9. The modules used to build the model are found on "templates".
o 10. In order to start the simulation, you must enter the command "run".
o 11. The number of servers in your model is equal to the number of trucks that are used.

Name ________________________

The appropriate values have not yet been entered into the dialogue windows shown below!

Loading Area

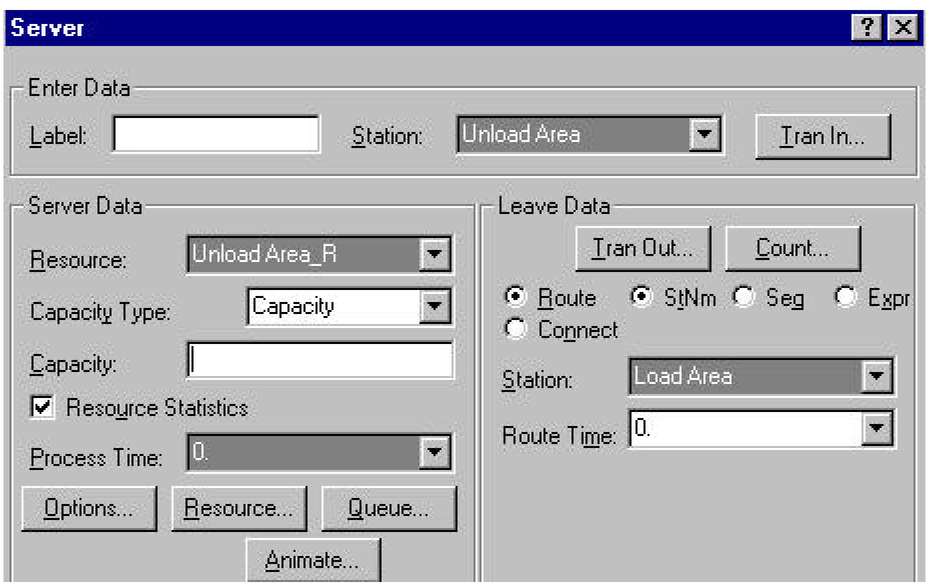


o 12. The capacity should be equal to the number of trucks.

o 13. Process Time is TRIA(2,3,5)

o 14. Route Time should be zero.

Unloading Area



\+ 15. The capacity should be equal to the number of trucks.

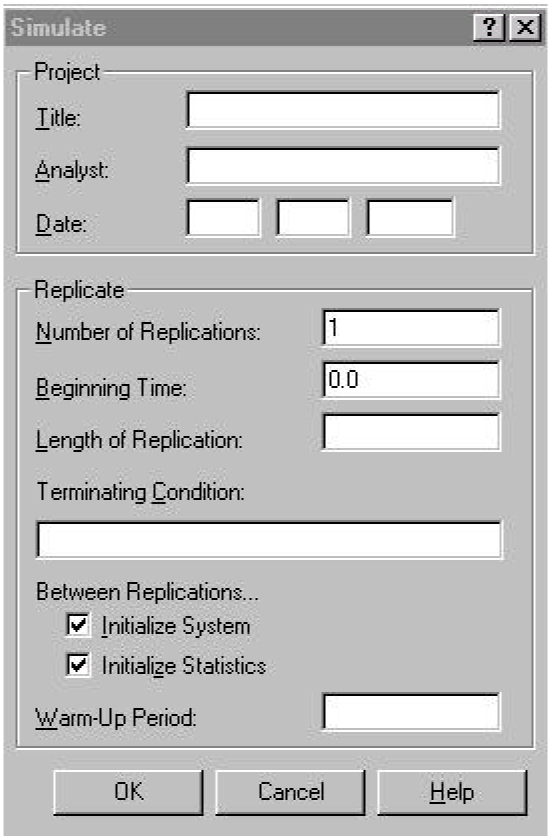
\+ 16. Process Time should be TRIA(1,2,4)

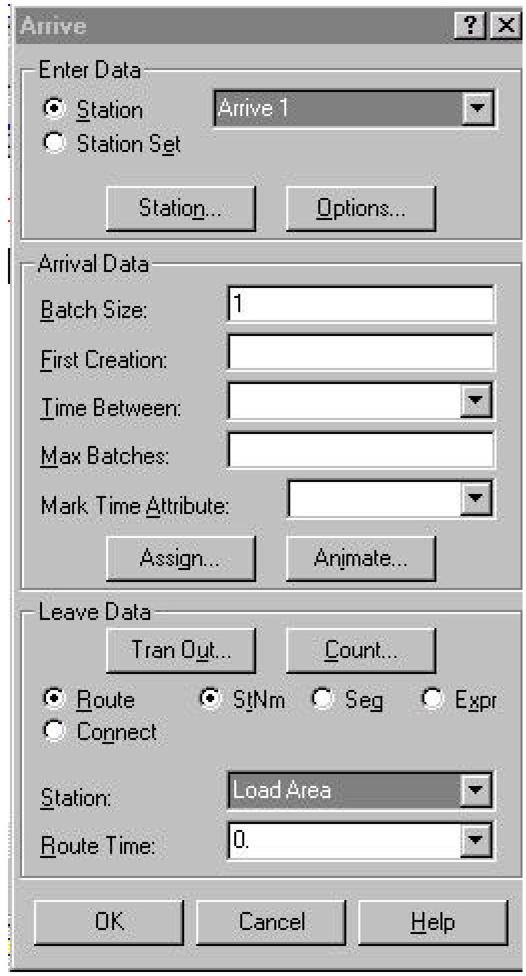
o 17. Route time should be 0

o 18. In the *Simulate* module, the length of replication should be 2400.
\+ 19. In the *Arrive* module, *Max Batches* is equal to the number of trucks
\+ 20. If the loader could be kept busy continually, about 28 days would be required to move all of the dirt

Refer to the ARENA simulation output:
\+ 21. The loader is kept busy about 62% of the time.
\+ 22. The total number of trucks unloaded is 22.

Name ________________________


Simulate
Project
Title:
Analyst:
Date:
Replicate
Number of Replications:
1
Beginning Time:
0.0
Length of Replication:
Terminating Condition:
Between Replications...
Initialize System
Initialize Statistics
Warm-Up Period:
OK
Cancel
Help


Arrive
Enter Data
Station
Station Set
Arrive 1
Station...
Options...
Arrival Data
Batch Size:
1
First Creation:
Time Between:
Max Batches:
Mark Time Attribute:
Assign...
Animate...
Leave Data
Tran Out...
Count...
Route
StNm
Seg
Expr
Connect
Station:
Load Area
Route Time:
0.
OK
Cancel
Help

Name ________________________

ARENA Simulation Results for the number of trucks = 3

```
                        Summary for Replication 1 of 1

Project:  Bectol Inc.Probl                      Run execution date :   2/15/2000
Analyst:  Hansuk Sohn                           Model revision date:   2/15/2000

Replication ended at time      : _______

                                 TALLY VARIABLES

Identifier                Average   Half Width  Minimum    Maximum   Observations
_______________________________________________________________________________

Unload Area_R_Q Queue    .00000     (Insuf)    .00000     .00000         22
Load Area_R_Q Queue Ti   .00000     (Insuf)    .00000     .00000         23

                           DISCRETE-CHANGE VARIABLES

Identifier                Average   Half Width  Minimum    Maximum   Final Value
_______________________________________________________________________________

# in Unload Area_R_Q     .00000     (Insuf)    .00000     .00000     .00000
Unload Area_R Availabl   3.0000     (Insuf)    3.0000     3.0000     3.0000
Load Area_R Busy         .61683     (Insuf)    .00000     1.0000     1.0000
# in Load Area_R_Q       .00000     (Insuf)    .00000     .00000     .00000
Load Area_R Available    1.0000     (Insuf)    1.0000     1.0000     1.0000
Unload Area_R Busy       .09783     (Insuf)    .00000     1.0000     .00000


Simulation run time: 0.00 minutes.
Simulation run complete.
```

Name ___________________________

57:022 Principles of Design II
Quiz #5 -- Spring 2000

___ 1. If you use the Minitab program to fit a line, it will find the straight line which minimizes the sum of the absolute values of the errors, i.e., the sum of the vertical distances between each data point and the line.

___ 2. If F(t) is the CDF of the interarrival time for a Poisson process, the expected number of arrivals E_i which fail in the time interval $[t_{i-1}, t_i]$ is $F(t_i) - F(t_{i-1})$

___ 3. In the chi-square goodness-of-fit test, the number of degrees of freedom is never more than the number of "cells" of the histogram.

___ 4. In a Poisson process, the time between arrivals has an exponential distribution.

___ 5. The mean and standard deviation of the exponential distribution are always equal.

___ 6. In a Poisson process with arrival rate l/minute, the number of arrivals in one minute is random, with a exponential distribution having mean l.

___ 7. The Erlang distribution is a special case of the exponential distribution.

The time *between* arrivals of exactly forty vehicles are measured. The number of observations O_i falling within each half-minute interval is shown in the table below. The average is computed by weighting the midpoint of each interval by its number of observations: 0.25x9 + 0.75x4 + 1.25x5 + ... = 2.225 minutes. We wish to test the "goodness of fit" of the exponential distribution having mean 2.225 minutes.

i	Interval	O_i	p_i	E_i	$(E_i-O_i)^2/E_i$
1	0.0 - 0.5	9	0.2015	8.0594	0.1098
2	0.5 - 1.0	4	0.1609	6.4355	0.9217
3	1.0 - 1.5	5	0.1285	5.1389	0.0038
4	1.5 - 2.0	3	0.1026	4.1035	0.2967
5	2.0 - 2.5	7	0.0819	3.2767	4.2308
6	2.5 - 3.0	3	0.0654	2.6165	0.0562
7	> 3.0	9	0.2592	10.3696	0.1809

The sum of the values in the last column is D = 5.8.

deg.of freedom	Chi-square Dist'n $P\{D \geq \chi^2\}$ 99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

Name ________________________

Indicate "+" for true, "o" for false:

____ 8. The quantity E_i is a random variable with approximately Poisson distribution.

____ 9. The parameter of the exponential distribution is assumed to be $\lambda = 1/2.225\text{min.} = 0.45/\text{min.}$

____ 10. The probability p_i that a car arrives in an interval #i, $[t_1, t_2]$, is $F(t_1) - F(t_2)$

____ 11. The CDF of the distribution of interarrival times is assumed to be $F(t) = 1 - \lambda e^{-\lambda t}$

____ 12. The number of observations, O_i, in an interval should have a binomial distribution, with n=40.

____ 13. The quantity D is assumed to have the chi-square distribution.

____ 14. The quantity $(E_i\text{-}O_i)^2/E_i$ is assumed to have the normal N(0,1) distribution.

____ 15. The chi-square distribution for this test will have 7 "degrees of freedom".

____ 16. The number of observations O_i in interval # i is a random variable with approximately Poisson distribution.

____ 17. If it is true that T has the exponential distribution with mean 2.225 minutes, then the probability that D exceeds 5.8 should be less than 10%.

____ 18. The exponential distribution with mean 2.225 minutes should be rejected as a model for the interarrival times of the vehicles.

____ 19. The chi-square distribution for this test will have 6 "degrees of freedom".

____ 20. The quantity D is assumed to have approximately a Normal distribution.

____ 21. The degrees of freedom is reduced by 2 because (i) the total number of observations is fixed, and (ii) the data was used to estimate one parameter for the distribution being tested.

____ 22. The smaller the value of D, the worse the fit for the distribution being tested.

____ 23. The quantity E_i is the expected number of observations in interval #i

____ 24. The sum of several N(0,1) random variables has chi-square distribution.

Name ____________________________

57:022 Principles of Design II
Quiz #5 Solutions -- Spring 2000

_+_ 1. In the chi-square goodness-of-fit test, the number of degrees of freedom is never more than the number of "cells" of the histogram.

_+_ 2. The mean and standard deviation of the exponential distribution are always equal.

_o_ 3. The Erlang distribution is a special case of the exponential distribution.

_o_ 4. In a Poisson process with arrival rate l/minute, the number of arrivals in one minute is random, with an exponential distribution having mean l.

_o_ 5. If you use the Minitab program to fit a line, it will find the straight line which minimizes the sum of the absolute values of the errors, i.e., the sum of the vertical distances between each data point and the line.

_+_ 6. If F(t) is the CDF of the interarrival time for a Poisson process, the expected number of arrivals E_i which fail in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$

_o_ 7. In a Poisson process, the time between arrivals has a Poisson distribution.

The time *between* arrivals of exactly forty vehicles are measured. The number of observations O_i falling within each half-minute interval is shown in the table below. The average is computed by weighting the midpoint of each interval by its number of observations: 0.25x9 + 0.75x4 + 1.25x5 + ... = 2.225 minutes. We wish to test the "goodness of fit" of the exponential distribution having mean 2.225 minutes.

i	Interval	O_i	p_i	E_i	$(E_i-O_i)^2/E_i$
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The sum of the values in the last column is D = 5.8.

deg.of	Chi-square Dist'n $P\{D\geq\chi^2\}$					
freedom	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
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7	1.239	2.167	2.833	12.017	14.067	18.475

Name ____________________________

Indicate "+" for true, "o" for false:

_o__ 8. The smaller the value of D, the worse the fit for the distribution being tested.

_o_ 9. The chi-square distribution for this test will have 6 "degrees of freedom".

_o_ 10. The chi-square distribution for this test will have 7 "degrees of freedom".

_o_ 11. The number of observations O_i in interval #i is a random variable with approximately Poisson distribution.

_+_ 12. The quantity D is assumed to have the chi-square distribution.

_o_ 13. The quantity D is assumed to have approximately a Normal distribution.

_+_ 14. The degrees of freedom is reduced by 2 because (i) the total number of observations is fixed, and (ii) the data was used to estimate one parameter for the distribution being tested.

_+_ 15. The quantity E_i is the expected number of observations in interval #i

_o_ 16. The sum of several N(0,1) random variables has chi-square distribution.

_o_ 17. The quantity $(E_i-O_i)^2/E_i$ is assumed to have the normal N(0,1) distribution.

_o_ 18. If it is true that T has the exponential distribution with mean 2.225 minutes, then the probability that D exceeds 5.8 should be less than 10%.

_o_ 19. The probability p_i that a car arrives in an interval #i, $[t_1, t_2]$, is $F(t_1) - F(t_2)$

_o_ 20. The CDF of the distribution of interarrival times is assumed to be $F(t) = 1 - \lambda e^{-\lambda t}$

_o_ 21. The exponential distribution with mean 2.225 minutes should be rejected as a model for the interarrival times of the vehicles.

_+_ 22. The parameter of the exponential distribution is assumed to be λ = 1/2.225min. = 0.45/min.

_+_ 23. The number of observations, O_i, in an interval should have a binomial distribution, with n=40.

_o_ 24. The quantity E_i is a random variable with approximately Poisson distribution.

Name ________________________

57:022 Principles of Design II
Quiz #6 -- Spring 2000

Indicate "+" for true, "o" for false:

___ 1. The quantity R(t) is the fraction of the motors which we expect to have failed at time t (or earlier).

___ 2. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

___ 3. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has failed at or before time t.

___ 4. We assumed in this HW (#6) that the number of motor failures at time t , $N_f(t)$, has a Weibull distribution.

___ 5. According to the results of this homework exercise, the failure rate of the motors is increasing rather than decreasing.

___ 6. Given only the coefficient of variation for the Weibull distribution (the ratio σ/μ), the parameter k can be determined.

___ 7. The fraction of the machines which are expected to fail in the time interval $[t_{i-1}, t_i]$ is $F(t_i) - F(t_{i-1})$ where F(t) is the CDF of the failure time distribution.

___ 8. A positive value of k indicates an increasing failure rate, and negative k indicates a decreasing failure rate.

___ 9. The method used in this HW (#6) to estimate the Weibull parameters u & k does <u>not</u> require that the motors be tested until <u>all</u> have failed.

___ 10. The CDF of the failure time of a motor is assumed to be $F(t) = 1 - e^{-(t/u)^k}$ for some parameters u & k.

___ 11. $\Gamma(n) = n!$ if n is an integer.

___ 12. The time between the failures in the batch of 200 motors is assumed to have the Weibull distribution.

___ 13. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

___ 14. The Reliability of a device with random failure time T is defined as

a. $R(t) = P\{t\}$	b. $R(x) = P\{T \mid t\}$	c. $R(x) = P\{T = t\}$
d. $R(t) = P\{t \mid T\}$	e. $R(t) = P\{T \geq t\}$	f. $R(x) = P\{T \leq t\}$

Select the letter below which indicates each correct answer:

In order to estimate the Weibull parameters by the method of today's homework,

___ 16. The variable plotted on the horizontal axis should be ...

___ 17. The variable plotted on the vertical axis should be ...

___ 18. The slope of the line should be approximately ...

___ 19. The vertical intercept of the line should be approximately ...

a. t	b. R_t	c. shape parameter k
d. ln t	e. ln R_t	f. scale parameter u
g. ln $1/t$	h. ln $1/R_t$	i. mean value μ
j. ln ln t	k. ln ln R_t	l. standard deviation σ
m. ln ln $1/t$	n. ln ln $1/R_t$	o. – ln u
p. –k ln u	q. – u ln k	r. ln k

57:022 Principles of Design II
Quiz #6 Solutions -- Spring 2000

Indicate "+" for true, "o" for false:

_o_ 1. The quantity R(t) is the fraction of the motors which we expect to have failed at time t (or earlier). *Note: R(t) is the fraction we expect to survive until time t.*

_+_ 2. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

_+_ 3. The Weibull CDF, i.e., F(t), gives, for each motor, the probability that it has failed at or before time t.

_o_ 4. We assumed in this HW (#6) that the number of motor failures at time t , $N_f(t)$, has a Weibull distribution. *Note:* $N_f(t)/N$ *is assumed to have Weibull distribution.*

_+_ 5. According to the results of this homework exercise, the failure rate of the motors is increasing rather than decreasing. *Note: k >1 indicates increasing failure rate, and in this case k>3.*

_+_ 6. Given only the coefficient of variation for the Weibull distribution (the ratio σ/μ), the parameter k can be determined.

_+_ 7. The fraction of the machines which are expected to fail in the time interval $[t_{i-1}, t_i]$ is $F(t_i) - F(t_{i-1})$ where F(t) is the CDF of the failure time distribution.

_o_ 8. A positive value of k indicates an increasing failure rate, and negative k indicates a decreasing failure rate. *Note: k>1 indicates increasing failure rate, k<1 indicates decreasing failure rate.*

_+_ 9. The method used in this HW (#6) to estimate the Weibull parameters u & k does not require that the motors be tested until all have failed.

_+_ 10. The CDF of the failure time of a motor is assumed to be $F(t) = 1 - e^{-\left(t/u\right)^k}$ for some parameters u & k.

_o_ 11. $\Gamma(n) = n!$ if n is an integer. *Note:* $\Gamma(1 + n) = n!$

_o_ 12. The time between the failures in the batch of 200 motors is assumed to have the Weibull distribution. *Note: the lifetime is assumed to have the Weibull distribution.*

_+_ 13. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

_e_ 14. The Reliability of a device with random failure time T is defined as

a. $R(t) = P\{t\}$ b. $R(x) = P\{T \mid t\}$ c. $R(x) = P\{T = t\}$
d. $R(t) = P\{t \mid T\}$ e. $R(t) = P\{T \geq t\}$ f. $R(x) = P\{T \leq t\}$

Select the letter below which indicates each correct answer:

In order to estimate the Weibull parameters by the method of today's homework,

| ln t | 16. The variable plotted on the horizontal axis should be ...

| $\ln \ln {}^1/_{Rt}$ | 17. The variable plotted on the vertical axis should be ...

| k | 18. The slope of the line should be approximately ...

| $-k \ln u$ | 19. The vertical intercept of the line should be approximately ...

a. t
b. R_t
c. shape parameter k
d. ln t
e. ln R_t
f. scale parameter u
g. $\ln {}^1/_t$
h. $\ln {}^1/_{Rt}$
i. mean value μ
j. ln ln t
k. ln ln R_t
l. standard deviation σ
m. $\ln \ln {}^1/_t$
n. $\ln \ln {}^1/_{Rt}$
o. – ln u
p. –k ln u
q. – u ln k
r. ln k

Name ____________________

❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍

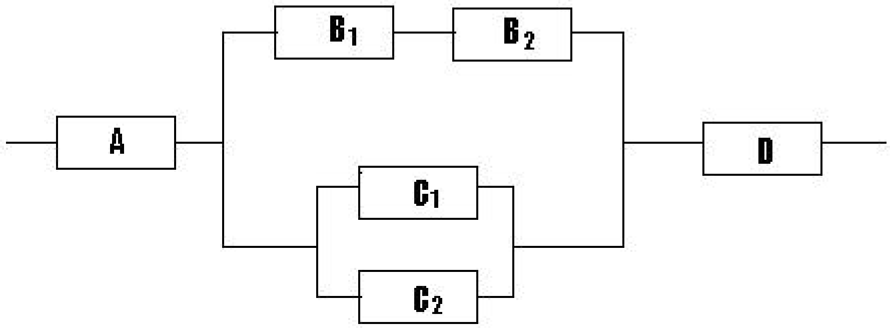
57:022 Principles of Design II

Quiz #7

Friday, March 24, 2000

❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍ ❍

1. A system contains 4 types of devices, with the system reliability represented schematically by



It has been estimated that the lifetime probability distributions of the device C is Exponential, with mean 2000 days.

1. For each scenario, indicate in the "System" (last) column whether the system fails ("X" indicates component failure):

Scenario	A	B1	B2	C1	C2	D	System
a		X				X	
b	X						
c			X	X			
d			X		X		
e		X		X			

___ 2. Suppose that the lifetime probability distributions of the device C is Exponential, with mean 2000 days. Then the reliability of device **C1** above for a designed system lifetime of 1000 days is:

a. $1 - e^{-1000}$
b. $1 - e^{-1}$
c. e^{-2}
d. e^{-1}
e. $1 - e^{-0.5}$
f. $e^{-0.5}$
g. *None of the above*

___ 3. Suppose the following component reliabilities:

A: 80% B1&B2: 90% C1&C2: 70% D. 90%

Then the system reliability is:

a. $0.8 \times \left[(0.9)^2 \left(1 - (0.7)^2\right)\right] \times 0.9 = 0.297432$

b. $0.8 \times \left(1 - (0.9)^2\right) \times (0.3)^2 \times 0.9 = 0.012312$

c. $0.8 \times \left[1 - (0.9)^2 \left(1 - (0.3)^2\right)\right] \times 0.9 = 0.124488$

d. $0.8 \times \left[1 - \left(1 - (0.9)^2\right)(0.3)^2\right] \times 0.9 = 0.707688$

e. $0.8 \times \left(1 - (0.9)^2\right) \times (0.7)^2 \times 0.9 = 0.067032$

f. *None of the above*

Name ____________________

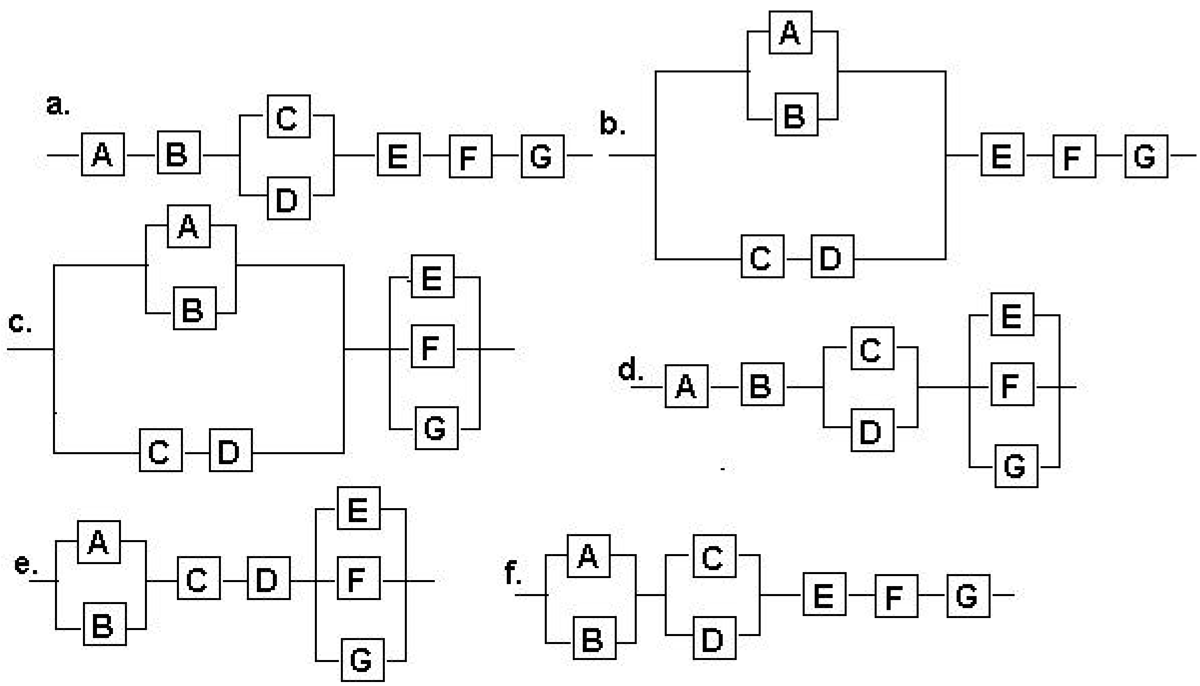
A system has 6 components which are subject to failure, each having lifetimes with *exponential* distributions. The average lifetimes are:

Component	Average Lifetime
A	2000 days
B	3000 days
C	800 days
D	800 days
E	500 days
F	500 days
G	500 days

The system design is such that the system will fail if any one of the following occur:

- Both A and B fail
- Either C or D
- All of E, F, &G fail

___4. Which diagram below represents the system above?

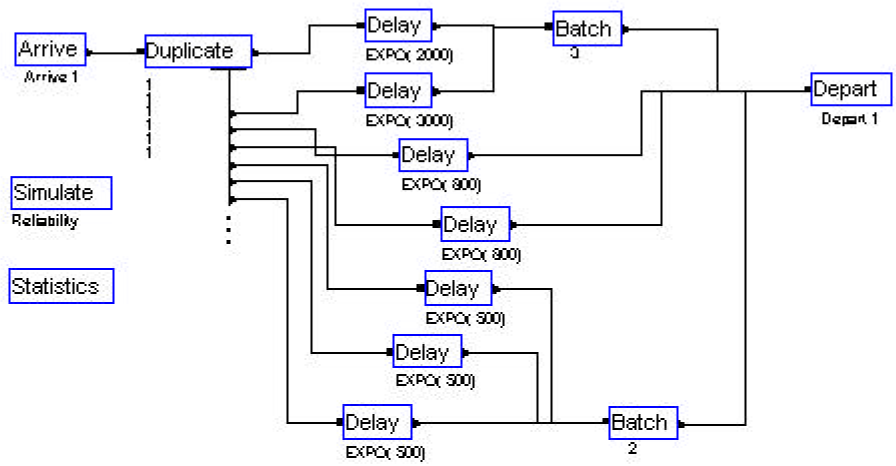


g. *None of the above*

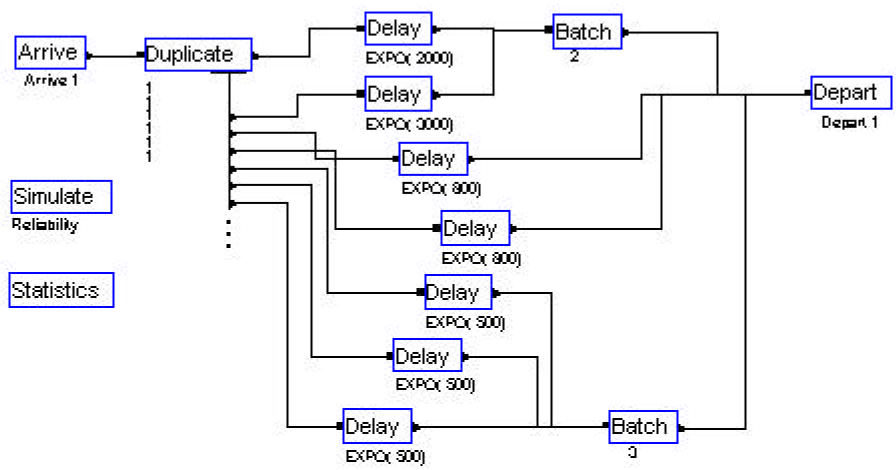
Name ____________________

___ 5. Which of the ARENA models below would be appropriate for this system?

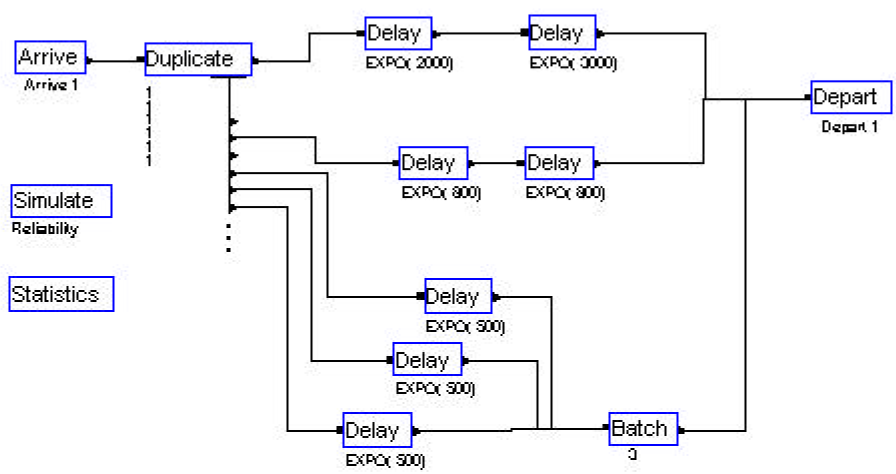
a.



b.

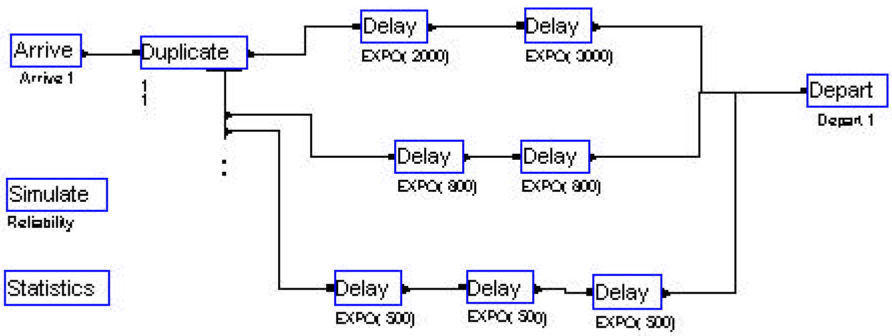


c.



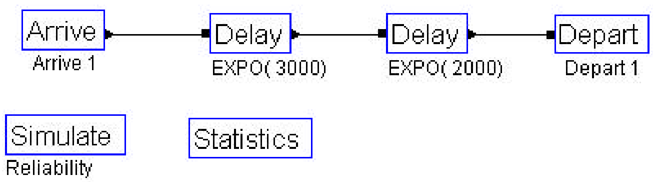
Name ____________________

d.

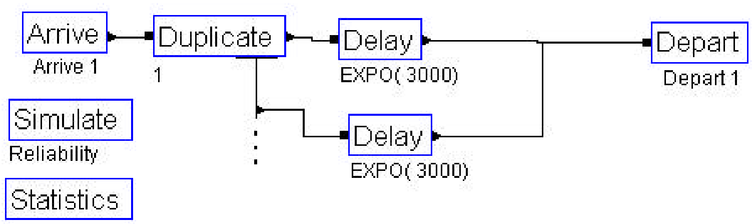


6. Match the three ARENA models below to the diagrams:

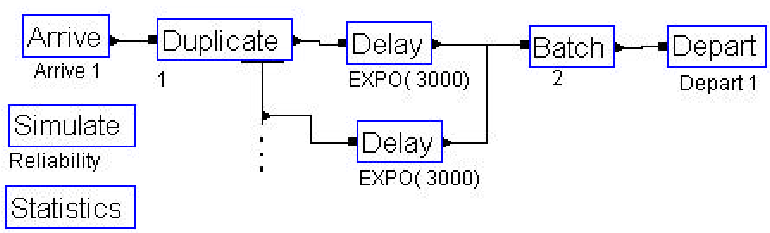
___a.

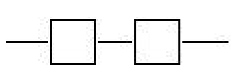
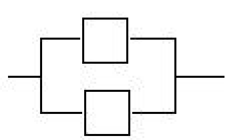
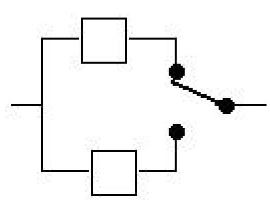


___b.



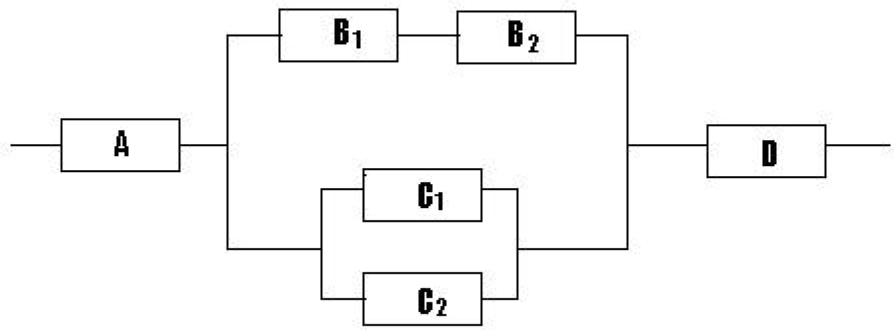
___c.



DIAGRAMS:1.  2.  3. 

57:022 Principles of Design II
Quiz #7
Friday, March 24, 2000

1. A system contains 4 types of devices, with the system reliability represented schematically by



1. For each scenario, indicate in the "System" (last) column whether the system fails ("X" indicates component failure):

Scenario	A	B1	B2	C1	C2	D	System
a		X				X	**X**
b	X						**X**
c			X	X			
d			X		X		
e		X		X			

_f_ 2. Suppose that the lifetime probability distributions of the device C is Exponential, with mean 2000 days. Then the reliability of device **C1** above for a designed system lifetime of 1000 days is:

a. $1 - e^{-1000}$ b. $1 - e^{-1}$ c. e^{-2}
d. e^{-1} e. $1 - e^{-0.5}$ f. $e^{-0.5}$
g. *None of the above*

_d_ 3. Suppose the following component reliabilities:
A: 80% B1&B2: 90% C1&C2: 70% D. 90%

Then the system reliability is:

a. $0.8 \times \left[(0.9)^2\left(1 - (0.7)^2\right)\right] \times 0.9 = 0.297432$

b. $0.8 \times \left(1 - (0.9)^2\right) \times (0.3)^2 \times 0.9 = 0.012312$

c. $0.8 \times \left[1 - (0.9)^2\left(1 - (0.3)^2\right)\right] \times 0.9 = 0.124488$

d. $\boxed{0.8 \times \left[1 - \left(1 - (0.9)^2\right)(0.3)^2\right] \times 0.9 = 0.707688}$

e. $0.8 \times \left(1 - (0.9)^2\right) \times (0.7)^2 \times 0.9 = 0.067032$

f. *None of the above*

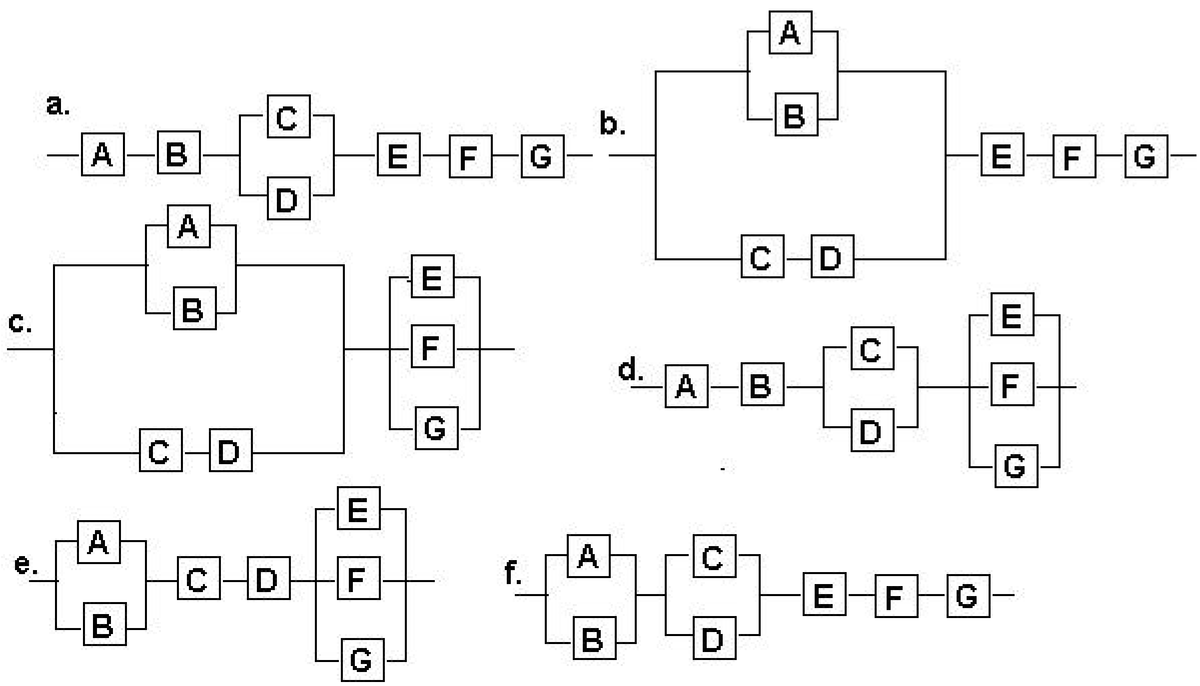
A system has 6 components which are subject to failure, each having lifetimes with *exponential* distributions. The average lifetimes are:

Component	Average Lifetime
A	2000 days
B	3000 days
C	800 days
D	800 days
E	500 days
F	500 days
G	500 days

The system design is such that the system will fail if any one of the following occur:

- Both A and B fail
- Either C or D
- All of E, F, &G fail

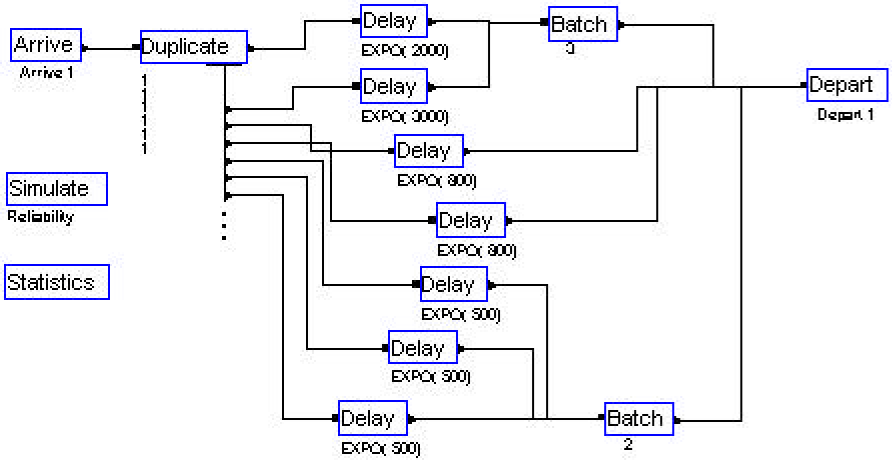
_e_4. Which diagram below represents the system above?



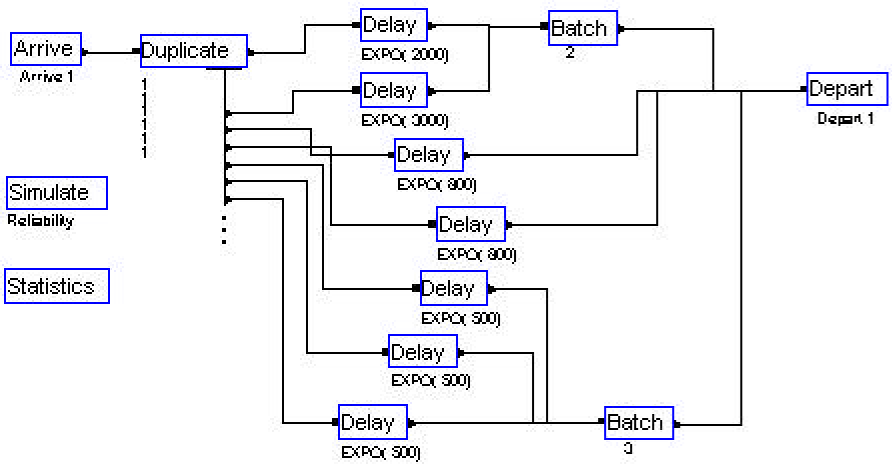
g. *None of the above*

_b_ 5. Which of the ARENA models below would be appropriate for this system?

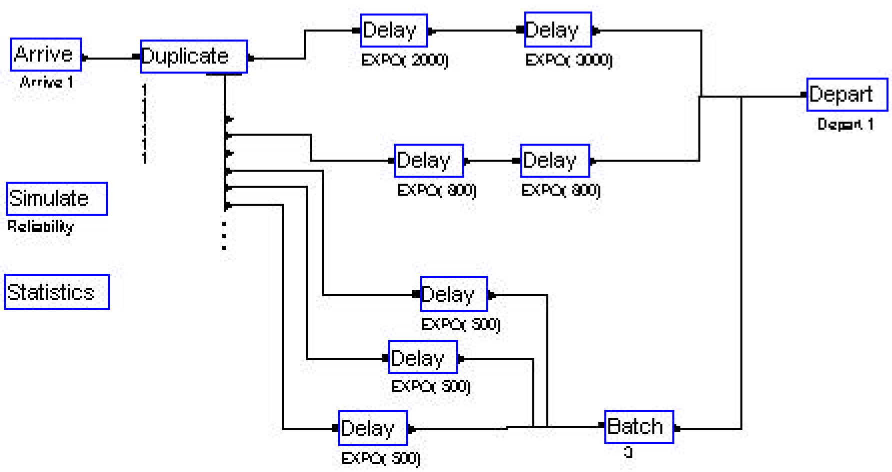
a.



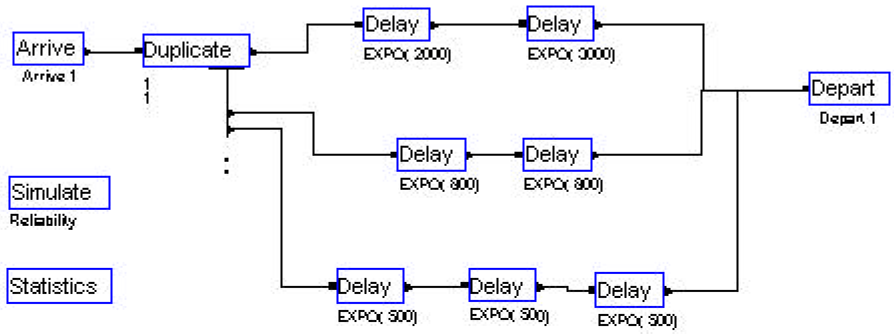
b.



c.

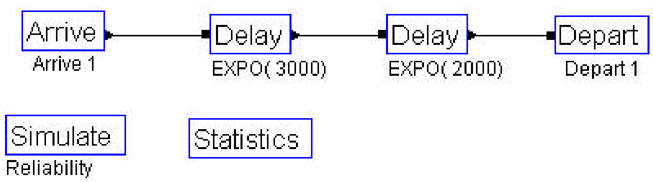


d.

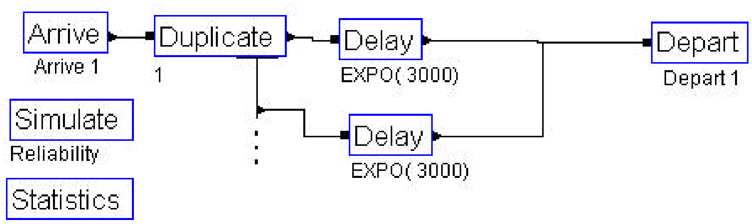


6. Match the three ARENA models below to the diagrams:

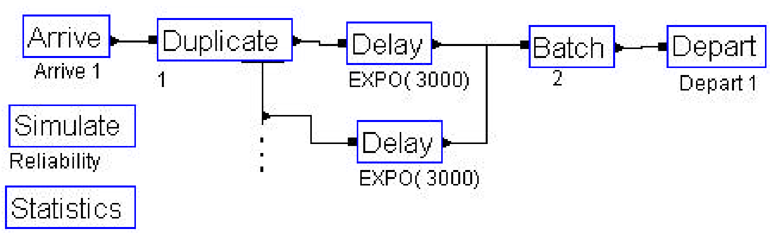
_3_a.

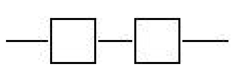
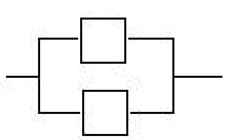
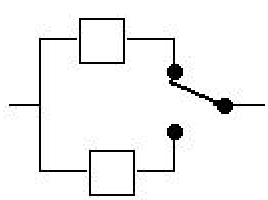


_1_b.



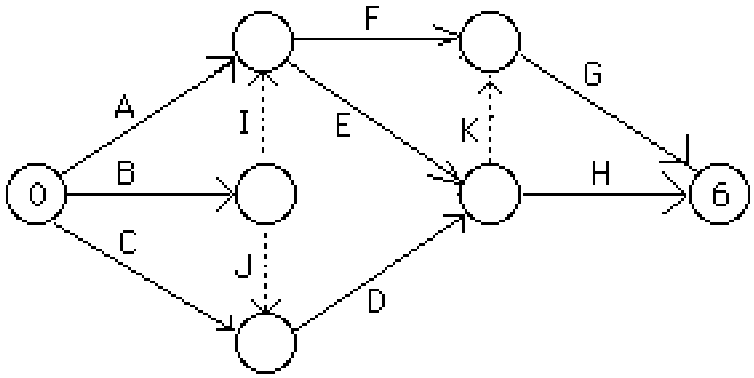
_2_c.



DIAGRAMS:1.  2.  3. 

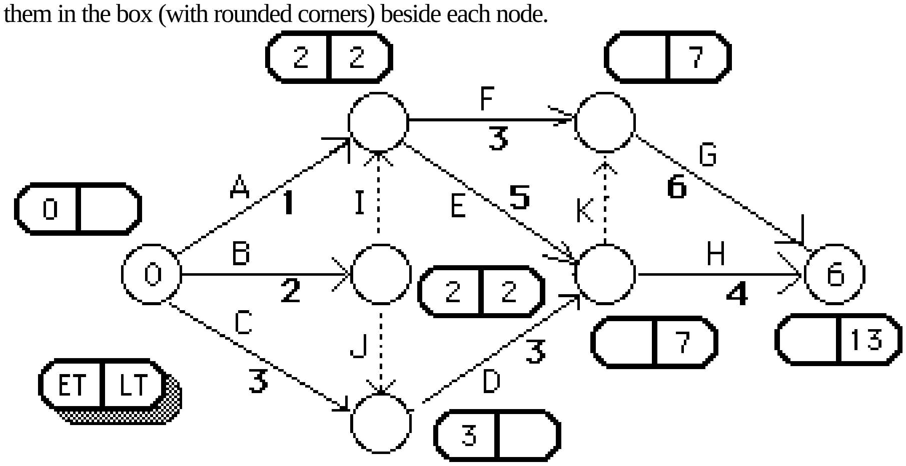
Name ___________________________

❍❍❍ 57:022 Principles of Design II ❍❍❍
❍❍❍ Quiz #8 - April 10, 2000 ❍❍❍



a. Complete the labeling of the nodes on the A-O-A project network above.

b. The activity durations are given below on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node.

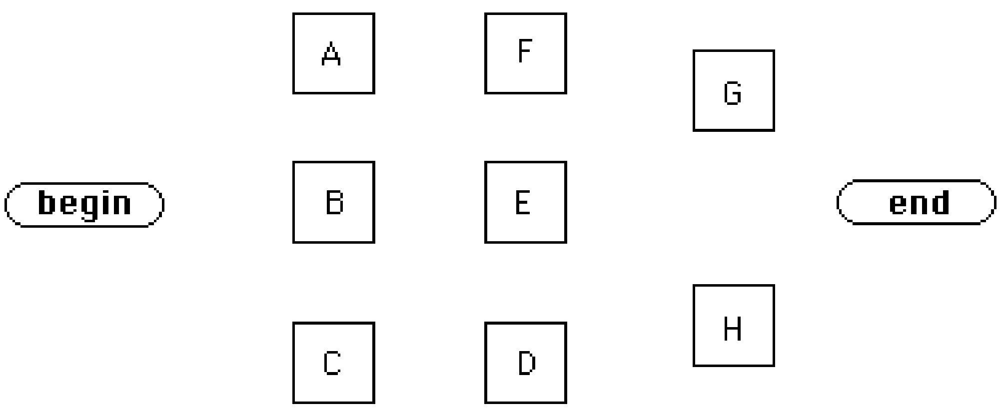


d. Find the slack ("total float") for activity D. _________

e. Which activities are critical? (circle: A B C D E F G H I J K)

f. What is the earliest completion time for the project? ___________

g. Complete the A-O-N (activity-on-node) network below for this same project. (Add any "dummy" activities which are necessary.)

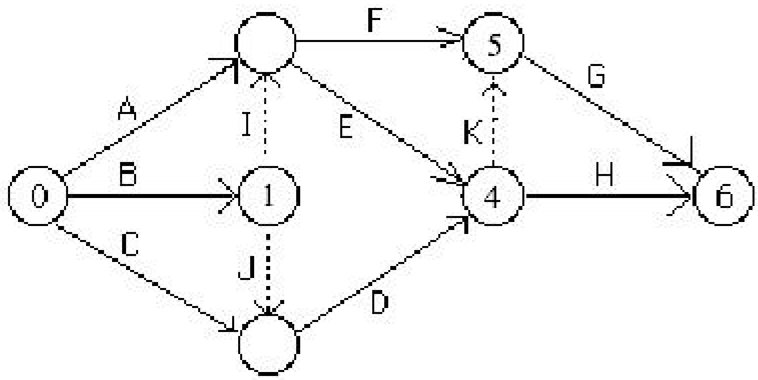


Suppose that the durations of the activities are all random variables with the expected values as given, and standard deviations equal to 1.

i. According to PERT, the duration of the project will have Normal distribution with mean _____ and standard deviation ____ .

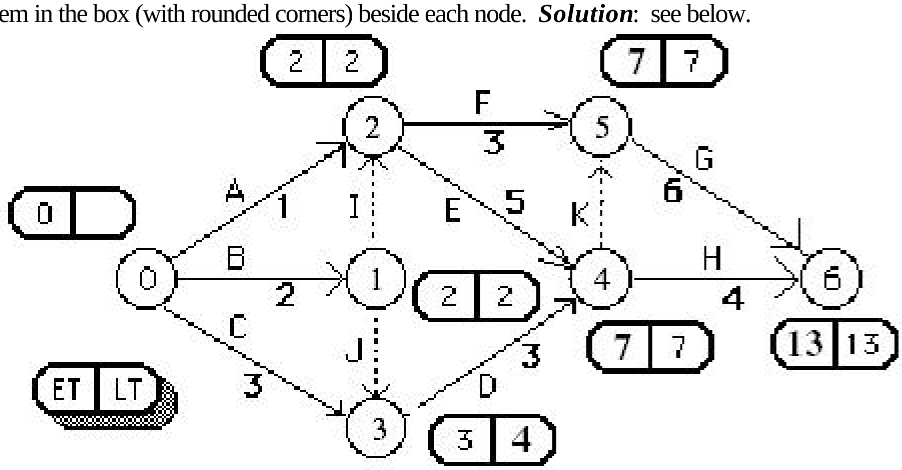
h. In the ARENA model to simulate this project, there should be ___ DUPLICATE nodes and ___ BATCH nodes.

❍❍❍ 57:022 Principles of Design II ❍❍❍
❍❍❍ Quiz #8 Solutions -- Spring 2000 ❍❍❍



a. Complete the labeling of the nodes on the A-O-A project network above.
Solution: see above. *Note: one of the two nodes not labeled above should be #2 and the other #3.*

b. The activity durations are given below on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node, writing them in the box (with rounded corners) beside each node. ***Solution***: see below.

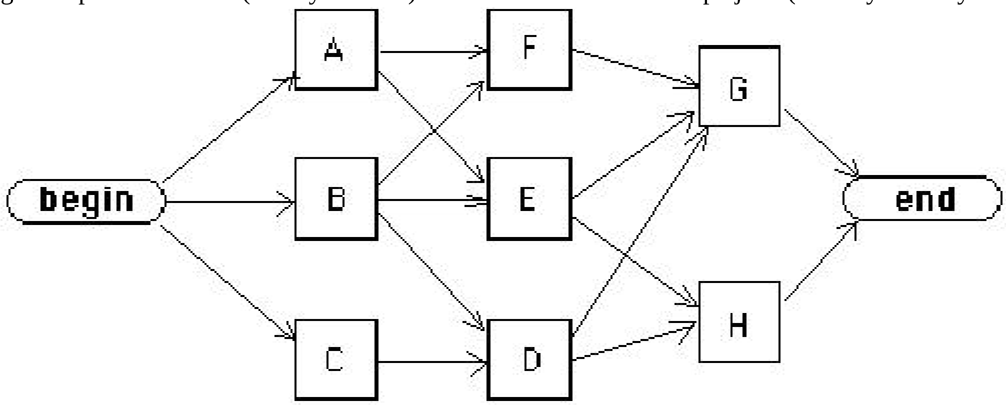


d. Find the slack ("total float") for activity D. __1 day__
Solution: Early Start for D is ET(3)=3, and Late Finish is LT(4)=7. Since the duration is 3 days, Late Start of D is (Late Finish of D) – 3=6 days. Therefore Slack of D is Early Start – Late Start = 7 – 6 = 1 day.

e. Which activities are critical? (circle: A [B] C D [E] F [G] H [I] J [K])

f. What is the earliest completion time for the project? __13 days__

g. Complete the A-O-N (activity-on-node) network below for this same project. (Add any "dummy" activities which are necessary.)



***Solution**:* see above. *Note that although "dummy" activities might be used, corresponding to the dummy activities in the AOA network, they are not necessary.*

Suppose that the durations of the activities are all random variables with the expected values as given, and standard deviations equal to 1.

i. According to PERT, the duration of the project will have Normal distribution with mean_13 days_ and standard deviation _1.732 .

Solution: If CP denotes the set of activities on the critical path, then since the variance of the sum is the sum of the variances,

$$\sigma^2_{total} = \sum_{j \in CP} \sigma_j^2 = 3 \Rightarrow \sigma_{total} = \sqrt{3}$$

h. In the ARENA model to simulate this project, there should be _5_DUPLICATE nodes and _6_ BATCH nodes.

Solution: Duplicate nodes will be required at exit of "Begin" node as well as A, B, D & E (where more than one arrow leaves the node). Batch nodes will be required at entrance to nodes D, E, F, G, H, & "End"

Name ____________________________________

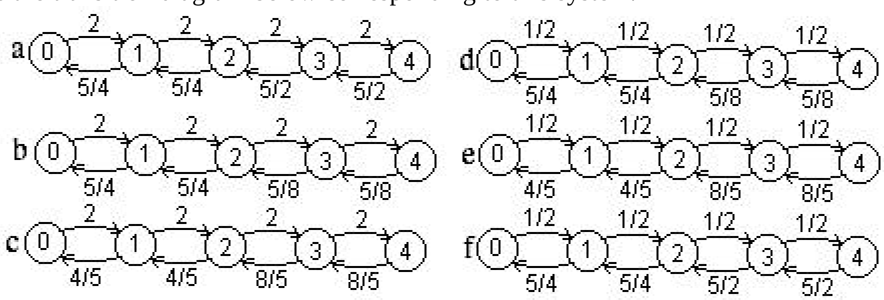
<><><><><> 57:022 Principles of Design II <><><><><>

Quiz #9 -- April 17, 2000

Consider the following situation:

- A neighborhood grocery store has only one check-out counter.
- Customers arrive at the check-out at a rate of one per 2 minutes.
- The grocery store clerk requires an average of one minute and 15 seconds to serve each customer. However, as soon as the waiting line exceeds 2 customers, *including* the customer being served, the manager helps by packing the groceries, which reduces the average service time by 50%. *(Note that still only one customer at a time is being served!)*
- Assume a Poisson arrival process and exponentially-distributed service times. Assume for ease of computation neglible probability that the queue includes more than 3 customers (four, counting the one being served).

____ 1. Choose the transition diagram below corresponding to this system.



g. None of the above

____ 2. The steady-state probability π_0 is computed by the formula:

a. $\frac{1}{\pi_0} = 1 + \frac{2}{4/5} + \frac{2}{4/5} + \frac{2}{8/5} + \frac{2}{8/5}$

b. $\frac{1}{\pi_0} = 1 + \frac{2}{4/5} + \frac{2}{4/5} \times \frac{2}{4/5} + \frac{2}{4/5} \times \frac{2}{4/5} \times \frac{2}{8/5} + \frac{2}{4/5} \times \frac{2}{4/5} \times \frac{2}{8/5} \times \frac{2}{8/5}$

c. $\frac{1}{\pi_0} = 1 + \frac{2}{5/4} + \frac{2}{5/4} \times \frac{2}{5/4} + \frac{2}{5/4} \times \frac{2}{5/4} \times \frac{2}{5/8} + \frac{2}{5/4} \times \frac{2}{5/4} \times \frac{2}{5/8} \times \frac{2}{5/8}$

d. $\frac{1}{\pi_0} = 1 + \frac{1/2}{5/4} + \frac{1/2}{5/4} + \frac{1/2}{5/8} + \frac{1/2}{5/8}$

e. $\frac{1}{\pi_0} = 1 + \frac{1/2}{4/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} \times \frac{1/2}{8/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} \times \frac{1/2}{8/5} \times \frac{1/2}{8/5}$

f. $\frac{1}{\pi_0} = 1 + \frac{1/2}{5/4} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} \times \frac{1/2}{5/8} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} \times \frac{1/2}{5/8} \times \frac{1/2}{5/8}$

g. *None of the above*

The steady-state probabilities for this system are:

π_0=46%, π_1=29%, π_2=18%, π_3=6% & π_4=2%.

____ 3. What fraction of the day will the checkout area be empty? *Choose nearest answer:*

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 4. What fraction of the day will the manager be working in the checkout area? *Choose nearest answer:*

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

Name ___________________________________

____ 5. What is the average number of customers in the checkout area? *Choose nearest answer:*

a. 0.2	c. 0.6	e. 1.0	g. 1.4
b. 0.4	d. 0.8	f. 1.2	h. 1.6

____ 6. What is the average number of customers waiting to be served? (Choose nearest answer.)

a. 0.2	c. 0.6	e. 1.0	g. 1.4
b. 0.4	d. 0.8	f. 1.2	h. 1.6

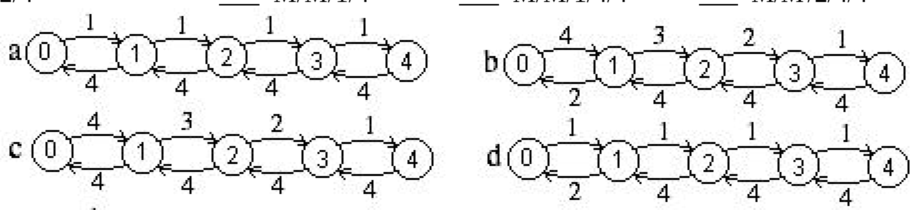
Suppose that the average arrival rate in steady state is approximately one every 2 minutes *(not the actual value)*.

____ 7. According to Little's Formula, the average total time spent by a customer in the checkout area is *(choose nearest value):*

a. 1 minute	c. 1.5 minutes	e. 2 minutes	g. 2.5 minutes
b. 1.25 minutes	d. 1.75 minutes	f. 2.25 minutes	h. > 2.5 minutes

Match the birth/death diagram with the queue classification:

___ M/M/2/4 ___ M/M/1/4 ___ M/M/1/4/4 ___ M/M/2/4/4



Name ___________________________________

<>\<><><> 57:022 Principles of Design II <><><><>

Quiz #9 -- April 17, 2000

Consider the following situation:

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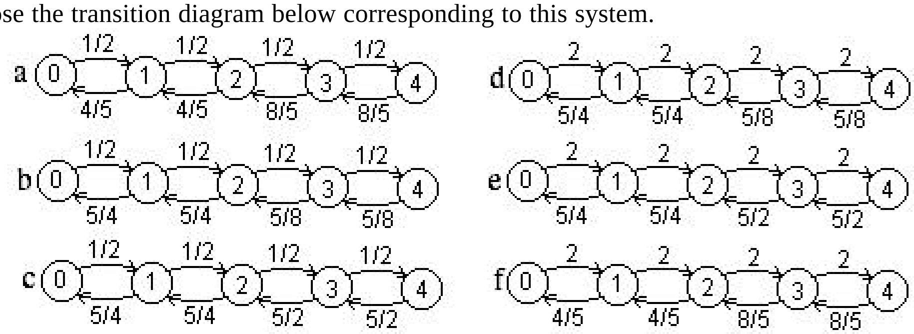
- Customers arrive at the check-out at a rate of one per 2 minutes.
- The grocery store clerk requires an average of one minute and 15 seconds to serve each customer. However, as soon as the waiting line exceeds 2 customers, *including* the customer being served, the manager helps by packing the groceries, which reduces the average service time by 50%. *(Note that still only one customer at a time is being served!)*
- Assume a Poisson arrival process and exponentially-distributed service times. Assume for ease of computation that the queue never includes more than 4 customers (five, counting the one being served).

____ 1. Choose the transition diagram below corresponding to this system.



g. None of the above

____ 2. The steadystate probability π_0 is computed by the formula:

a. $\frac{1}{\pi_0} = 1 + \frac{1/2}{5/4} + \frac{1/2}{5/4} + \frac{1/2}{5/8} + \frac{1/2}{5/8}$

b. $\frac{1}{\pi_0} = 1 + \frac{1/2}{5/4} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} \times \frac{1/2}{5/8} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} \times \frac{1/2}{5/8} \times \frac{1/2}{5/8}$

c. $\frac{1}{\pi_0} = 1 + \frac{1/2}{4/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} \times \frac{1/2}{8/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} \times \frac{1/2}{8/5} \times \frac{1/2}{8/5}$

d. $\frac{1}{\pi_0} = 1 + \frac{2}{4/5} + \frac{2}{4/5} \times \frac{2}{4/5} + \frac{2}{4/5} \times \frac{2}{4/5} \times \frac{2}{8/5} + \frac{2}{4/5} \times \frac{2}{4/5} \times \frac{2}{8/5} \times \frac{2}{8/5}$

e. $\frac{1}{\pi_0} = 1 + \frac{2}{5/4} + \frac{2}{5/4} \times \frac{2}{5/4} + \frac{2}{5/4} \times \frac{2}{5/4} \times \frac{2}{5/8} + \frac{2}{5/4} \times \frac{2}{5/4} \times \frac{2}{5/8} \times \frac{2}{5/8}$

f. $\frac{1}{\pi_0} = 1 + \frac{2}{4/5} + \frac{2}{4/5} + \frac{2}{8/5} + \frac{2}{8/5}$

g. *None of the above*

Suppose that the steady-state probabilities for this system are:

π_0=46%, π_1=29%, π_2=18%, π_3=6% & π_4=2%.

____ 3. What fraction of the day will the checkout area be empty? *Choose nearest answer:*

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 4. What fraction of the day will the manager be working in the checkout area? *Choose nearest answer:*

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

____ 5. What is the average number of customers in the checkout area? *Choose nearest answer:*

a. 0.2	c. 0.6	e. 1.0	g. 1.4
b. 0.4	d. 0.8	f. 1.2	h. 1.6

____ 6. What is the average number of customers waiting to be served? (Choose nearest answer.)

a. 0.2	c. 0.6	e. 1.0	g. 1.4
b. 0.4	d. 0.8	f. 1.2	h. 1.6

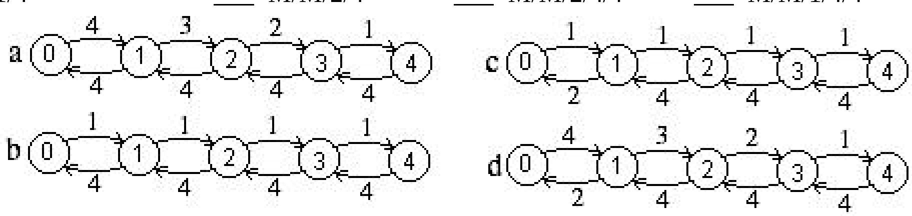
Suppose that the average arrival rate in steady state is approximately one every 2 minutes *(not the actual value)*.

____ 7. According to Little's Formula, the average total time spent by a customer in the checkout area is *(choose nearest value):*

a. 1 minute	c. 1.5 minutes	e. 2 minutes	g. 2.5 minutes
b. 1.25 minutes	d. 1.75 minutes	f. 2.25 minutes	h. > 2.5 minutes

Match the birth/death diagram with the queue classification:

___ M/M/1/4 ___ M/M/2/4 ___ M/M/2/4/4 ___ M/M/1/4/4

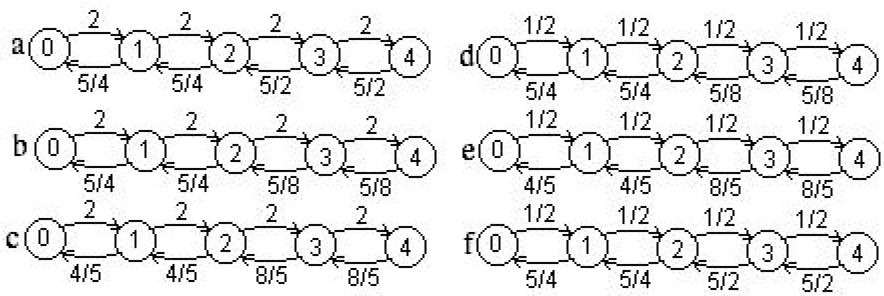


<><><><> 57:022 Principles of Design II <><><><>
Quiz #9 Solutions -- April 17, 2000

Consider the following situation:

- A neighborhood grocery store has only one check-out counter.
- Customers arrive at the check-out at a rate of one per 2 minutes.
- The grocery store clerk requires an average of one minute and 15 seconds to serve each customer. However, as soon as the waiting line exceeds 2 customers, *including* the customer being served, the manager helps by packing the groceries, which reduces the average service time by 50%.
- Assume a Poisson arrival process and exponentially-distributed service times. Assume for ease of computation that the queue never includes more than 4 customers (five, counting the one being served).

__e_ 1. Choose the transition diagram below corresponding to this system.



g. None of the above

_f__ 2. The steadystate probability π_0 is computed by the formula:

a. $\frac{1}{\pi_0} = 1 + \frac{2}{4/5} + \frac{2}{4/5} + \frac{2}{8/5} + \frac{2}{8/5}$

b. $\frac{1}{\pi_0} = 1 + \frac{2}{4/5} + \frac{2}{4/5} \times \frac{2}{4/5} + \frac{2}{4/5} \times \frac{2}{4/5} \times \frac{2}{8/5} + \frac{2}{4/5} \times \frac{2}{4/5} \times \frac{2}{8/5} \times \frac{2}{8/5}$

c. $\frac{1}{\pi_0} = 1 + \frac{2}{5/4} + \frac{2}{5/4} \times \frac{2}{5/4} + \frac{2}{5/4} \times \frac{2}{5/4} \times \frac{2}{5/8} + \frac{2}{5/4} \times \frac{2}{5/4} \times \frac{2}{5/8} \times \frac{2}{5/8}$

d. $\frac{1}{\pi_0} = 1 + \frac{1/2}{5/4} + \frac{1/2}{5/4} + \frac{1/2}{5/8} + \frac{1/2}{5/8}$

e. $\frac{1}{\pi_0} = 1 + \frac{1/2}{5/4} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} \times \frac{1/2}{5/8} + \frac{1/2}{5/4} \times \frac{1/2}{5/4} \times \frac{1/2}{5/8} \times \frac{1/2}{5/8}$

f. $\frac{1}{\pi_0} = 1 + \frac{1/2}{4/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} \times \frac{1/2}{8/5} + \frac{1/2}{4/5} \times \frac{1/2}{4/5} \times \frac{1/2}{8/5} \times \frac{1/2}{8/5}$

g. *None of the above*

The steady-state probabilities for this system are:

π_0=46%, π_1=29%, π_2=18%, π_3=6% & π_4=2%.

__e_ 3. What fraction of the day will the checkout area be empty? *Choose nearest answer:*
Solution: π_0=46%,

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

__a_ 4. What fraction of the day will the manager be working in the checkout area? *Choose nearest answer:* ***Solution:*** $\pi_3 + \pi_4$=8%.

a. 10%	c. 30%	e. 50%	g. 70%
b. 20%	d. 40%	f. 60%	h. *NOTA*

__e_ 5. What is the average number of customers in the checkout area? *Choose nearest answer:* ***Solution:*** $L = 0\times\pi_0 + \pi_1 + 2\pi_2 + 3\pi_3 + 4\pi_4 = 0.91$

a. 0.2	c. 0.6	e. 1.0	g. 1.4
b. 0.4	d. 0.8	f. 1.2	h. 1.6

__b_ 6. What is the average number of customers waiting to be served? (Choose nearest answer.) ***Solution:*** $L_q = 0\times\pi_0 + 0\times\pi_1 + 1\times\pi_2 + 2\pi_3 + 3\pi_4 = 0.36$

a. 0.2	c. 0.6	e. 1.0	g. 1.4
b. 0.4	d. 0.8	f. 1.2	h. 1.6

Suppose that the average arrival rate in steady state is approximately one every 2 minutes *(not the actual value)*.

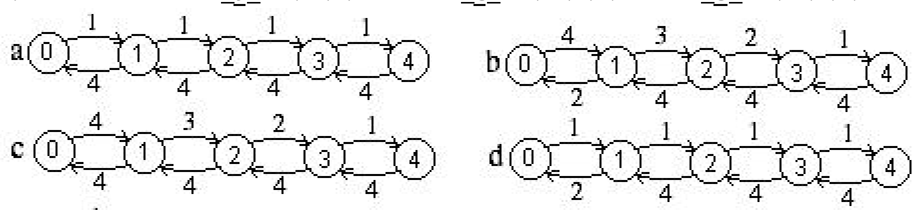
__d_ 7. According to Little's Formula, the average total time spent by a customer in the checkout area is *(choose nearest value):*

Solution: $L=\lambda W \Rightarrow W = L/\lambda = 0.91/0.5 = 1.82$ minutes

a. 1 minute	c. 1.5 minutes	e. 2 minutes	g. 2.5 minutes
b. 1.25 minutes	d. 1.75 minutes	f. 2.25 minutes	h. > 2.5 minutes

Match the birth/death diagram with the queue classification:

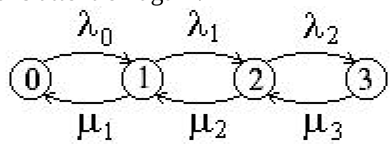
_d_ M/M/2/4 _a_ M/M/1/4 _c_ M/M/1/4/4 _b_ M/M/2/4/4



Name ___________________________________

<><><><> 57:022 Principles of Design II <><><><>
Quiz #10 -- April 24, 2000

Consider the following situation:
A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading) has exponential distribution with mean 15 minutes. The machine will then run unattended for an average of 1 hour (but with actual time having exponential distribution) before it requires the operator's attention again.



_____ 1. The Markov chain model diagrammed above is *(select one or more)*:
- a. an M/M/1/3/3 queue
- b. a Poisson process
- c. a Birth-Death process
- d. an M/M/1 queue
- e. an M/M/3 queue
- f. an M/M/1/3 queue

___ 2. The value of λ_2 is
- a. 1/hr.
- b. 2/hr.
- c. 3/hr.
- d. 4/hr.
- e. 0.5/hr.
- f. *none of the above*

___ 3. The value of μ_2 is
- a. 1/hr.
- b. 2/hr.
- c. 3/hr.
- d. 4/hr.
- e. 0.5/hr.
- f. *none of the above*

___ 4. The value of λ_0 is
- a. 1/hr.
- b. 2/hr.
- c. 3/hr.
- d. 4/hr.
- e. 0.5/hr.
- f. *none of the above*

___ 5. The steady-state probability π_0 is computed by solving
- a. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 \approx \frac{1}{0.366}$
- b. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2}\times\frac{3}{4} + \frac{1}{4}\times\frac{1}{2}\times\frac{3}{4} \approx \frac{1}{0.451}$
- c. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \approx \frac{1}{0.4}$
- d. $\frac{1}{\pi_0} = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.753}$
- e. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.496}$
- f. *none of the above*

___ 6. The operator will be busy what fraction of the time? *(choose nearest value)*
- a. 30%
- b. 35%
- c. 40%
- d. 45%
- e. 50%
- f. 55%
- g. 60%
- h. 65%
- i. 70%

___ 7. What fraction of the time will the operator be busy but with no machine waiting to be serviced? *(choose nearest value)*
- a. 30%
- b. 35%
- c. 40%
- d. 45%
- e. 50%
- f. 55%
- g. 60%
- h. 65%
- i. 70%

___ 8. Approximately 2.2 machines per hour require the operator's attention. What is the average length of time that a machine waits before the operator begins to ready the machine for the next job? *(select nearest value)*
- a. 0.1 hr. (i.e., 6 min.)
- b. 0.15 hr. (i.e., 9 min.)
- c. 0.2 hr. (i.e., 12 min.)
- d. 0.25 hr. (i.e., 15 min.)
- e. 0.3 hr. (i.e., 18 min.)
- f. greater than 0.33 hr. (i.e., >20 min.)

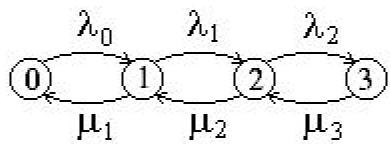
___ 9. What will be the utilization of this group of 3 machines? *(choose nearest value)*
- a. 30%
- b. 35%
- c. 40%
- d. 45%
- e. 50%
- f. 55%
- g. 60%
- h. 65%
- i. 70%

<><><><> 57:022 Principles of Design II <><><><>

Quiz #10 Solution -- Spring 2000

Consider the following situation:

A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading) has exponential distribution with mean 15 minutes. The machine will then run unattended for an average of 1 hour (but with actual time having exponential distribution) before it requires the operator's attention again.



_b,c,g 1. The Markov chain model diagrammed above is *(select one or more):*

a. a discrete-time Markov chain b. a continuous-time Markov chain
c. a Birth-Death process d. an M/M/1 queue
e. an M/M/3 queue f. an M/M/1/3 queue
g. an M/M/1/3/3 queue h. a Poisson process

Note: in the answers below, the state of the system is defined to be the number of machines which require the operator's attention.

_a_ 2. The value of λ_2 is

a. 1/hr. b. 2/hr.
c. 3/hr. d. 4/hr.
e. 0.5/hr. f. *none of the above*

_d_ 3. The value of μ_2 is

a. 1/hr. b. 2/hr.
c. 3/hr. d. 4/hr.
e. 0.5/hr. f. *none of the above*

_c_ 4. The value of λ_0 is

a. 1/hr. b. 2/hr.
c. 3/hr. d. 4/hr.
e. 0.5/hr. f. *none of the above*

_b_ 5. The steady-state probability π_0 is computed by solving

a. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 \approx \frac{1}{0.366}$ b. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2}\times\frac{3}{4} + \frac{1}{4}\times\frac{1}{2}\times\frac{3}{4} \approx \frac{1}{0.451}$

c. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \approx \frac{1}{0.4}$ d. $\frac{1}{\pi_0} = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.753}$

e. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.496}$ f. *none of the above*

_f_ 6. The operator will be busy what fraction of the time? *(choose nearest value)*

a. 30% b. 35% c. 40%
d. 45% e. 50% f. 55%
g. 70% h. 65% i. 70%

_b_ 7. What fraction of the time will the operator be busy but with no machine waiting to be serviced? *(choose nearest value)*

a. 30%	b. 35%	c. 40%
d. 45%	e. 50%	f. 55%
g. 70%	h. 65%	i. 70%

Note: $\pi_1 = \pi_0(3/4) = 34\%$, etc.

i.e., $\pi_0 = 0.4507$, $\pi_1 = 0.338$, $\pi_2 = 0.169$, $\pi_3 = 0.04225$

_f_ 8. Approximately 2.2 machines per hour require the operator's attention. What is the average length of time that a machine waits before the operator begins to ready the machine for the next job? *(select nearest value)*

a. 0.1 hr. (i.e.,6 min.)	b. 0.15 hr. (i.e., 9 min.)
c. 0.2 hr. (i.e., 12 min.)	d. 0.25 hr. (i.e., 15 min.)
e. 0.3 hr. (i.e., 18 min.)	f. greater than 0.33 hr. (i.e., >20 min.)

Note: $L = \sum_{n=0}^{3} n\pi_0 = 0.8$, $W = L/\lambda = 0.8/2.2 = 0.365$

_i_9. What will be the utilization of this group of 3 machines? *(choose nearest value)*

a. 30%	b. 35%	c. 40%
d. 45%	e. 50%	f. 55%
g. 60%	h. 65%	i. 70%

Note: The average number of machines in operation is 3-L = 2.197. Hence, each machine is in use about 2.197/3 = 73% of the time.