57:022 Principles of Design II

Quizzes Fall 1996

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A telephone exchange contains 6 lines. A line can be busy or available for calls and all lines act independently. Each line is busy 75% of the noon period (so that the probability that a line will be busy at any given time during the noon period is 75%).

___ 1. What is the expected number of free lines at any given time during the noon period?

a. 0.75 c. 1.25 e. 1.5
b. 2 d. 2.5 f. None of the above

___ 2. What is the name of the probabity distribution of the number of free lines at any given time?

a. Bernouilli c. Geometric e. Pascal
b. Binomial d. Poisson f. None of the above

___ 3. What is the probability that at any time you call this exchange, all six lines are busy?

a. 0 c. $(0.25)^6 = 0.00024$ e. $(0.75)^6 = 0.178$
b. 0.75 d. 0.25 f. None of the above

Suppose that you call the phone exchange's number exactly once per minute until you get a free line.

___ 4. What is the expected number of calls which you will make (including the final successful call) ?

a. 1 c. 6(0.25) = 1.5 e. $^1/_{0.25} = 2.5$
b. 6(0.75) = 4.5 d. $^6/_{0.25} = 24$ f. None of the above

___ 5. What is the name of the probabity distribution of the total number of calls which you will make?

a. Bernouilli c. Geometric e. Exponential
b. Binomial d. Poisson f. None of the above

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The foreman of a casting section in a certain factory finds that on the average, 1 in every 8 castings made is defective. Today, the factory's output is 10 castings. Let N_{10} be the number of defects in these ten castings.

___ 6. What is the name of the probabity distribution of N_{10} ?

a. Exponential   c. Geometric   e. Pascal
b. Binomial   d. Poisson   f. None of the above

___ 7. What is the expected (mean) value of N_{10} ?

a. $(^1/_8)(1-{}^1/_8) = 0.1094$ c. $8\,(^1/_{10}) = 0.8$ e. 1
b. $10\,(^1/_8) = 1.25$ d. 8   f. None of the above

___ 8. What is the probability that exactly two defects are found when inspecting the ten castings?

a. $\frac{(2)^{10/8}}{2!}\, e^{-2} = 0.1476$ c. $\binom{8}{2}\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^8 = 0.1503$ e. $\frac{(10/8)^2}{2!}\, e^{-10/8} = 0.2054$
b. $\binom{10}{2}\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^8 = 0.2416$ d. $\binom{10}{2}\left(\frac{2}{10}\right)^2\left(\frac{8}{10}\right)^8 = 0.3020$ f. None of the above

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A light bulb in an apartment entrance fails randomly, with an expected lifetime of 20 days, and as soon as it has failed, it is replaced immediately by the custodian. Assume that this bulb's lifetime (denoted by T_1) has an exponential distribution.

___ 9. What is the parameter (λ) for the distribution of T_1?

a. $^1/_{20}$ bulb/day   c. 20 days/bulb   e. 20 days
b. $^1/_{20}$ d. 1 bulb   f. None of the above

For budgeting purposes, the apartment manager is interested in the number N_{yr} of bulbs required during each calendar year.

___ 10. What is the name of the probabity distribution of N_{yr} ?

a. Erlang   c. Geometric   e. Exponential
b. Binomial   d. Poisson   f. None of the above

Suppose that the bulb was installed ten days ago, and has not yet failed.

___ 11. What is now the expected lifetime of the bulb (from installation to failure)?

a. 10 days   c. 15 days   e. 20 days
b. 25 days   d. 30 days   f. None of the above

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Along a certain highway in Iowa, the probability that each passing car stops to pick up a hitchhiker is p=4%; different drivers, of course, make their decisions whether to stop or not to stop independently of each other.

Suppose further that the arrivals of the cars form a Poisson process, at the average rate of 15 per minute. Define the random variables

1. ___ X_i = 1 if car #i stops to pick up the hitchhiker, otherwise 0
2. ___ T_i = time of arrival of car #i (which does not necessarily stop!)
3. ___ Y_1 = time of the <u>first</u> car to stop.
4. ___ τ_i = time between arrival of car #(i-1) and car #i
5. ___ N_t = number of cars which have stopped between time zero and t.
6. ___ Z_1 = number of the first car to stop for the hitchhiker

For each of the six random variables above, indicate (by letter) from the list below the name of their probability distribution:

a. Poisson	d. Exponential	g. Binomial
b. Bernouilli	e. Uniform	h. Erlang (with k>1)
c. Pascal with (k>1)	f. Geometric	i. None of the above

___ 7. The probability that the first car to stop is #3 is

a. $(0.04)^2(0.96) = 0.001536$	d. $(0.04)(0.96) = 0.0384$
b. $e^{-3} = 0.04978$b. 2	e. $1 - e^{-3} = 0.9502$
c. $(0.96)^2(0.04) = 0.03686$	f. None of the above

___ 8. The expected number of the first car to stop is

a. 5	d. 20	g. 35
b. 10	e. 25	h. 40
c. 15	f. 30	i. None of the above

___ 9. The probability that the hitchhiker must wait no more than one minute for a ride is

a. $1 - e^{-15}$	d. e^{-15}	g. $1 - \frac{0.6^{15}}{15!} e^{-15}$
b. $\frac{0.6^{15}}{15!} e^{-15}$	e. $\frac{0.6^{1}}{1!} e^{-0.6}$	h. $\frac{0.6^{-1}}{1!} e^{-0.6}$
c. $1 - e^{-0.6}$	f. $e^{-0.6}$	i. None of the above

Suppose that a procedure for generating uniformly-distributed random numbers has yielded the value R=0.353. We want to generate a random value for Y_1, i.e., the time at which the first car stops.

___ 10. Using the Inverse Transformation method, then according to the table below the nearest value of Y_1 should be

a. 0.25 minute	e. 1.25 minute	i. 2.5 minutes

b. 0.5 minute   f. 1.5 minutes   j. 2.75 minutes
c. 0.75 minute   g. 2 minutes   k. 3 minutes
d. 1 minute   h. 2.25 minutes   l. greater than 3 min.

___ 11. Suppose that the next uniformly-generated random number is 0.619. Then the corresponding arrival time of the second car is (choose nearest value):

a. 0.25 minute   e. 1.25 minute   i. 2.5 minutes
b. 0.5 minute   f. 1.5 minutes   j. 2.75 minutes
c. 0.75 minute   g. 2 minutes   k. 3 minutes
d. 1 minute   h. 2.25 minutes   l. greater than 3 min.

• •

Distribution of Y_1:

x	P{Y≤x}	Δp	P{Y>x}
0	0.00000000	0.00000000	1.00000000
0.25	0.13929202	0.13929202	0.86070798
0.5	0.25918178	0.11988976	0.74081822
0.75	0.36237185	0.10319007	0.63762815
1	0.45118836	0.08881652	0.54881164
1.25	0.52763345	0.07644508	0.47236655
1.5	0.59343034	0.06579689	0.40656966
1.75	0.65006225	0.05663191	0.34993775
2	0.69880579	0.04874354	0.30119421
2.25	0.74075974	0.04195395	0.25924026
2.5	0.77686984	0.03611010	0.22313016
2.75	0.80795009	0.03108025	0.19204991
3	0.83470111	0.02675102	0.16529889

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Consider the proposed design for two drive-up bank teller windows where the arrival rate of customers has increased to 20/hour (one every 3 minutes). The SLAM output appears below. Refer to this output to answer the following questions:

1. How much time did the average customer spend waiting in the queue? ___________

2. What was the average number of busy tellers during the day? ___________

3. What was the average time spent by a customer in the system (i.e. from arrival to departure)? ___________

4. What was the maximum time that any customer spent in the system? ___________

5. How many cars attempted but were unable to enter the queue each day, because the queue was filled to capacity? ___________

6. What percent of the customers spent less than 5 minutes (from arrival to departure) at the bank? ___________

7. How many customers were served during the day? ___________

8. What percent of the time was each teller busy each day, i.e., the utilization of each teller? ___________

9. At the end of the simulated day, how many customers were in the system, i.e., either in the queue or at one of the teller windows? ___________

10. What was the maximum number of cars in the system (including those who might be at one of the teller windows)? ___________

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        ••••••••••••••••••••••••••••••••••••••••
         S L A M   I I   S U M M A R Y   R E P O R T

SIMULATION PROJECT BANKTELLERS              BY BRICKER
DATE  9/12/1996                             RUN NUMBER    1 OF    1

CURRENT TIME    .4800E+03
STATISTICAL ARRAYS CLEARED AT TIME   .0000E+00
```

****STATISTICS FOR VARIABLES BASED ON OBSERVATION****

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	.237E+01	.260E+01	.110E+01	.117E-01	.236E+02	179
OVERFLOW			NO VALUES RECORDED			

****FILE STATISTICS****

FILE NUMBER	LABEL/TYPE	AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	QUEUE	.099	.355	4	0	0.266
2		.000	.000	0	0	.000
3	CALENDAR	1.785	.794	4	1	2.164

****SERVICE ACTIVITY STATISTICS****

ACT NUM	ACT LABEL OR START NODE	SER CAP	AVERAGE UTIL	STD DEV	CUR UTIL	AVERAGE BLOCK	MAX IDL TME/SER	MAX BSY TME/SER	ENT CNT
1	QUEUE	2	.785	.79	0	.00	2.00	2.00	179

****HISTOGRAM NUMBER 1****

CUSTOMER_TIME

```
OBS  RELA   UPPER
FREQ FREQ  CELL LIM 0        20        40        60        80       100
                    +    +    +    +    +    +    +    +    +    +    +
 27  .151  .500E+00 +********                                         +
 30  .168  .100E+01 +********       C                                 +
 28  .156  .150E+01 +********               C                         +
 16  .089  .200E+01 +****                       C                     +
 17  .095  .250E+01 +*****                           C                +
 11  .061  .300E+01 +***                                C             +
 11  .061  .350E+01 +***                                   C          +
  9  .050  .400E+01 +***                                      C       +
  9  .050  .450E+01 +***                                        C     +
  4  .022  .500E+01 +*                                           C    +
  3  .017  .550E+01 +*                                            C   +
  2  .011  .600E+01 +*                                             C  +
  2  .011  .650E+01 +*                                             C  +
  2  .011  .700E+01 +*                                              C +
  2  .011  .750E+01 +*                                              C +
  1  .006  .800E+01 +                                                C+
  0  .000  .850E+01 +                                                C+
  1  .006  .900E+01 +                                                C+
  1  .006  .950E+01 +                                                C+
  0  .000  .100E+02 +                                                C+
  1  .006  .105E+02 +                                                C+
  2  .011    INF    +*                                               C
---                 +    +    +    +    +    +    +    +    +    +    +
179                 0        20        40        60        80       100
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Goodness-of-Fit test: The number of vehicles arriving during each of 25 one-minute intervals was recorded. The mean value of O_i was computed to be 1.76. We wish to test the "goodness of fit" of the Poisson distribution having mean 1.76. Denote the sum of the last column by “D”. (A “Chi-square” table appears below.) Note that we are ignoring the advice to aggregate cells to avoid very small numbers of observations.)

i	O_i	p_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
0	4	0.17204	4.30112	0.02108
1	6	0.30280	7.56997	0.32560
2	11	0.26646	6.66158	2.82544
3	2	0.15633	3.90813	0.93163
4	0	0.06878	1.71958	1.71958
5	2	0.02421	0.60529	3.21369
sum	25	0.99063	24.76566	9.03703

Indicate "+" for true, "o" for false:

____ 1. The smaller the value of D, the better the fit of the distribution being tested.

____ 2. The quantity O_i is a random variable with approximately Poisson distribution (assuming the fitted distribution is correct).

____ 3. The chi-square distribution for this goodness-of-fit test will have 5 "degrees of freedom".

____ 4. The quantity E_i is the expected number of observations of i arrivals (assuming the fitted distribution is correct).

____ 5. The CDF of the distribution of interarrival times is assumed to be F(t) = $1 - e^{-1.76t}$ (assuming the fitted distribution is correct).

____ 6. In this test, the “degrees of freedom” is reduced by 2 because (i) the total number of observations is fixed at 25, and (ii) the data was used to estimate the mean of the distribution being tested.

____ 7. The quantity $(E_i-O_i)^2/E_i$ is assumed to have the normal N(0,1) distribution.

____ 8. The number of observations O_i in interval #i is a random variable with approximately binomial distribution.

____ 9. The parameter of the Poisson distribution is assumed to be λ = 1.76/minute.

____ 10. The chi-square distribution for this test will have 6 "degrees of freedom".

____ 11. The number of observations O_i above should have a binomial distribution, with n (number of “trials”) = 25 and p (probability of “success”) = p_i (assuming the fitted distribution is correct)..

____ 12. The quantity D is assumed to have approximately a Normal distribution.

____ 13. The probability p_2 that no more than 2 cars arrive during a one-minute interval, under the assumption that the arrival process is Poisson, is $e^{-1.76}\frac{1.76^2}{2!}$ (assuming the fitted distribution is correct).

____ 14. The sum of the squares of several N(0,1) random variables has a chi-square distribution.

____ 15. If a chi-square random variable has 7 degrees of freedom, then according to the chi-square table, there is a 10% probability that it will exceed 12.017.

____ 16. The quantity D is assumed to have the chi-square distribution.

____ 17. The time between arrivals in a Poisson process has exponential distribution.

____ 18. If it is true that this is a Poisson arrival process with rate 1.76 /minute, then the probability that D exceeds 9.03 is less than α = 10%.

____ 19. The time of the second arrival in a Poisson process has a Pascal distribution.

____ 20. If we choose α = 10%, the Poisson distribution with mean 1.76 may be accepted as a model for the interarrival times of the vehicles.

deg.of freedom	Chi-square Dist'n $P\{D \geq \chi^2\}$ 99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

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Regression Analysis. Tests on the fuel consumption of a vehicle traveling at different speeds yeilded the following results:

Speed s (mph)	20	30	40	50	60	70	80	90
Consumption C (mile/gal.)	11.4	17.9	22.1	25.5	26.1	27.6	29.2	29.8

(Note: the above data is completely fictitious!)

It is suggested that the relationship between the two variables is of the form $C = a + b/s$.

Cricket Graph Output:

Fig. A:

	1	2	3	4	5
	C	s	1/s	ln C	ln s
1	11.4	20	0.050	2.434	2.996
2	17.9	30	0.033	2.885	3.401
3	22.1	40	0.025	3.096	3.689
4	25.5	50	0.020	3.239	3.912
5	26.1	60	0.017	3.262	4.094
6	27.6	70	0.014	3.318	4.248
7	29.2	80	0.013	3.374	4.382
8	29.8	90	0.011	3.395	4.500

Fig. B:

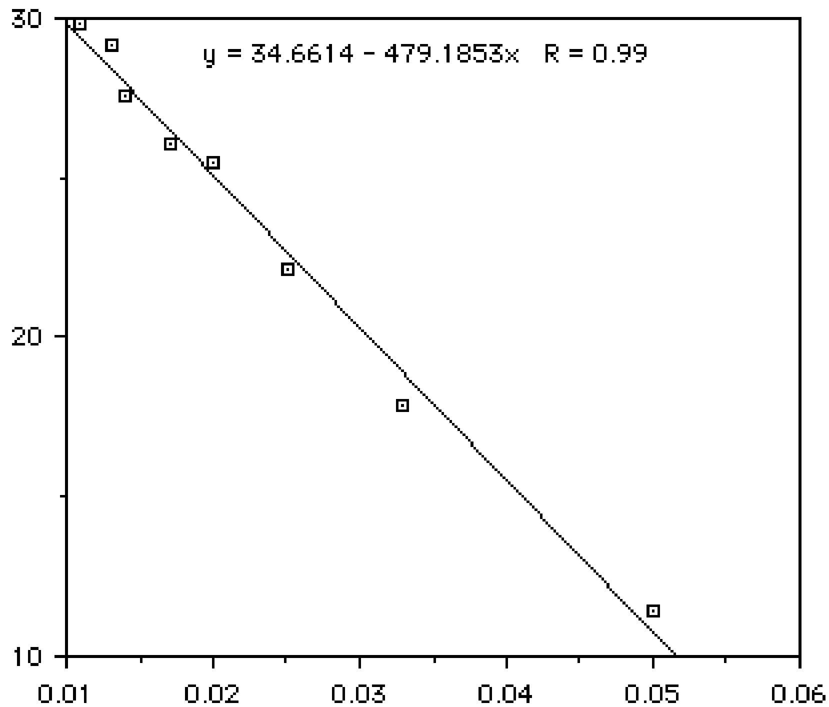
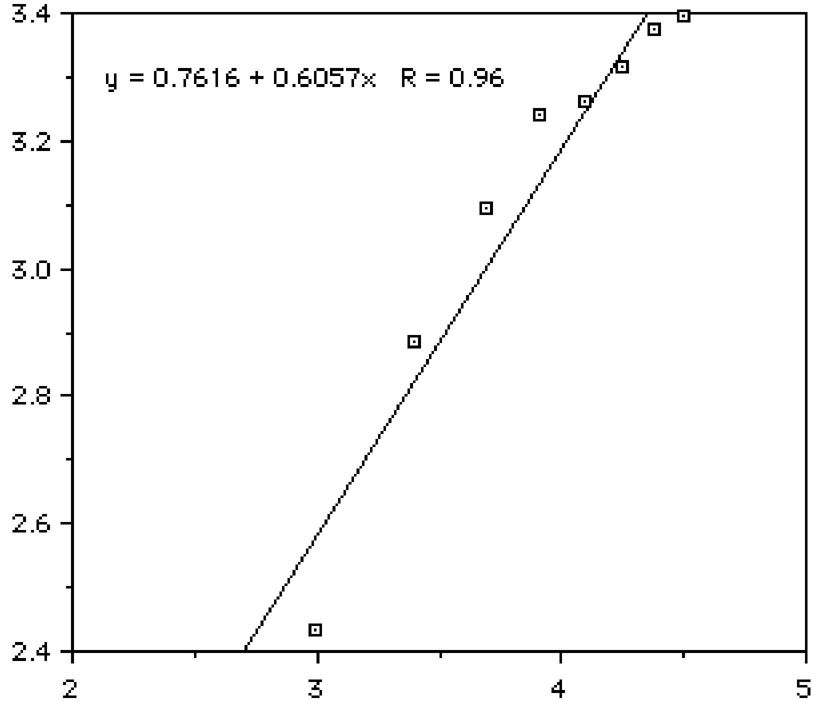

y = 34.6614 - 479.1853x R = 0.99
30
20
10
0.01
0.02
0.03
0.04
0.05
0.06

Fig. C:



Curve Fit 0.7e Output:

Fig.D:

Untitled Data

X Coord.	Y Coord.	Plot	Fit
2.00000E+01	1.14000E+01	☒	☒
3.00000E+01	1.79000E+01	☒	☒
4.00000E+01	2.21000E+01	☒	☒
5.00000E+01	2.55000E+01	☒	☒
6.00000E+01	2.61000E+01	☒	☒
7.00000E+01	2.76000E+01	☒	☒
8.00000E+01	2.92000E+01	☒	☒
9.00000E+01	2.98000E+01	☒	☒

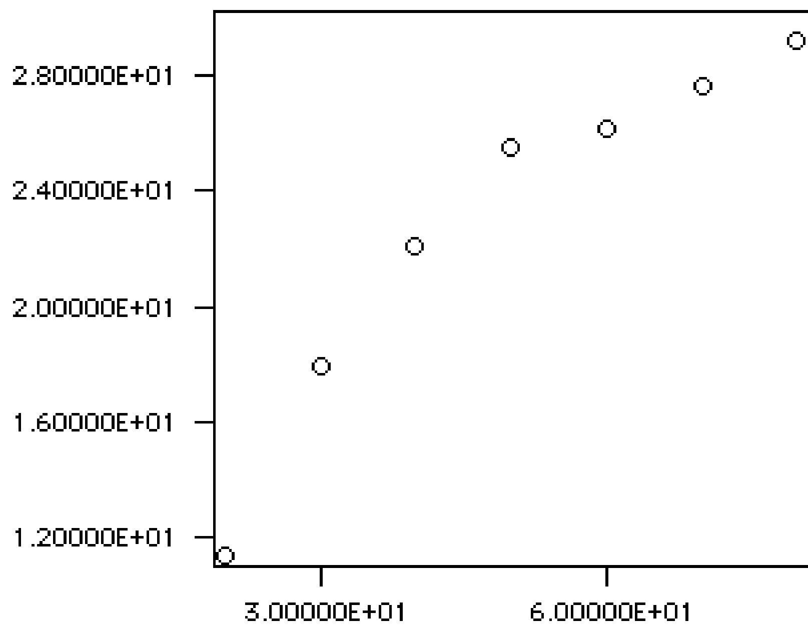
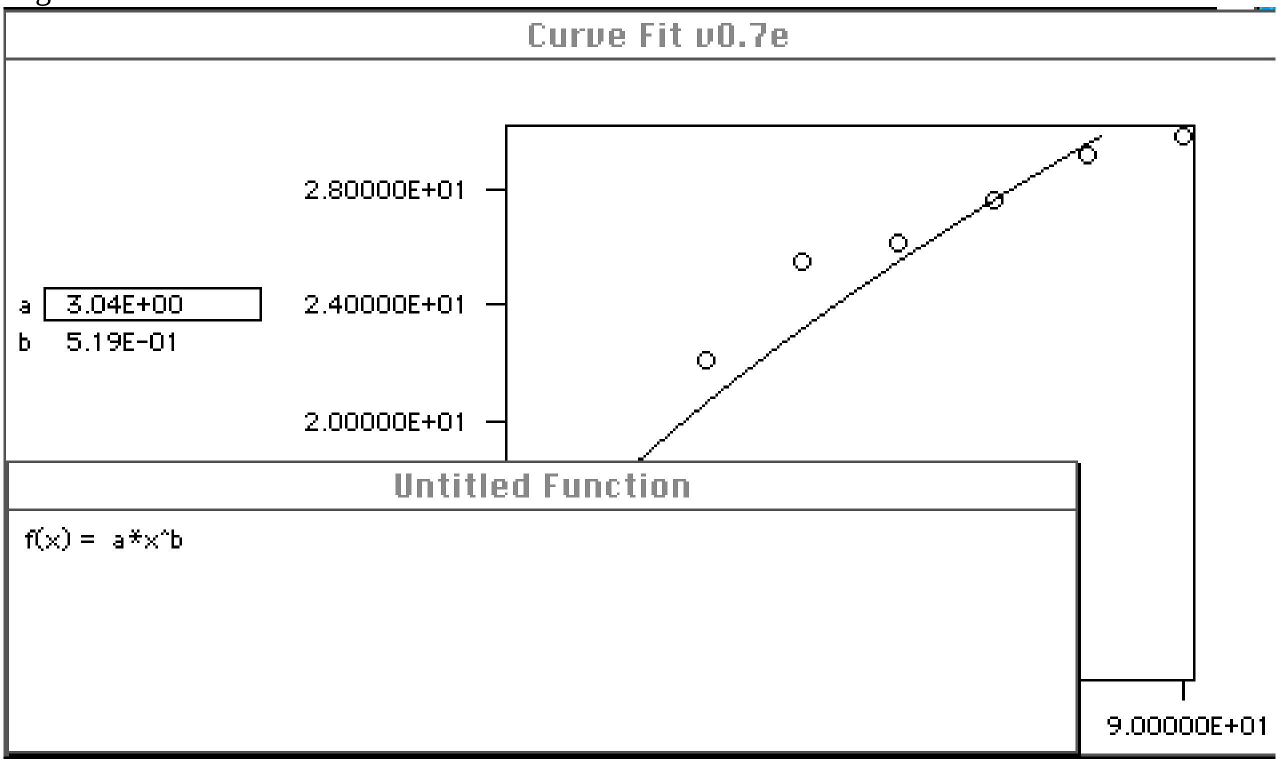


Fig. E



In #1,2, &3 below, state the fitted curve (with numerical values inserted):

1. What is the curve which is fit in Figure B? C = ___________________________

2. What is the curve which is fit in Figure C? C = ___________________________

3. What is the curve which is fit in Figure E? C = ___________________________

Consider the case of the power function $C=as^b$. From the list below, select the objective function which is minimized in each case:

a. $\sum_{i=1}^{8} C_i - as_i^b$

b. $\sum_{i=1}^{8} \left|C_i - as_i^b\right|$

c. $\sum_{i=1}^{8} \left(C_i - as_i^b\right)^2$

d. $\operatorname{Max}_{i=1,\ldots 8} \left\{C_i - as_i^b\right\}$

e. $\operatorname{Max}_{i=1,\ldots 8} \left\{\left|C_i - as_i^b\right|\right\}$

f. $\sum_{i=1}^{8} \left(\ln C_i - \left[\ln a + b\times\ln s_i\right]\right)^2$

g. $\operatorname{Max}_{i=1,\ldots 8} \left\{\left|\ln C_i - \left[\ln a + b\times\ln s_i\right]\right|\right\}$

h. $\sum_{i=1}^{8} 2\left(\ln C_i - \left[\ln a + b\times\ln s_i\right]\right)$

i. $\sum_{i=1}^{8} \left|\ln C_i - \left[\ln a + b\times\ln s_i\right]\right|$

j. None of the above

4. Cricket Graph will minimize expression _____ above when fitting the curve $C=as^b$.
5. Curve Fit 0.7e will minimize expression _____ above when fitting the curve $C=as^b$.

6. The LP software package LINDO, given an appropriate formulation, could minimize expression _______ above in order to fit the curve $C=as^b$.

___ 7. Which (one or more) methods can be used to find the optimal fitted curve in the Curve Fit 0.7e software package?

a. Simplex method b. Steepest descent method
c. Newton's method d. Quasi-Newton method
e. Binary search method f. None of the above

8. Which of the curves that you specified in #1, 2, & 3 provides the best fit?
C =
(Note: To answer this question, you would have to recall the solution of a similar question in HW #5.)

___ 9. In Fig. B, what is plotted on the horizontal axis?
a. C b. s c. 1/s
d. ln C e. ln s f. None of the above

___ 10. In Fig. B, what is plotted on the vertical axis?
a. C b. s c. 1/s
d. ln C e. ln s f. None of the above

___ 11. In Fig. C, what is plotted on the horizontal axis?
a. C b. s c. 1/s
d. ln C e. ln s f. None of the above

___ 12. In Fig. C, what is plotted on the vertical axis?
a. C b. s c. 1/s
d. ln C e. ln s f. None of the above

___ 13. If you were to use Cricket Graph to fit the curve $C = ae^{bs}$ to the data, what would you plot on the horizontal axis?
a. C b. s c. 1/s
d. ln C e. ln s f. None of the above

___ 14. If you were to use Cricket Graph to fit the curve $C = ae^{bs}$ to the data, what would you plot on the vertical axis?
a. C b. s c. 1/s
d. ln C e. ln s f. None of the above

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Some statements below refer specifically to today's homework assignment (HW#6). Indicate "+" for true, "o" for false:

___ 1. We assumed in this HW#6 that the lifetime of the device has a Weibull distribution.
___ 2. The Weibull distribution is often an appropriate model for the minimum of a large number of nonnegative random variables.
___ 3. A positive value of k indicates an increasing failure rate, and negative k indicates a decreasing failure rate for the Weibull probability model.
___ 4. If we were given a coefficient of variation for the Weibull distribution (the ratio σ/μ), the parameter k can be determined.
___ 5. The y-intercept of the line fit by Cricket Graph is the Weibull parameter u.
___ 6. The exponential distribution is a special case of the Weibull distribution, with a constant failure rate.

___ 7. According to the results of HW#6, the failure rate of the mechanical device is increasing.

___ 8. The Weibull CDF, i.e., F(t), gives, for each unit of the device, the probability that it has failed at or before time t.

___ 9. The exponential distribution with parameter λ is a special case of the Weibull distribution, with parameters k=1 and u=λ .

___ 10. We assumed in HW#6 that the number of failures at time t , $N_f(t)$, has a Weibull distribution.

___ 11. The error is defined to be the vertical distance between the data point and a point on the curve.

___ 12. To estimate the time at which 90% of the units will have failed, we evaluate 1 - F(0.90), where F is the CDF of the failure-time distribution.

___ 13. The method used in HW#6 to estimate the Weibull parameters u & k does not require that the units of the device be tested until all have failed.

___ 14. If 10 units of this device are installed in a system, the number still functioning after 500 hours is assumed to have a Weibull distribution.

___ 15. The quantity R_t is the fraction of the devices which have survived until time t.

___ 16. If the failure rate is increasing, it may be more appropriate to use the Gumbel distribution than the Weibull.

___ 17. If the assumption of Weibull distribution were correct, a plot of the data points(t, $N_f(t)$) for t=50, 100, ... , 250 should lie approximately on a straight line.

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In order to estimate Weibull parameters k & u in HW#6, Cricket Graph was used to fit a straight line to some plotted data points. Select the letter below which indicates each correct answer:

___ 18. The label on the horizontal axis should be ...

___ 19. The vertical intercept of the line fit by Cricker Graph should be approximately ...

___ 20. The slope of the line fit by Cricket Graph should be approximately ...

___ 21. The label on the vertical axis should be ...

a. t	h. R_t	o. ln ln R_t
b. ln t	i. ln R_t	p. ln ln $^1/_{R_t}$
c. ln $^1/_t$	j. ln $^1/_{R_t}$	q. mean value μ
d. ln ln t	k. shape parameter k	r. standard deviation σ
e. ln ln $^1/_t$	l. ln k	s. coefficient of variation σ/μ
f. -k ln u	m. -u ln k	t. scale parameter u
g. ln u	n. k ln u	u. -ln k

◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊

___ 22. The "Cumulative Distribution Function" (CDF) of any random variable X is defined to be

a. f(x) = P{x | X}	c. F(x) = P{X≥x}	e. F(x) = P{X≤x}
b. f(x) = P{x}	d. f(x) = P{X|x}	f. F(x) = P{X=x}

◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊◊

Choose the answer to the next two questions from the list below:

a. $\sum_t \left(\ln\ln\frac{1}{R_t} - \left[a+b\ln t\right]\right)$

b. $\sum_t \left(\ln\ln\frac{1}{R_t} - \left[a+b\ln t\right]\right)^2$

c. $\sum_t \left(R_t - \exp\left[-\left(t/u\right)^k\right]\right)^2$

d. $\sum_t \left(F_t - \left[-\left(t/u\right)^k\right]\right)^2$

e. $\sum_t \left(F_t - \exp\left[-\left(t/u\right)^k\right]\right)^2$

f. $\sum_t \left(R_t - \exp\left[-\left(t/u\right)^k\right]\right)$

g. $\sum_t \left(\ln F_t - \left[-\left(t/u\right)^k\right]\right)^2$

h. none of the above

___ 23. The Cricket Graph program fits a straight line which minimizes...

___ 24. The Curve Fit program fits a curve which minimizes ...

• • • • • • • • • • • • • • • Quiz #7 • • • • • • • • • • • • • • • • • •

◊◊◊◊◊◊◊◊◊◊◊ Part One ◊◊◊◊◊◊◊◊◊◊◊

A system consists of five components (A,B,C,D, &E). The probability that each component fails during the first year of operation is 10% for A, B, and C, and 20% for D and E. For each alternative of (a) through (e), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)
- the SLAM network which would simulate the system lifetime

Diagram	Reliability	SLAM network

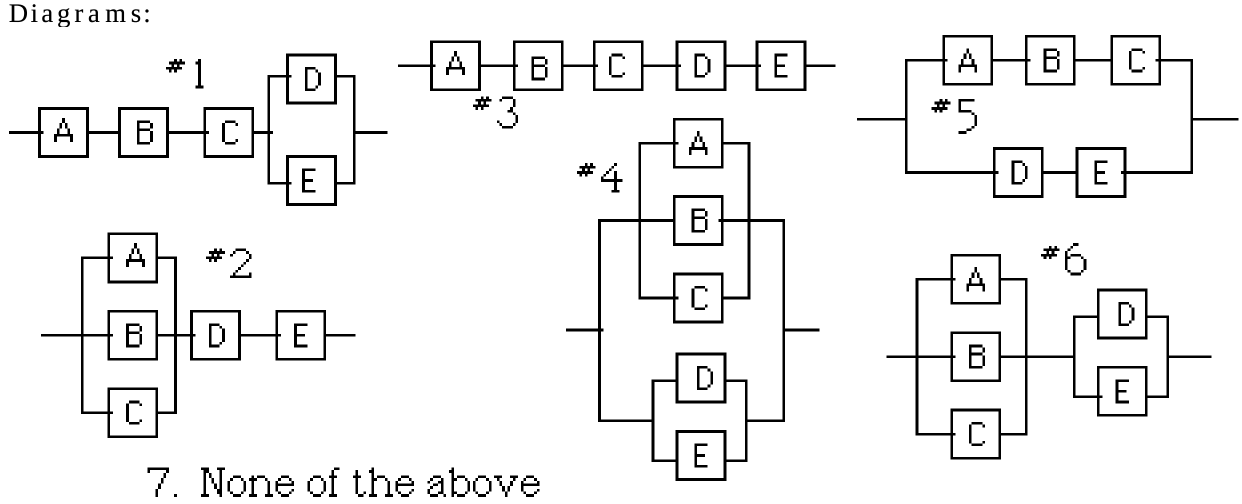
___ ___ ___ a. The system can function if <u>all</u> of A, B, and C function or if <u>both</u> D and E function.

___ ___ ___ b. The system requires <u>all</u> of A, B, & C, and <u>at least one</u> of D & E.

___ ___ ___ c. The system requires that D & E <u>both</u> function, and <u>at least one</u> of A, B, & C function.

___ ___ ___ d. The system requires <u>at least one</u> of A ,B, & C, and <u>at least one</u> of D & E .

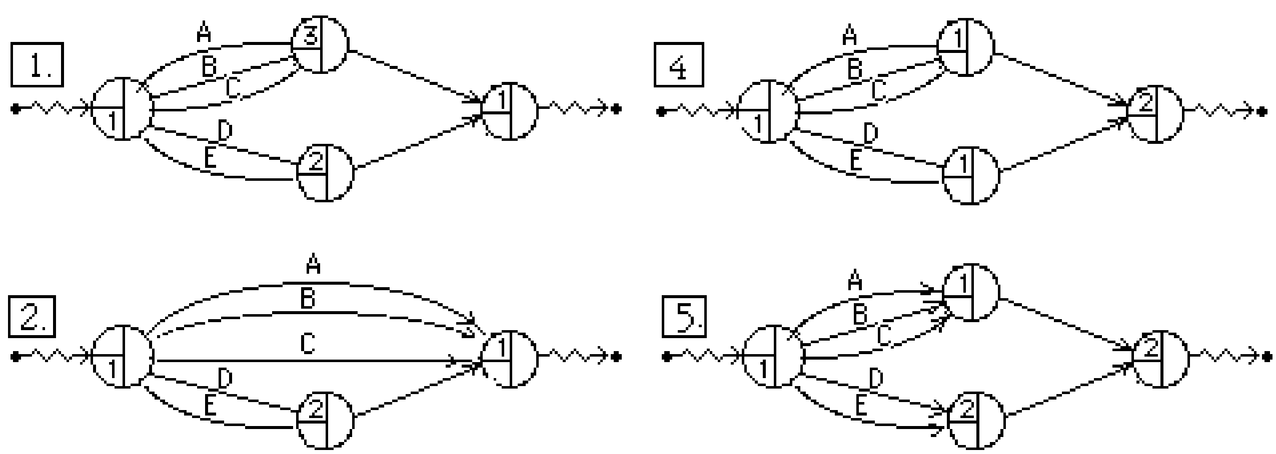
Diagrams:

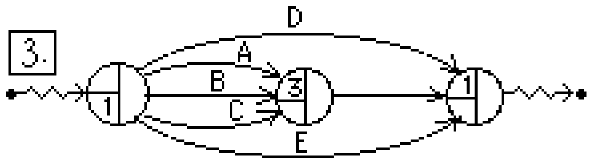


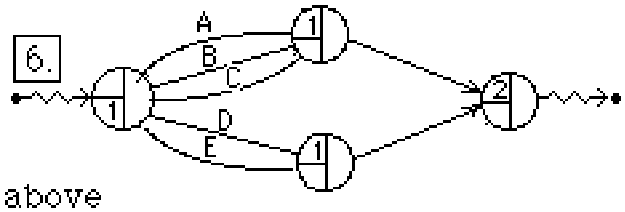
Reliabilities:

1. $[1-0.1^3]\ [1-0.2^2]$ = 95.9%
2. $1 - (0.1)^3(0.2)^2$ = 99.99%
3. $1-(0.9)^3(0.8)^2$ = 53.34%
4. $1- [1-0.9^3]\ [1-0.8^2]$ = 90.24%
5. $0.9^3[1-0.2^2]$ = 69.98%
6. $[1-0.1^3]\ (0.8)^2$ = 63.94%
7. None of the above

SLAM networks:





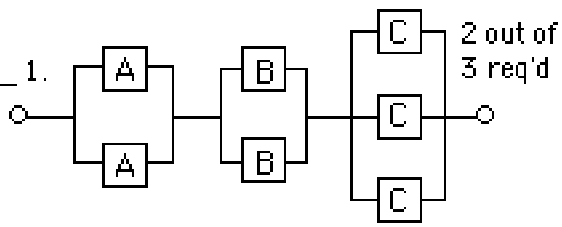


7. None of the above

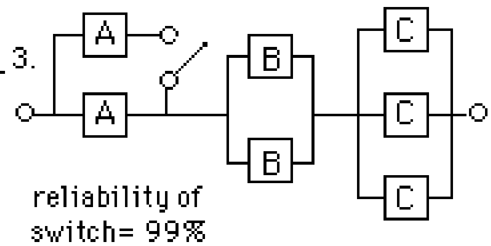
◊◊◊◊◊◊◊◊◊◊◊ Part Two ◊◊◊◊◊◊◊◊◊◊◊

For each system 1-4 below, write the letter (A-H) of the SLAM model which simulates the system lifetime. The switch in the diagram indicates that the back-up copy of A is switched into the system (possibly with less than 100% reliability) when the first copy of A fails. Assume that A, B, C, etc. in the SLAM network represent the lifetime distributions of devices A, B, C, etc.

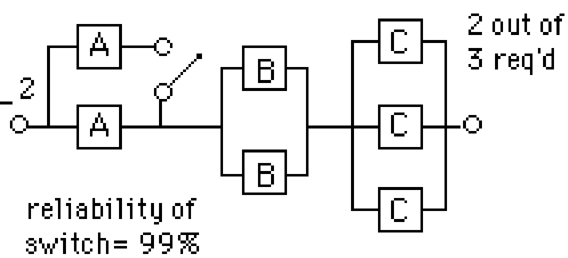
_____ 1.



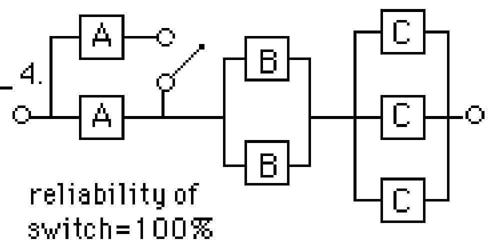
_____ 3.

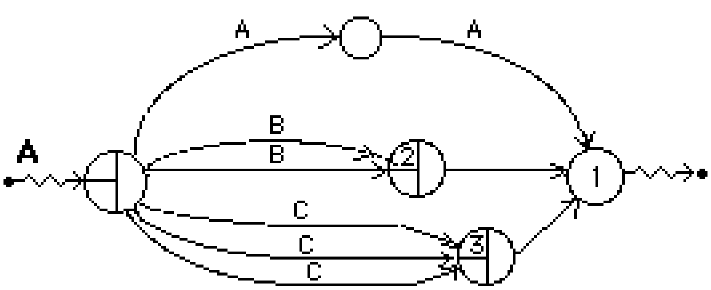


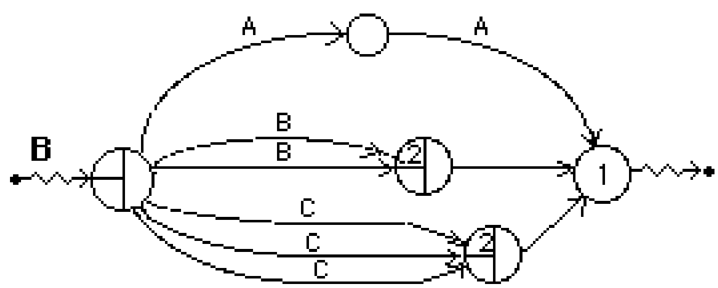
_____ 2

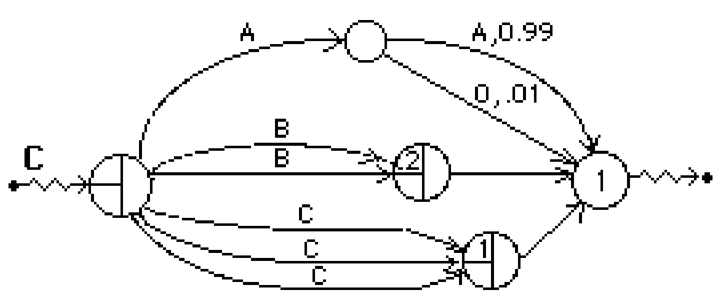


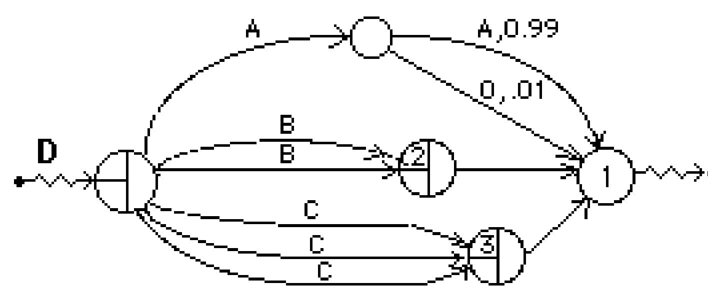
_____ 4.

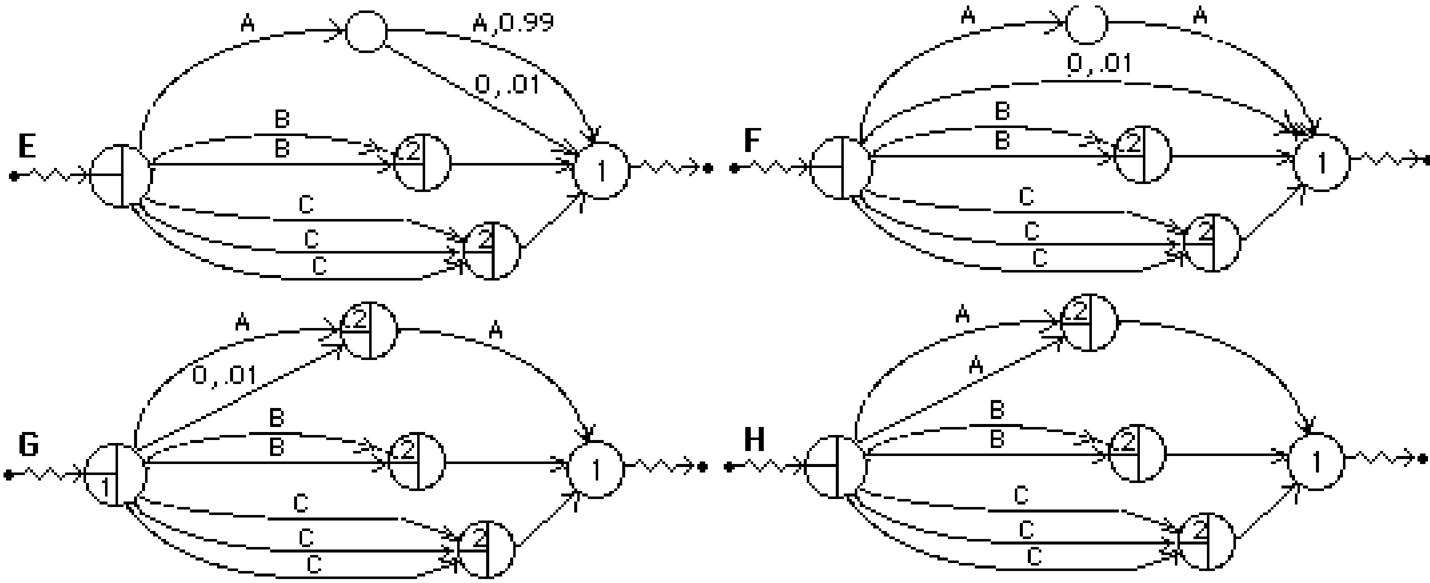












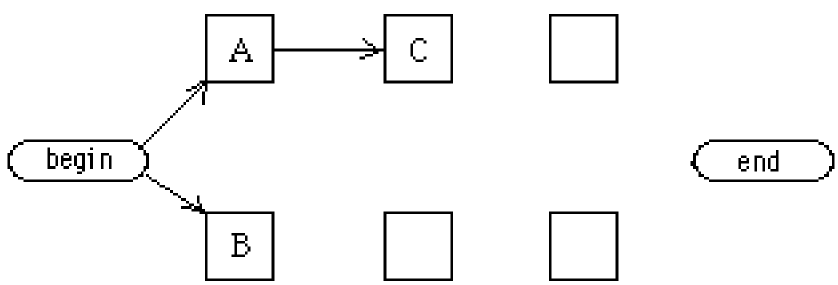
I. None of the above

• • • • • • • • • • • • • • • Quiz #8 • • • • • • • • • • • • • • • • • •

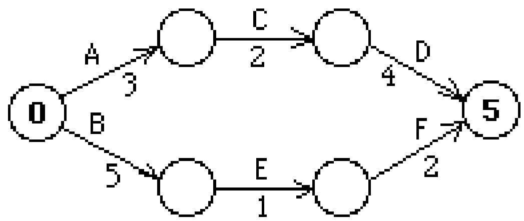
Project Scheduling.   Consider the following project:

Activity	Predecessor Activities	Duration (days) Mean	Std Dev
A	none	3	1
B	none	5	2
C	A	2	1
D	C	4	2
E	B	1	0
F	C, E	2	1

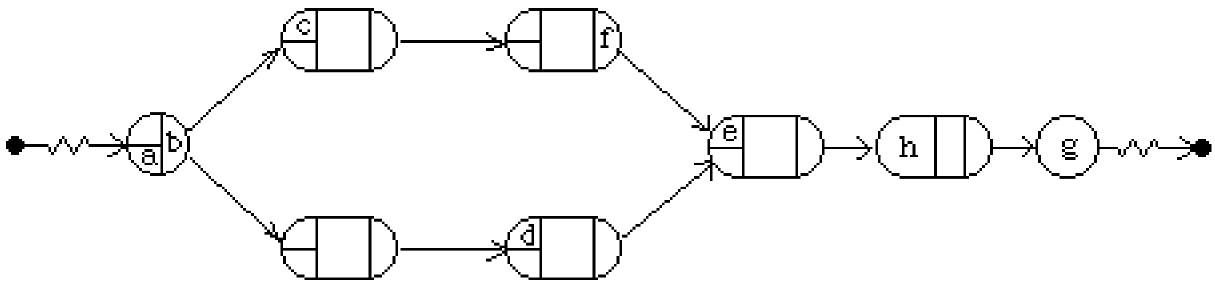
1. Complete the AON network by labeling the nodes and inserting arrows:



2. Complete the AOA network below by inserting any dummy activities which are necessary, and labelling the nodes:

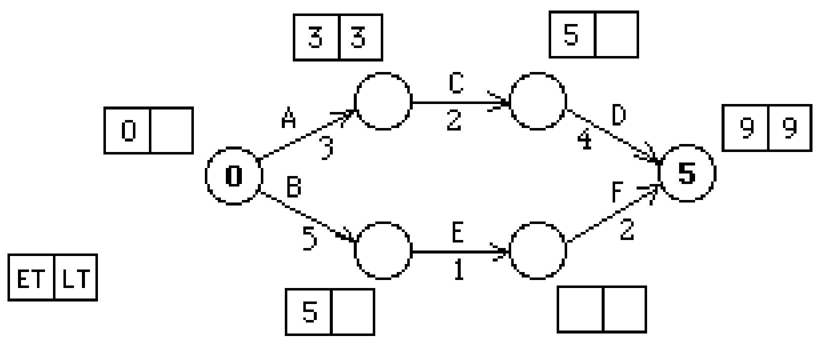


3. Give numerical values (0, 1, 2, 3, 4, or ∞) for parameters "a" - "g" and a statistic type for "h" on the SLAM network below which would simulate the project and collect statistics on the completion time. (You needn't insert the probability distributions for the durations!)



(a)___ (b) ___ (c) ___ (d) ___ (e) ___ (f) ___ (g) ___
(h) circle: (FIRST) (LAST) (BETWEEN) (INTVL) (ALL)

4. Complete the ETs (earliest times) & LTs (latest times) in the network below, using the expected activity durations, as indicated. Don't forget any "dummy" activities which you entered above!



5. What is the "total slack" or "total float" in activity C? _______

6. What is the "total slack" ("total float") in activity E? _______

7. What are the critical activities? Circle: A B C D E F

8. Are any "dummy" activities critical?_______

9. What is the expected length of the project's critical path? _______
...the standard deviation of the length of the project's critical path? _______

• • • • • • • • • • • • • • • • Quiz #9 • • • • • • • • • • • • • • • • • • •

Project Scheduling. Indicate whether true (+) or false (o):

___1. PERT assumes that each activity's duration has a Normal distribution.
___2. If an average of 1000 random project completion times is desired, the TERMINATE node in a SLAM model should indicate that the simulation should stop after 1000 entities have arrived at that node.
___3. In a SLAM model of a project, a node with two activities entering it should be an ACCUMULATE node.
___4. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
___5. In a SLAM model of a project, an entity will leave a node with two activities entering it as soon as the first entity arrives.
___6. A SLAM model of a project employs the "Activity-on-Node" rather than "Activity-on-Arrow" representation of the project.
___7. An ACCUMULATE node may be used to accumulate statistics in successive runs of a simulation model.
___8. If a project is simulated twice by SLAM, each simulation run may have a different critical path of activities.
___9. PERT assumes that the project duration has a beta distribution.
___10. Both the triangular and beta distributions for an activity have the property that they are uniquely specified when you give the minimum & maximum durations and the mode, i.e., a most likely duration.
___11. PERT assumes that the critical path is always the same as that found by assuming all activity durations have their expected values.
___12. If the duration of an activity has triangular distribution, then its expected value is its most likely value, i.e., its mode.
___13. The expected value of the sum of random variables equals the sum of their expected values.
___14. PERT assumes that the project duration has a Normal distribution.
___15. The standard deviation of the sum of random variables equals the sum of their expected values.
___16. In a SLAM simulation of a project, the CREATE node should simultaneously create one entity for every activity in the project.
___17. In a SLAM model of a project, a node with a single activity entering it may be either a GOON or ACCUMULATE node.
___18. Doubling the standard deviation of the duration of each activity in a project will double the standard deviation of the project duration.
___19. In a SLAM model of a project with N activities, the simulation is not terminated until N entities have arrived at the TERMINATE node.
___20. The critical path of a project is the <u>longest</u> path from beginning to end.

• • • • • • • • • • • • • • • •Quiz #10 • • • • • • • • • • • • • • • • • •

Discrete-time Markov Chains

Part I: Consider an inventory system in which the number of items on the shelf is checked at the end of each day. The maximum number on the shelf is 8. If 3 or fewer units are on the shelf, the shelf is refilled overnight. The demand distribution is as follows:

x	0	1	2	3	4	5	6
P{D=x}	.1	.15	.25	.25	.15	.05	.05

The system is modeled as a Markov chain, with the state defined as the number of units on the shelf at the end of each day. The probability transition matrix is:

from \ to	1	2	3	4	5	6	7	8	9
1	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
2	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
3	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
4	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1
5			0.25	0.15	0.1	0	0	0	0
6	0.1	0.15	0.25	0.25	0.15	0.1	0	0	0
7	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0	0
8	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1	0
9	0	0	0.05	0.05	0.15	0.25	0.25	0.15	0.1

The steady-state distribution of the above Markov chain is:

i	π_i
1	0.06471513457
2	0.07698357218
3	0.1304613771
4	0.1355295351
5	0.16322964
6	0.1698706746
7	0.1384131423
8	0.0754980776
9	0.04529884656

The mean first passage matrix is:

from \ to	1	2	3	4	5	6	7	8	9
1	15.45	12.98	7.66	7.37	5.69	5.24	6.39	12.66	22.07
2	15.45	12.98	7.66	7.37	5.69	5.24	6.39	12.66	22.07
3	15.45	12.98	7.66	7.37	5.69	5.24	6.39	12.66	22.07
4	15.45	12.98	7.66	7.37	5.69	5.24	6.39	12.66	22.07
5	12.27	10.49	6.64	7.25	6.12	6.35	7.50	13.77	23.18
6	14.31	11.51	6.47	6.42	5.85	5.88	7.68	13.96	23.37
7	14.63	12.44	7.01	6.24	5.25	5.79	7.22	14.30	23.71
8	15.22	12.18	7.62	6.77	5.19	5.29	7.10	13.24	24.12
9	15.45	12.98	7.66	7.37	5.69	5.24	6.39	12.66	22.07

Note: "NOTA" = "None of the above"

___ 1. The value of P_{51} in the transition probability matrix above is

a. 0 b. 0.05 c. 0.1
d. 0.15 e. 0.25 f. NOTA

___ 2. The value of P_{52} in the transition probability matrix above is

a. 0 b. 0.05 c. 0.1
d. 0.15 e. 0.25 f. NOTA

___ 3. Which of the equations below may be used to compute the steadystate probabilities?

a. $\pi_1=0.05\pi_3+0.05\pi_4+0.15\pi_5+0.25\pi_6+0.25\pi_7+0.15\pi_8+0.1\pi_9$

b. $0.05\pi_3+0.05\pi_4+0.15\pi_5+0.25\pi_6+0.25\pi_7+0.15\pi_8+0.1\pi_9 = 0$

b. $\pi_9=0.1\pi_1+0.1\pi_2+0.1\pi_3+0.1\pi_4+0.1\pi_9$

c. $0.1\pi_1+0.1\pi_2+0.1\pi_3+0.1\pi_4+0.1\pi_9 = 1$

d. NOTA

___ 4. The average stock on hand at the end of the day (before restocking occurs) is

a. $\sum_{i=1}^{4} \pi_i = 0.4076$ b. $\sum_{i=1}^{9} (i-1)\pi_i = 3.968$ c. $\sum_{i=1}^{4} i\pi_i = 1.152$

d. $\sum_{i=1}^{9} i\pi_i = 4.968$ e. $\sum_{i=1}^{4} (i-1)\pi_i = 0.744$ f. NOTA

___ 5. If the shelf begins in the full state, what is the expected number of days until the first stockout (i.e., empty shelf) occurs. Choose nearest number!

a. 6 days b. 8 days c. 13 days
d. 15days e. 20 days f. 24 days

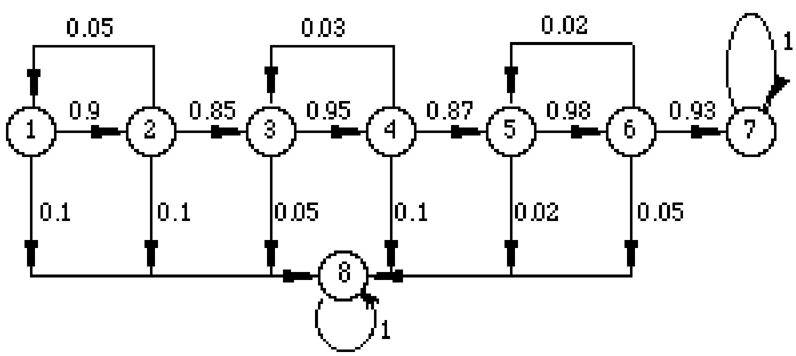
Part II: 2. Consider a manufacturing process in which raw parts (blanks) are machined on three machines, and inspected after each machining operation. The relevant data is as follows:

Station	Machine Operation			Inspection			
i	T	C	S	T	C	S	R
1	0.5	20	10	0.1	15	10	5
2	0.75	20	5	0.2	15	10	3
3	0.25	20	2	0.25	15	5	2

Pack & Ship: 0.1 hrs at 10 $/hr
Cost per blank: $50; Scrap Value: $10

T = time (hrs) per operation
C = cost ($/hr) of operation
S = scrap rate (%)
R = rework rate (%)

The Markov chain model of this manufacturing system is



State	Location
1	Machine1
2	Inspection station1
3	Machine2
4	Inspection station2
5	Machine3
6	Inspection station3
7	Pack-&-Ship Dept.
8	Scrap bin

The Markov chain model of a part moving through this system has "absorption probability" and "expected # visits" matrices:

Absorption Probability Matrix

	OK	Scrap
1	0.6335	0.3665
2	0.7039	0.2961
3	0.7909	0.2091
4	0.8325	0.1675
5	0.9296	0.07038
6	0.9486	0.05141

E = Expected No. Visits

	1	2	3	4	5	6
1	1.047	0.9424	0.8245	0.7833	0.6951	0.6812
2	0.05236	1.047	0.9162	0.8704	0.7723	0.7569
3	0	0	1.029	0.9779	0.8678	0.8504
4	0	0	0.03088	1.029	0.9134	0.8952
5	0	0	0	0	1.02	0.9996
6	0	0	0	0	0.0204	1.02

___ 6. What percent of the parts which are started are expected to be scrapped? Choose nearest number!

a. 10% b. 20% c. 30%
d. 40% e. 50% f. 60%

___ 7. If a part passes the inspection following machine #2, what is the probability that it will be successfully completed? Choose nearest number!

a. 70% b. 75% c. 80%
d. 85% e. 90% f. 95%

___ 8. What is the expected number of machining operations performed on a blank part (including multiple operations on the same machine)? Choose nearest number!

a. 2.0 b. 2.25 c. 2.5
d. 2.75 e. 3 f. 3.25

___ 9. What is the expected number of blanks which should be required to fill an order for 100 completed parts? Choose nearest number!

a. 100 b. 120 c. 140
d. 160 e. 180 f. 200

___ 10. The transient states of this Markov chain model are (circle):

1 2 3 4 5 6 7 8

• •