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57:022 Principles of Design II
Quizzes Fall '95

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<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #1**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Choose the answers to the questions below from the list of distributions, or the table of probabilities. (State "NONE" if the answer does not appear in either place!

A telephone exchange contains 10 lines. A line can be busy or available for calls and all lines act independently. Each line is busy 80% of the noon period (so that the probability that a line will be busy at any given time during the noon period is 80%). What is...

________1. the probability of there being at least *three* free lines at 12:18 pm, when you attempt to call?
________2. the expected number of free lines at any time during this period?
________3. the name of the probability distribution of the number of free lines at 12:18 pm?
________4. the probability that you get a busy signal on your first attempt to call?
________5. the probability that you require at least two tries in order to complete your call?

You need to make three calls to this exchange, and each time you receive a busy signal you try again. What is...

________6. the name of the probability distribution of the number of times you try in order to complete your *first* of the three calls?

The foreman of a casting section in a certain factory finds that, on the average, 1 in every 5 castings made is defective.

________7. If the section makes 10 castings a day, what is the probability that exactly 2 of these will be defective?
________8. What is the probability that 3 or more defective castings are made in one day?
________9. What's the name of the probability dist'n of the quality of casting #8 (either defective or OK).

Advertising states that, for a certain lottery ticket, "every fifth ticket carries a prize". If you buy ten tickets, what is...

________10. the probability that you get *exactly* one winning ticket?
________11. the probability that you get *at least* one winning ticket?

If, instead of deciding in advance how many tickets to buy, you continue buying tickets until you have a winning ticket...

________12. what's the name of the probability distribution of the number of tickets you buy ?

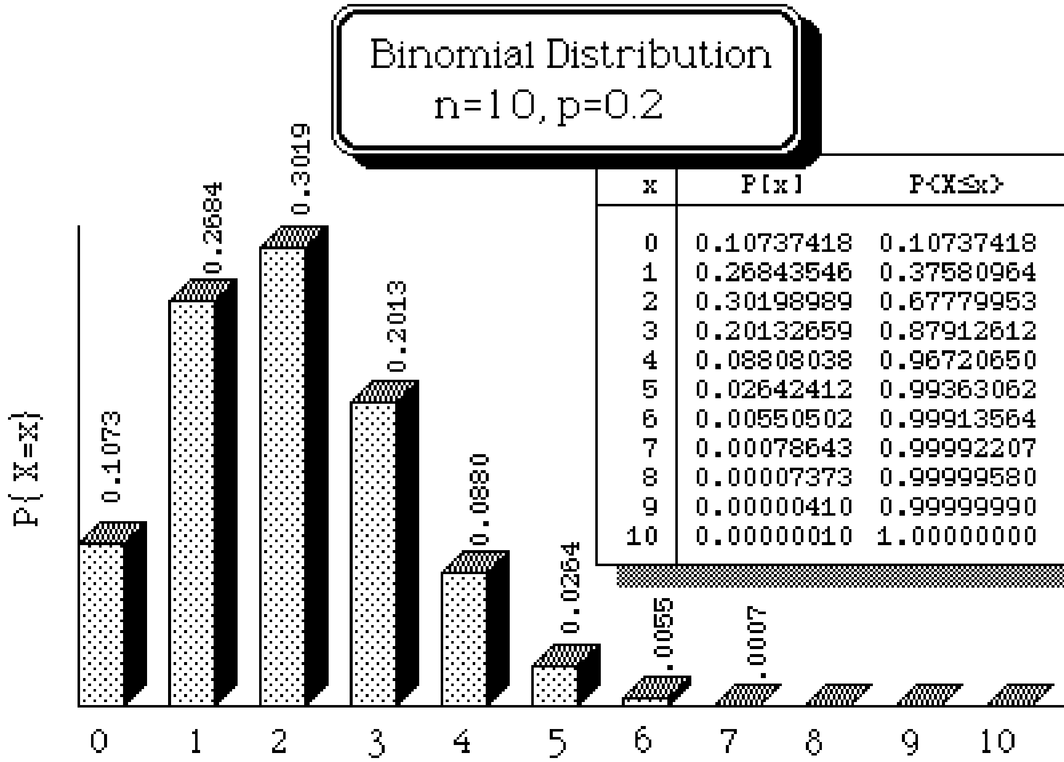
If you continue buying tickets until you have *two* winning tickets...

________13. what's the name of the probability distribution of the number of tickets you buy ?

Some common probability distributions:

a. Bernouilli	b. Random	c. Binomial
d. Poisson	e. Geometric	f. Normal

g. Exponential h. Erlang i. Pascal



x	P[x]	P{X≤x}	P{X>x}
0	0.10737418	0.10737418	0.89262582
1	0.26843546	0.37580964	0.62419036
2	0.30198989	0.67779953	0.32220047
3	0.20132659	0.87912612	0.12087388
4	0.08808038	0.96720650	0.03279350
5	0.02642412	0.99363062	0.00636938
6	0.00550502	0.99913564	0.00086436
7	0.00078643	0.99992207	0.00007793
8	0.00007373	0.99999580	0.00000420
9	0.00000410	0.99999990	0.00000010
10	0.00000010	1.00000000	0.00000000

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We wish to simulate the vehicles arriving at a toll booth on the freeway at the average rate of 5/minute in a completely random fashion. The vehicles are counted and arrival times are recorded, beginning at 12:00 noon. Ninety percent of the vehicles are cars, while the remainder are trucks, buses, etc.

Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all! When appropriate, you may answer NOTA (None of the Above).

___ 1. time of arrival of first vehicle
___ 2. time of arrival of vehicle #2
___ 3. time between arrival of vehicle #1 and vehicle #2
___ 4. number of vehicles arriving during the first 5 minutes
___ 5. vehicle# of the first vehicle which is *not* a car.
___ 6. the number of cars among the first 10 vehicles to arrive
___ 7. the vehicle# of the second vehicle which is *not* a car.
___ 8. an indicator for vehicle #n which is 1 if a car, 0 otherwise.

Some common probability distributions:

A. Bernouilli B. Exponential C. Uniform
D. Normal E. Beta F. Poisson
G. Lambda H. Erlang I. Pascal
J. Binomial K. Geometric L. Random

Write the alphabetic letter corresponding to the numerical value of the following quantities.

___ 9. P{*exactly* 6 vehicles arrive during the first minute}

___ 10. P{two of the first 6 vehicles are cars}
___ 11. P{the first vehicle arrives during the first $^1/_6$ minute}
___ 12. P{the sixth vehicle arrives during the first minute}
___ 13. P{the first non-car is vehicle #6}
___ 14. P{the second non-car is vehicle #6}
___ 15. P{vehicle #6 is not a car}

Numerical values:

M $1 - e^{-6}$

N. $(0.9^5)(0.1)$

O. $\sum_{x=0}^{5} \frac{6^x}{x!} e^{-6}$

P. $\frac{6^6}{6!} e^{-6}$

Q. 0.1

R. $1 - e^{-1}$

S. $\binom{6}{2} 0.1^4\ 0.9^2$

T. $\binom{5}{1} (0.9^4)(0.1^2)$

U. $1 - \sum_{x=0}^{5} \frac{6^x}{x!} e^{-6}$

V. $6e^{-6}$

W. *None of the above*

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___ 1. The "Cumulative Distribution Function" (CDF) of a random variable X is
- a. $F(x) = P\{X \geq x\}$
- b. $F(x) = P\{X \leq x\}$
- c. $F(x) = P\{X = x\}$
- d. $f(x) = P\{x \mid X\}$
- e. $f(x) = P\{X|x\}$
- f. $f(x) = P\{x\}$

___ 2. The arrival of parts to be processed by a machine is a Poisson process, with the rate 4/hour. The actual number of parts which arrive during the first hour has the
- a. Exponential distribution
- b. Normal distribution
- c. Uniform distribution
- d. Poisson distribution
- e. Binomial distribution
- f. *None of the above*

___ 3. The time between arrivals of parts in the preceding question has the
- a. Exponential distribution
- b. Normal distribution
- c. Uniform distribution
- d. Poisson distribution
- e. Binomial distribution
- f. *None of the above*

___ 4. The CDF of the distribution in (3) above, i.e., the inter-arrival times, is
- a. $4e^{-4t}$
- b. $1 - e^{-4t}$
- c. $4 - e^{-4t}$
- d. $1 - 4e^{-4t}$
- e. e^{-4t}
- f. *None of the above*

___ 5. An inter-arrival time T can be randomly generated by using a uniformly-generated random variable X and computing
- a. $T = e^{-4X}$
- b. $T = -\frac{\ln X}{4}$
- c. Either (a) or (d)
- d. $T = 1 - e^{-4X}$
- e. $T = -\frac{\ln(1-X)}{4}$
- f. Either (b) or (e)

The time between arrivals of fifty cars are measured. It is expected that these observations have an exponential distribution with mean of 4 minutes, although the actual average value of the observations was 3.68 minutes. We wish to decide whether the discrepancy between the assumed arrival rate (1 every 4 minutes) and the observed arrival rate (1 every 3.68 minutes) is so large as to disqualify our assumption. The number of observations O_i falling within each of several intervals is shown in the table below. We wish to test the "goodness of fit" of the exponential distribution with mean 4 minutes, and have prepared the table below:

i	Interval	O_i	p_i	$E_i = 50p_i$	$\frac{(E_i - O_i)^2}{E_i}$
1	0-1	12	0.221199	11.06	0.0798984
2	1-2	11	0.17227	8.61351	0.661212
3	2-3	6	0.134164	6.70821	0.0747674
4	3-5	6	0.185862	9.29309	1.16693
5	5-9	10	0.181106	9.05528	0.0985611
6	9-∞	5	0.105399	5.26996	0.0138291

total = 2.0952

A portion of a table of the chi-square distribution is given below:

deg.of freedom	Chi-square Dist'n $P\{D \geq \chi^2\}$					
	99%	95%	90%	10%		5%
	1%					
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

Indicate whether true or false, using "**+**" for true, "**o**" for false.

____ 6. The CDF of the inter-arrival time distribution is $F(t) = P\{T \leq t\}$

____ 7. The parameter of the exponential distribution was assumed to be λ = 1/4 min. = 0.25/minute.

____ 8. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D exceeds 2.0952 is more than 10%.

____ 9. The quantity $(E_i\text{-}O_i)^2/E_i$ is assumed to have a "chi-square" distribution.

____ 10. The smaller the value of D, the better the fit for the distribution being tested.

____ 11. The quantity E_i is the expected number of observations in interval #i, if the assumption is true.

____ 12. The probability p_i that a car arrives in interval #3, i.e., [2,3], is F(2) - F(3), where F(t) is the CDF of the interarrival times.

____ 13. The quantity D=2.0952 is assumed to have the chi-square distribution.

____ 14. If the assumption is correct, the arrivals of the cars forms a Poisson process.

____ 15. The chi-square distribution for this test will have 6 "degrees of freedom".

____ 16. Based upon these observations, the exponential distribution with mean 4 minutes should <u>not</u> be rejected as a model for the interarrival times of the vehicles.

____ 17. The chi-square distribution for this test will have 6 "degrees of freedom".

____ 18. The number of observations O_i in interval #i is a random variable with approximately binomial distribution with n=50 and probability of "success" $p=p_i$.

____ 19. The quantity E_i is a random variable with approximately a Poisson distribution.

____ 20. The quantity D is assumed to have approximately a Normal distribution.

____ 21. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D is less than 2.0952 is more than 10%.

____ 22. The degrees of freedom is reduced by 1 because the total number of observations is fixed at 50.

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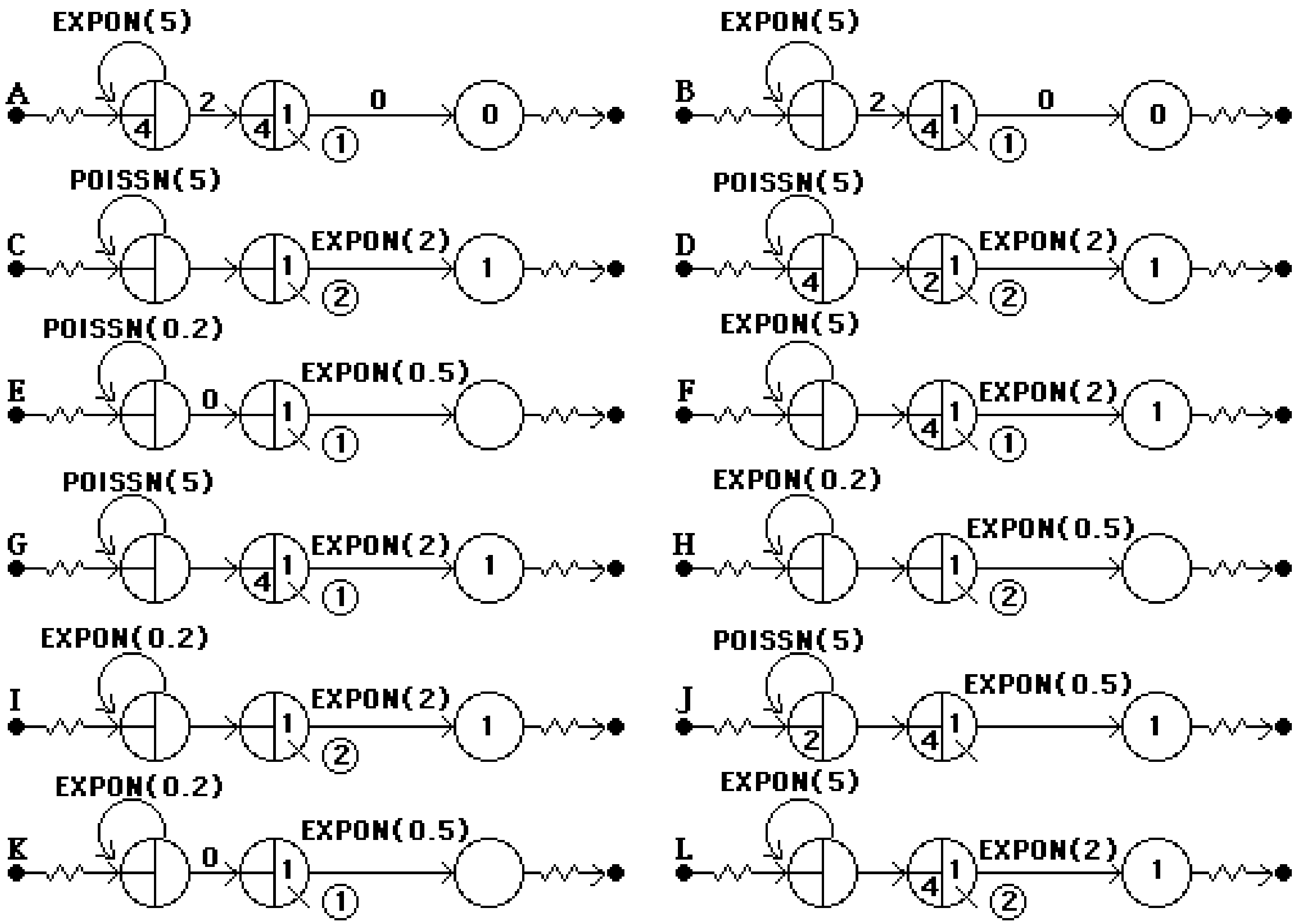
Homework #4 dealt with the proposed drive-up teller window (with a single teller). The arrival of customers (a Poisson process) occurs an average of one every five minutes, and customer service (exponentially distributed) requires an average of two minutes. Space will be provided for at most four autos, plus the one at the teller window.

1. Which of the twelve SLAM networks below best models this system? _______
2. Which of the twelve SLAM networks below best models the alternative proposal with two teller windows? ________



Use the SLAM output below to answer the questions which follow. (Note that this output is for a slightly different SLAM model, in which the overflow of the queue causes immediate termination of the simulation.)

STATISTICS FOR VARIABLES BASED ON OBSERVATION

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	0.303E+01	0.286E+01	0.944E+00	0.345E-01	0.110E+02	88
	0.408E+03	0.000E+00	0.000E+00	0.408E+03	0.408E+03	1

FILE STATISTICS

FILE NUMBER	LABEL/TYPE	AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	QUEUE	0.300	0.724	4	4	1.317
2	CALENDAR	1.439	0.496	3	2	2.669

SERVICE ACTIVITY STATISTICS

ACT NUM	ACT START NODE	LABEL OR SER CAP	AVERAGE UTIL	STD DEV	CUR UTIL	AVERAGE BLOCK	MAX IDL TME/SER	MAX BSY TME/SER	ENT CNT
1	QUEUE	1	0.439	0.50	1	0.00	17.35	29.23	88

3. What percent of the time was the bank teller busy serving customers? __________
4. What is the average total time that a customer spent at the bank? ______________
5. What is the maximum time that any customer spent at the bank? ______________
6. How many customers were served during this simulation? _______________
7. Based upon the chi-square test which you performed, is the statement that "the total time spent by a customer at the bank has exponential distribution" reasonable? _________
8. What is the average number of cars in the waiting line? _____________
9. What is the average time that a customer spent waiting to be served? ___________

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #5**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Statements below refer to today's homework assignment (HW#5).
Indicate "+" for true, "o" for false:

___ 1. The Cricket Graph program fits a line which minimizes the sum of the errors, i.e., the vertical distance between each data point and the line.

___ 2. The sum of the CDF (cumulative distribution function) and the Reliability function is always equal to 1 for *every* probability distribution.

___ 3. The quantity which is recorded in Cricket Graph as R_t is the fraction of the 500 bulbs which have failed at time t (or earlier).

___ 4. The Weibull CDF, i.e., F(t), gives, for each bulb, the probability that it has failed at or before time t.

___ 5. A positive value of k indicates an increasing failure rate, and negative k indicates a decreasing failure rate.

___ 6. The method used in this homework to estimate the Weibull parameters u & k does not require that the bulbs be tested until all 500 have failed.

___ 7. Given a coefficient of variation for the Weibull distribution (the ratio $^{\sigma}/_{\mu}$), both of the parameters u and k can be determined.

___ 8. According to the results of this homework exercise, the failure rate of the light bulbs is *increasing* rather than *decreasing*.

___ 9. If 6 bulbs are installed in an office's light fixtures, the number still functioning after 150 days has a Weibull distribution.

___ 10. If the failure rate is increasing, it may be more appropriate to use the Gumbel distribution than the Weibull.

___ 11. We assumed in this HW that the number of failures at time t , $N_f(t)$, has a Weibull distribution.

___ 12. If 10 bulbs are installed in an office's light fixtures, the number still functioning after 100 days has a Poisson distribution.

___ 13. The exponential distribution is a special case of the Weibull distribution, with constant failure rate.

___ 14. The "gamma" function has the property $\Gamma(1+x) = x!$ for all nonnegative integer values of x.

___ 15. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

___ 16. If k=1, then $\Gamma\left(1+\frac{1}{k}\right) = 1$.

Select the letter below which indicates each correct answer: When preparing a plot so as to estimate the Weibull parameters, ...

___ 17. The label on the horizontal axis should be ...

___ 18. The vertical intercept of the line fit by Cricker Graph should be approximately ...

___ 19. The slope of the line fit by Cricket Graph should be approximately ...

___ 20. The label on the vertical axis should be ...

a. t	b. ln t	c. -ln t
d. ln $^1/_t$	e. ln ln t	f. ln ln $^1/_t$
g. +k ln u	h. u ln k	i. +ln k
j. -k ln u	k. -u ln k	l. -ln k
m. R_t	n. ln R_t	o. - ln R_t
p. ln $^1/_{R_t}$	q. ln ln R_t	r. ln ln $^1/_{R_t}$
s. shape parameter k	t. scale parameter u	u. coefficient of variation $^\sigma/_\mu$
v. mean value μ	w. standard deviation σ	x. *None of the above*

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A system consists of five components (A,B,C,D, & E). The probability that each component *survives* the first year of operation is 70% for A, B, & C, and 80% for D & E. For each alternative of (1) through (4), indicate:

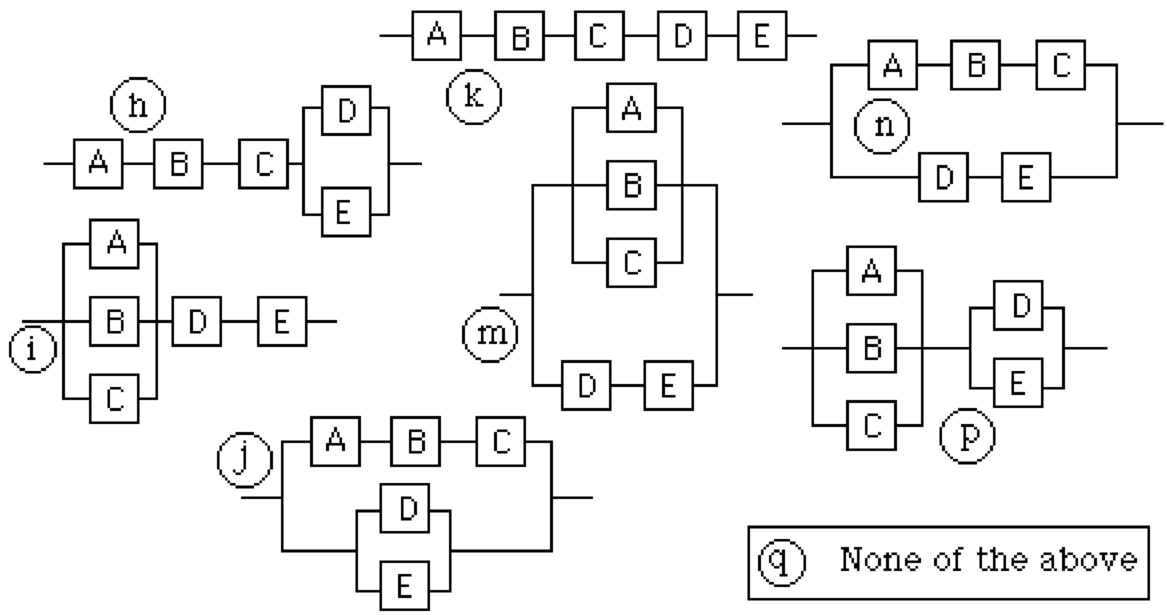
(i) the letter of the ***reliability diagram*** below which represents the system

(ii) the letter of the ***SLAM network*** model which represents the system

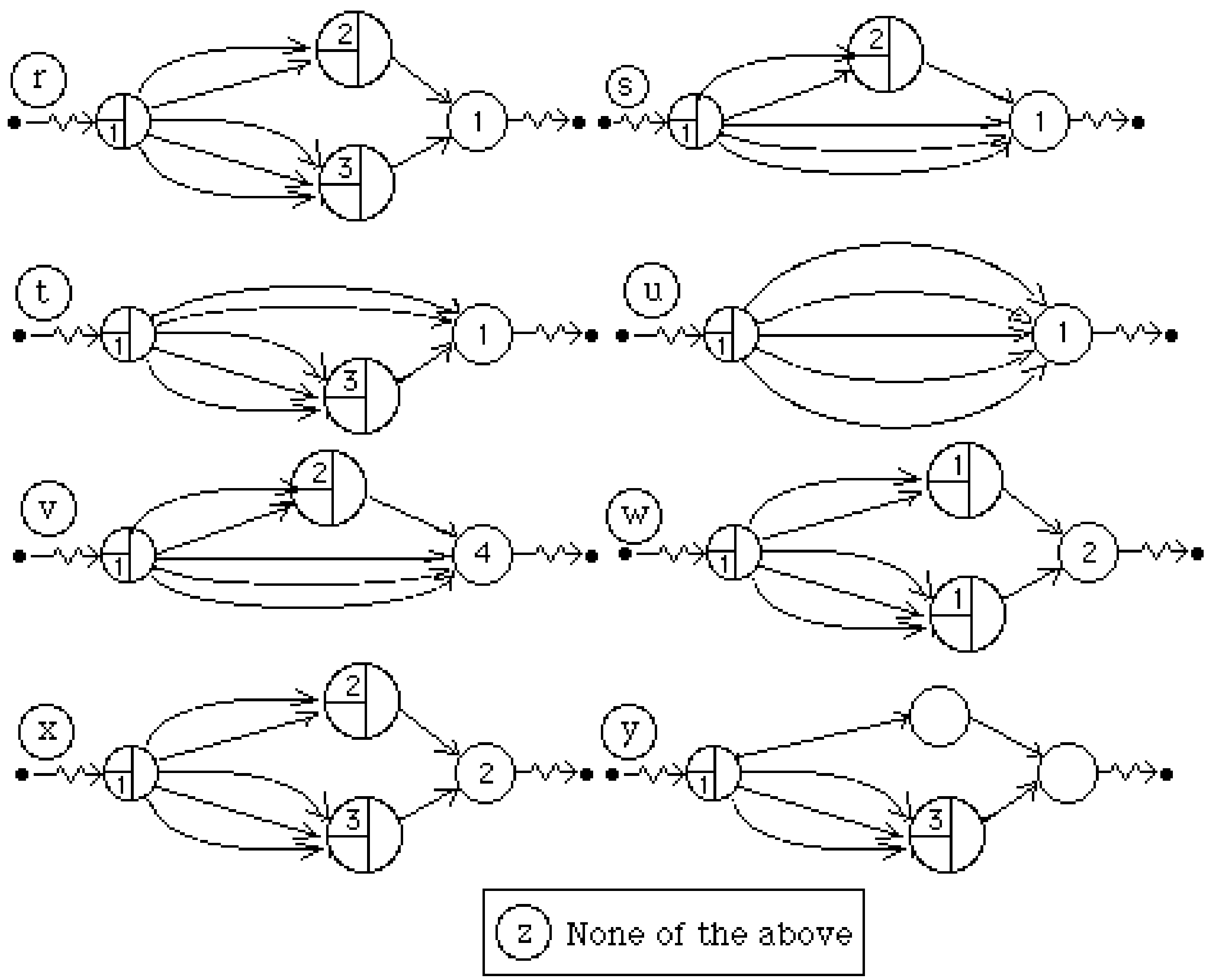
(iii) the letter with the ***computation*** of the 1-year reliability (i.e., survival probability)

i.	ii.	iii.	
___	___	___	1. The system requires both of D & E, and at least one of A, B, & C.
___	___	___	2. The system requires all of A, B, & C, and at least one of D & E.
___	___	___	3. The system requires at least one of A, B, & C, and at least one of D & E.
___	___	___	4. The system requires all of A, B, C, D, & E.

Diagrams:



SLAM networks:



Reliabilities:

a. $1 - (0.3)^3(0.2)^2 = 0.99892$
b. $1 - (0.3)^3[1 - (0.8)^2] = 0.99028$
c. $[1 - (0.3)^3]\,[1 - (0.2)^2] = 0.93408$
d. $(0.7)^3(0.8)^2 = 0.21952$
e. $(0.7)^3[1- (0.2)^2] = 0.32928$
f. $1 - (0.3)^3[1- (0.8)^2] = 0.99028$
g. $[1 - (0.3)^3]\,(0.2)^2 = 0.03892$
h. None of the above

<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #7**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

Discrete-Time Markov Chains -- Part I: Steady-state analysis

(s,S) Inventory System: Consider the following inventory system for a certain spare part for a company's 2 production lines. A maximum of four parts may be kept on the shelf. At the end of each day, the parts in use are inspected and, if worn, replaced with one off the shelf. The probability distribution of the number replaced each day is:

n=	0	1	2
P{n}=	0.3	0.5	0.2

To avoid shortages, the current policy is to **restock** the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there are *fewer than 2* parts on the shelf.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions: **Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.**

$$P=\begin{bmatrix} 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.5 & \boxed{A} & 0 & 0 \\ 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \end{bmatrix},\quad P^2=\begin{bmatrix} 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.06 & 0.15 & 0.23 & 0.35 & 0.21 \\ 0.1 & 0.31 & 0.34 & 0.19 & 0.06 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \end{bmatrix}$$

$$P^3=\begin{bmatrix} 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.046 & 0.185 & 0.328 & 0.315 & 0.126 \\ 0.068 & 0.208 & 0.291 & 0.292 & 0.141 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \end{bmatrix},\quad P^4=\begin{bmatrix} 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.227 & 0.327 & 0.273 & 0.107 \\ 0.058 & 0.203 & 0.316 & 0.296 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \end{bmatrix}$$

$$\sum_{n=1}^{4} P^n=\begin{bmatrix} 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.3716 & 1.062 & 1.1853 & 0.938 & 0.4431 \\ 0.2262 & 0.9219 & 1.4477 & 1.0781 & 0.3261 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \end{bmatrix}$$

$$M=\begin{bmatrix} 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 12.692 & 2.754 & 3.153 & 4 & 9.795 \\ 15 & 3.396 & 2.307 & 3.514 & 10.816 \\ 15.769 & 4.641 & 3.076 & 2.571 & \boxed{B} \end{bmatrix}$$

i	π_i
1	0.063
2	0.215
3	0.317
4	0.284
5	0.119

____ 1. the value $P_{5,4}$ is
a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. none of the above

____ 2. the value $P_{2,4}$ is
a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. none of the above

____ 3. the value $P_{3,1}$ is

a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. none of the above

____ 4. the numerical value **A** in the matrix above is
a. 0 b. 0.1 c. 0.2
d. 0.3 e. 0.4 f. 0.5

____ 5. the numerical value **B** in the mean-first-passage time matrix (M) above is *(select nearest value)*
a. 1 b. 2 c. 4
d. 6 e. 8 f. 10

____ 6. If the shelf is full Monday morning, the expected number of days until a stockout occurs is *(select nearest value):*
a. 2 b. 5 c. 10
d. 15 e. 20 f. more than 20

____ 7. If the shelf is full Monday morning, the probability that the shelf is full Wednesday night is *(select nearest value):*
a. 7% b. 8% c. 9%
d. 10% e. 11% f. more than 12%

____ 8. If the shelf is full Monday morning, the probability that the shelf is restocked Wednesday night is (select nearest value):
a. 10% b. 15% c. 20%
d. 25% e. 30% f. more than 30%

____ 9. If the shelf is full Monday morning, the expected number of nights that the shelf is restocked before Friday morning is (select nearest value):
a. 0.6 b. 0.7 c. 0.8
d. 0.9 e. more than once but less than twice f. more than 2

____ 10. The number of *transient* states in this Markov chain model is
a. zero b. 1 c. 2
d. 5 e. none of the above

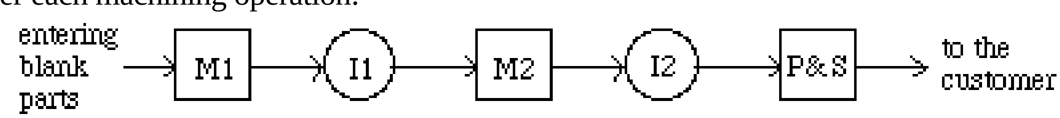
____ 11. The number of *recurrent* states in this Markov chain model is
a. zero b. 1 c. 2
d. 5 e. none of the above

____ 12. The number of absorbing states in this Markov chain model is
a. zero b. 1 c. 2
d. 5 e. none of the above

____ 13. Which (one or more) of the following equations are among those solved to compute the steady state probability distribution?
a. $\pi_1 = 0.2\pi_3$
b. $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$
c. $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$
d. $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$
e. $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

Discrete-Time Markov Chains -- Part II: Absorption analysis

Multistage Manufacturing System: Consider a system of 2 machines, with inspection of a part after each machining operation:



Relevant data are:

Operations	Time Rqmt. (man-hrs)	Operating Cost ($/hr.)	Scrap rate %	% sent back for rework
Machine 1	1	10	10	
Inspection 1	0.25	10	5	5
Machine 2	1	15	5	

Inspection 2	0.5	15	15	5
Pack & Ship	0.5	10		

The manufacturing system is modeled as a discrete-time Markov chain with 6 states and the transition probability matrix:

$$P = \left[\begin{array}{c|c} Q & R \\ \hline O & I \end{array}\right] = \begin{bmatrix} 0 & 0.9 & 0 & 0 & 0 & 0.1 \\ 0.05 & 0 & 0.9 & 0 & 0 & 0.05 \\ 0 & 0 & 0 & 0.95 & 0 & 0.05 \\ 0 & 0 & 0.05 & 0 & 0.8 & 0.15 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0 & 0.001823 & 0 & 0.07118 & 0.6156 & 0.3114 \\ 0.0001013 & 0 & 0.005777 & 0 & 0.7473 & 0.2469 \\ 0 & 0 & 0 & 0.002143 & 0.7961 & 0.2018 \\ 0 & 0 & 0.0001128 & 0 & 0.8398 & 0.1601 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = (I-Q)^{-1} = \begin{bmatrix} 1.047 & 0.9424 & 0.8905 & 0.8459 \\ 0.05236 & 1.047 & 0.9894 & 0.9399 \\ 0 & 0 & 1.05 & 0.9974 \\ 0 & 0 & 0.05249 & 1.05 \end{bmatrix}, A = ER = \begin{bmatrix} 0.6768 & 0.3232 \\ 0.7519 & 0.2481 \\ 0.7979 & 0.2021 \\ 0.8399 & 0.1601 \end{bmatrix}$$

____ 1. The number of transient states in this Markov chain is
a. zero b. two c. four
d. six e. None of the above

____ 2. The number of recurrent states in this Markov chain is
a. zero b. two c. four
d. six e. None of the above

____ 3. The percent of parts which must be scrapped is *(choose nearest value):*
a. 15% b. 20% c. 25%
d. 30% e. 35% f. more than 40%

____ 4. The expected number of blanks which must be processed in order to produce 100 parts is *(choose nearest value):*
a. 110 b. 120 c. 130
d. 140 e. 150 f. more than 160

____ 5. The probability that a part will be successfully completed if it must be reworked on the first machine is *(choose nearest value):*
a. 60% b. 65% c. 70%
d. 75% e. 80% f. less than 60%

____ 6. The expected number of man-hours at the first inspection station in order to successfully produce 100 parts is *(choose nearest value):*
a. 20 b. 25 c. 30
d. 35 e. 40 f. more than 45 man-hours

____ 7. The probability that a part is successfully completed with no reworking is *(choose nearest value):*
a. 45% b. 50% c. 55%
d. 60% e. 65% f. more than 70%

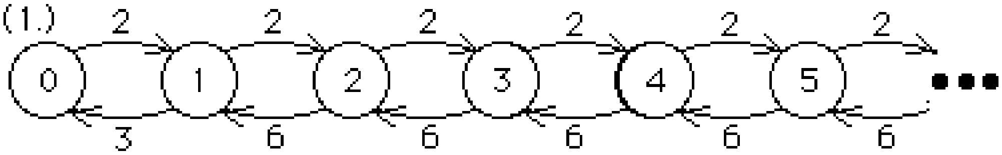
<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>**Quiz #8**<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>-<•>

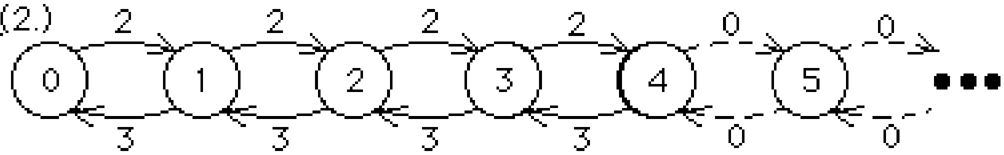
Part One

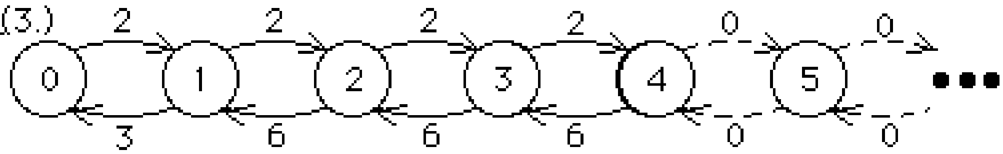
For each diagram of a Markov model of a queue in (1) through (5) below, indicate the correct Kendall's classification from among the following choices :

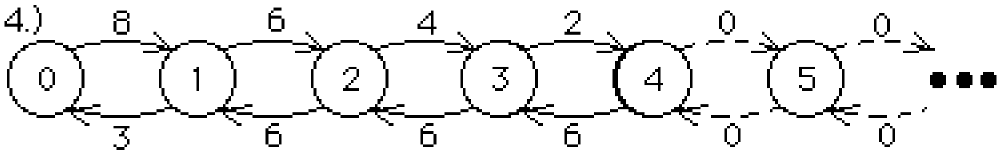
(a) M/M/1 (b) M/M/2 (c) M/M/1/4
(d) M/M/4 (e) M/M/2/4 (f) M/M/2/4/4

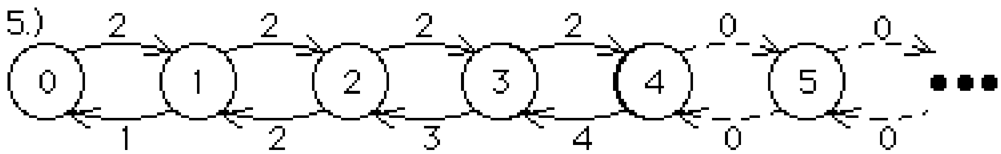
(g) M/M/1/2/4 (h) M/M/4/2 (i) M/M/4/4
(j) none of the above

___(1.)


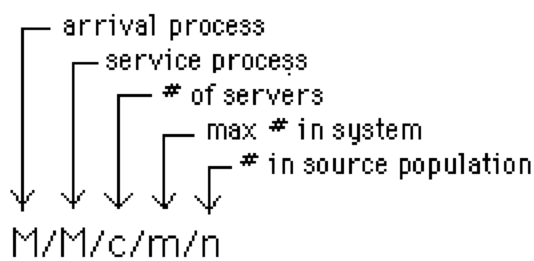
___(2.)


___(3.)


___(4.)


___(5.)


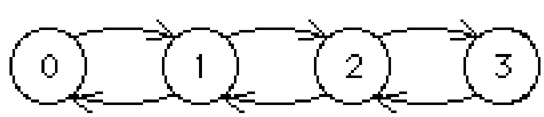
Note: Kendall's notation:



Part Two

Two mechanics work in an auto repair shop, with a capacity of 3 cars. If there are 2 or more cars in the shop, each mechanic works individually, each completing the repair of a car in an average of 4 hours (the actual time being random with exponential distribution). If there is only one car in the shop, both mechanics work together on it. When both work on the same car, the average repair time for that car is only 3 hours (exponentially distributed). Cars arrive randomly, according to a Poisson process, at the rate of one every two hours when tthe mechanics are idle, but one every 4 hours when the mechanics are busy. While 3 cars are in the shop, however, no cars arrive.

1. Label the transition diagram below with transition rates:



2. Which equation is used to compute the steady-state probability π_0? *(Note: The arithmetic is correct!)*

(a.) $\frac{1}{\pi_0} = 1 + \frac{1/2}{1/3} + \frac{1/4}{1/2} + \frac{1/4}{1/2} = \frac{1}{0.2857}$

(b.) $\frac{1}{\pi_0} = 1 + \frac{1/2}{1/3} + \frac{1/2 \times 1/4}{1/3 \times 1/2} + \frac{1/2 \times 1/4 \times 1/4}{1/3 \times 1/2 \times 1/2} = \frac{1}{0.2759}$

(c.) $\frac{1}{\pi_0} = 1 + \frac{1/3}{1/2} + \frac{1/3 \times 1/2}{1/2 \times 1/4} + \frac{1/3 \times 1/2 \times 1/2}{1/2 \times 1/4 \times 1/4} = \frac{1}{0.1765}$

(d.) $\frac{1}{\pi_0} = 1 + \frac{1/2}{1/3} + \left(\frac{1/4}{1/2}\right)^2 + \left(\frac{1/4}{1/2}\right)^3 = \frac{1}{0.3478}$

3. What fraction of the day will both mechanics be idle? *(Choose nearest answer.)*
   a. 20% b. 25% c. 30%
   d. 35% e. 40% f. 45%
4. What fraction of the day will both mechanics be working on the same car? *(Choose nearest answer.)*
   a. 20% b. 25% c. 30%
   d. 35% e. 40% f. 45%
5. The average number of cars in the shop is 1.14 and the average time between arrivals is 3.41 hours. According to Little's Law, what is the average turnaround time (i.e., total time both waiting and being repaired) of a car in the shop? *(Choose nearest answer.)*
   a. 3 hours b. 4 hours c. 5 hours
   d. 6 hours e. 7 hours f. 8 hours

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