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57:022 Principles of Design II - Quiz #5 Solutions
Spring 2002

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Part I: One hundred identical devices are tested, and the test is terminated after 50 days, at which time 42 of them have failed. ***Assumption***: The device has a lifetime with Weibull distribution.
*Indicate "+" for **true**, "O" for **false**.*

_o_ 1. If 10 units of this device are installed in a facility, the number still functioning after 50 days has a Weibull distribution.
_o_ 2. To estimate the time at which 90% of the devices will have failed, evaluate 1 - F(0.90).
_+_ 3. The quantity R_t is the fraction of the devices which have survived until time t.
_o_ 4. To estimate the Weibull parameters u & k for this particular device, we may use the "Method of Moments".
_o_ 5. Under our assumption above, the time *between* the failures will have the Gumbel distribution.
_o_ 6. The number of failures at time t , $N_f(t)$, is assumed to have a Weibull distribution.
_+_ 7. The Weibull CDF, i.e., F(t), gives, for each device, the probability that it has failed at time t.
_o_ 8. The time between the failures in the group of 100 units was assumed to have the Weibull distribution.
_+_ 9. The *secant method* is a method which is used to solve a nonlinear equation.
_+_ 10. The exponential distribution is a special case of the Weibull distribution, with $\lambda = 1/u$.
_o_ 11. The exponential distribution is a special case of the Weibull distribution, with $k=0$.
_+_ 12. A value of $k>1$ indicates an increasing failure rate, while $k<1$ indicates a decreasing failure rate.
_+_ 13. The slope of the straight line fit by linear regression to the data will be the estimate of the "shape" parameter k.
_+_ 14. In general, given only a coefficient of variation (i.e., the ratio σ/μ) for the Weibull distribution, the parameter k can be determined.

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Part II: Multiple choice:
From the list below, choose the most appropriate model for the following random variables:
Gumbel 15. The time at which there are no living individuals in a population of bacteria which is exposed to an antibiotic being tested.
Weibull 16. The strength of a chain used on a crane for loading & unloading cargo.
Normal 17. The total weight of the university football team.
Weibull 18. The failure time of a television.
Gumbel 19. The annual peak flow of the Mississippi River at Davenport.

a. Normal	b. Gumbel	c. Exponential
d. Weibull	e. Erlang	f. Chi-square

Let Y_i be the i^{th} of N observations, and $F_i = i/N$.
When plotting the points to fit a straight line in order to estimate α & u for the *Gumbel* distribution,
–ln(–ln F) 20. The vertical axis should represent ...
Y 21. The horizontal axis should represent ...
α 22. The slope of the line should be approximately ...

a. Y	b. F	c. ln Y	d. ln F	e. –ln F	f. –ln(–ln F)
g. α	h. u	i. – α u	j. coefficient of variation	k. *None of the above*	

GUMBEL DISTRIBUTION

CDF	$F_Y(y) = \exp\{-\exp[-\alpha(y-u)]\}$
density	$f_Y(y) = \alpha \exp\{-\alpha(y-u) - \exp[-\alpha(y-u)]\}$
mean value	$\mu_Y = u + \frac{0.577}{\alpha}$
standard deviation	$\sigma_Y = \frac{1.282}{\alpha}$

WEIBULL DISTRIBUTION

CDF	$F_T(t) = 1 - e^{-\left(t/u\right)^k}$
Mean value	$\mu_T = u\Gamma\left(1+\frac{1}{k}\right)$
Standard deviation	$\sigma_T = u\sqrt{\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)}$
Coefficient of variation	$\frac{\sigma_T}{\mu_T} = \sqrt{\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1}$

$\Gamma(x) = (x-1)!$ for integer x>0 $\Rightarrow \Gamma\left(1+\frac{1}{k}\right) = \left(\frac{1}{k}\right)!$ for k=1, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$