57:022 Principles of Design II
Quiz #2 Solution -- Spring 2002

Production of parts by a machine is a Poisson process, at the average rate of 2 parts per hour. Inspection will find that 20% of the processed parts are defective.

Match the name of the distribution to the random variable:

Poisson	1. the number of parts which are produced during the first eight hours?
Exponential	2. the time between production of parts?
Poisson	3. the number of defective parts which are produced during the first eight hours?
2-Erlang	4. the time that the second defective part is produced?
Binomial	5. the number of defective parts among the first eight which are produced?

In each case below, use the tables and select the *nearest* numerical value.

6. The probability that the first part is completed during the first half-hour.
   Solution: $P\{T_1 \le 0.5\} = 0.63212$, where T_1 has exponential distribution with $\lambda = 2$/hour
7. The probability that exactly two of the first eight parts are defective.
   Solution: $P\{N8=2\} = 0.2936$, where N_8 has binomial distribution with n=8, p=0.2
8. The probability that exactly two parts are completed during the first hour
   Solution: $P\{N_1=2\} = 0.270671$, where N_1 has Poisson distribution with $\lambda t = 2/\text{hr} \times 1$ hr.
9. The probability that the second part is completed during the first hour.
   Solution: $P\{T_2 \le 1\} = P\{N_1 \ge 2\} = 0.593994$, where N_1 has Poisson distribution with $\lambda t = 2/\text{hr} \times 1$ hr.

Binomial Distribution (n= 8, p= 0.2)

x	P{x}	P{X ≤ x}	P{X > x}
0	0.16777	0.16777	0.83222
1	0.33554	0.50331	0.49668
2	0.29360	0.79691	0.20308
3	0.14680	0.94371	0.05628
4	0.04587	0.98959	0.01040
5	0.00917	0.99876	0.00123
6	0.00114	0.99991	0.00008
7	0.00008	0.99997	0.00003
8	0.00003	1.00000	0.00000

Exponential Dist'n, Lambda = 2/hour

t	P{T≤t}	P{T>t}
0	0	1
0.25	0.393469	0.606531
0.5	0.632121	0.367879
0.75	0.77687	0.22313
1	0.864665	0.135335
1.25	0.917915	0.082085
1.5	0.950213	0.0497871
1.75	0.969803	0.0301974
2	0.981684	0.0183156

Poisson Distribution, expected value 2

x	P{X=x}	P{X≤x}	P{X>x}
0	0.135335	0.135335	0.864665
1	0.270671	0.406006	0.593994
2	0.270671	0.676676	0.323324
3	0.180447	0.857123	0.142877
4	0.090223	0.947347	0.052653
5	0.036089	0.983436	0.016563
6	0.012029	0.995466	0.004533
7	0.003437	0.998903	0.001096
8	0.000859	0.999763	0.000237