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57:022 Principles of Design II
Quiz #3 Solutions—Spring 2002

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Indicate "+" for true, "O" for false.

_+_ 1. The "Cumulative Distribution Function" (CDF) of a random variable *X* is defined as $F(x) = P\{X \leq x\}$.

_O_ 2. The rejection method for generating a random number *x* having a CDF F(x) requires that you derive the inverse function $F^{-1}(\bullet)$, obtain two random numbers (*x,y*) having *uniform* distribution in [0,1]. If $y \leq F^{-1}(x)$ then we accept *x* as the random number, else repeat. *Note*: Rejection method: If $y \leq f(x)$ where *f* is the *density* function, then we accept *x* as the random number, else repeat.

_+_ 3. The inverse transformation method can always be used to generate a random number with distribution function F, provided you can calculate its inverse $F^{-1}(\bullet)$.

_+_ 4. The inverse transformation method (if it can be used) will always require fewer uniformly-generated random numbers than the rejection method. *Note*: the inverse transformation method requires 1 uniformly-generated random number, whereas the rejection method requires one pair *(x,y)* if the first *x* is not rejected, and usually more pairs.

_O_ 5. If the random variable R is uniformly distributed in [0,1], then $-\frac{\ln(1-R)}{\lambda}$ has Poisson distribution with parameter λ. *Note*: this has exponential distribution!

_O_ 6. In a Poisson process, the time between arrivals has a Poisson distribution. *Note*: interarrival time has exponential distribution!

_+_ 7. The inverse transformation method to generate a random number can be used to simulate interarrival times for a Poisson process.

_+_ 8. In a Poisson process with arrival rate λ/minute, the number of arrivals in t minutes is random, with a Poisson distribution having mean λt.

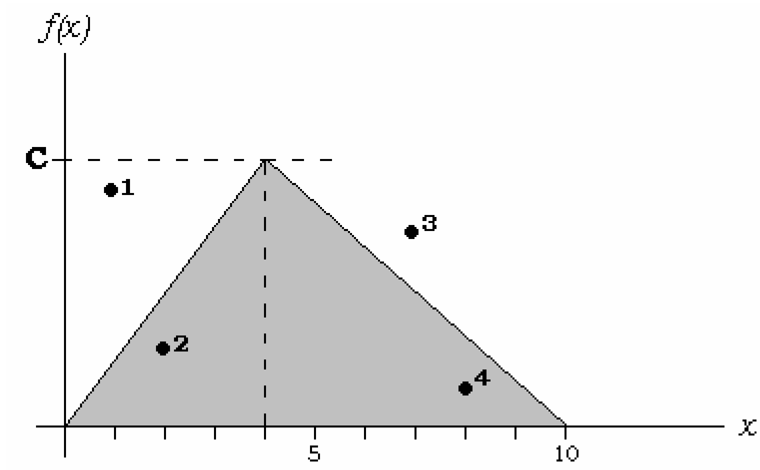
_+_ 9. The exponential distribution is a special case of the Erlang distribution.

_+_ 10. If F(t) is the CDF of the interarrival time for a Poisson process, the expected number of arrivals E_i which fail in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$

_+_ 11. The inverse transformation method could be used for generating random numbers having an Erlang distribution.

_O_ 12. If F is the CDF of a random variable X, then F(0) = 1. *Note*: F(∞)=1.

Consider the probability distribution with density function *f* shown on the right:



_b_ 13. The value of C must be *(choose nearest value):*

a. 0.1 b. 0.2 c. 0.3 d. 0.4
e. 0.5 f. 0.6 g. 0.7 h. 0.8
i. 0.9 j. 1.0 k. 10 l. ≥ 20

Note: area of triangle must be 1, therefore $0.5\times10\times C=1$

_d_ 14. Suppose that ***four*** pairs *(x,y)* of random numbers were generated, with *x* uniformly distributed between 0 and 10, and *y* between 0 and C, and that the four pairs were plotted as shown above. Which sequence of random numbers would have the desired distribution?

a. 1, 2, … b. 1, 7, … c. 2, 7, … d. 2, 8, …

_e_ 15. This method for generating random numbers is known as

a. inverse transformation method b. triangular method
c. decomposition method d. composition method
e. rejection method f. *none of the above*