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57:022 Principles of Design II
Quiz #1 Solution -- Spring 2002

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The foreman of a casting section in a certain factory finds that on the average, 1 in every 5 castings that are made is defective, and that the quality of a casting is independent of whether a defect was found in the previous casting. Let

X_i = 1 if the ith casting is defective, 0 otherwise
N_i = # of defective castings among the first *i* castings which are produced.
T_k = number of the kth defective casting.

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Suppose that 10 castings are made, with quality:
"++d+d++++d" where "+" indicates OK, and "d" indicates a defect.
Indicate the numerical values of the random variables in this case:

_2_ 1. N_5 _1_ 2. X_5 _3_ 3. T_1 _5_ 4. T_2

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Select the answer to #5-8 from the options below:

Binomial 5. If the section makes 8 castings a day, the probability distribution of the number of defective castings is
Geometric 6. The probability dist'n of the number of castings before the first defective one is produced (i.e., T_1) is
Pascal 7. The probability distribution of the number of castings before the second defective one (i.e., T_2) is….
Geometric 8. The probability distribution of $T_2 - T_1$, i.e., the number of additional castings after the first defect until the second defect is found, is …

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Choose an expression below for each quantity in #5-7

$\frac{10!}{2!8!}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8$ 9. If the section makes 10 castings, what is probability that exactly 2 of these will be defective?

$\frac{1}{5}$ 10. What is the expected value of X_{10}?

$\frac{1}{5}\times 10$ 11. What is the expected value of N_{10}?

$\left(\frac{1}{5}\right)^2$ 12. What is the probability that the first two castings are both defective?

$\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$ 13. What is the probability that the fifth casting is the first one which is defective?

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Indicate + = "true", o = "false"

_+_ 14. The sequence $\{N_1, N_2, N_3, N_4, \ldots\}$ is a stochastic process.
_+_ 15. The sequence $\{X_1, X_2, X_3, X_4, \ldots\}$ is a Bernouilli process.

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