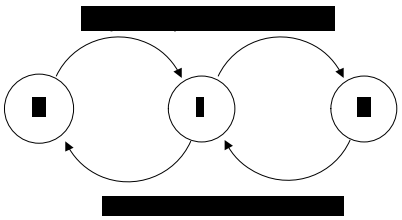
[redacted] 57:022 Principles of Design II [redacted]
Quiz #10 – 3 May 2002

A total of 10,000,000 cu ft of dirt is needed to construct a dam. The dirt is moved via dumpers to the dam site. Only one loader and two dumpers are available. Each dumper can hold 1000 cu ft of dirt. It takes an average of ***six*** minutes to load a dumper with dirt, and it takes each dumper an average of ***ten*** minutes to deliver the dirt to the dam and return to the loader. Making appropriate assumptions about exponentiality so as to obtain a birth/death model, we wish to determine the total expected time required to move the dirt needed to build the dam.



Letting the state be the number of dumpers at the loading site, we model this as a birth-death process, as in the diagram on the right.

1. Match the values

_e_ λ_1	a. 1/hr	b. 2/hr	c. 3/hr
_g_ λ_0	d. 5/hr	e. 6/hr	f. 10/hr
_f_ μ_1	g. 12/hr	h. 15/hr	i. 20/hr
_f_ μ_2	j. 30/hr	k. *None of the above*	

_e_ 2. If the loader were fully utilized, the length of time required to complete moving the dirt would be *(choose nearest value):*

a. 100 hr.	b. 500 hr.	c. 1000 hr.	d. 5000 hr.
e. 10,000 hr.	f. 50,000 hr.	g. 100,000 hr.	h. $\geq$ 500,000 hr.

_d_ 3. Indicate the equation used to compute the probability that the loading area is idle:

a. $\frac{1}{\pi_0} = 1 + \frac{12}{10} + \left(\frac{6}{10}\right)^2 = 2.56$ b. $\frac{1}{\pi_0} = 1 + \frac{10}{6} + \left(\frac{10}{6}\right)^2 = 5.44$ c. $\frac{1}{\pi_0} = 1 + \frac{6}{10} + \left(\frac{6}{10}\right)^2 = 1.96$

d. $\frac{1}{\pi_0} = 1 + \frac{12}{10} + \frac{12}{10} \times \frac{6}{10} = 2.92$ e. $\frac{1}{\pi_0} = 1 + \frac{20}{6} + \frac{20}{6} \times \frac{10}{6} = 9.88$ f. *None of the above*

_k_ 4. The utilization of the loader (fraction of time it is busy) is *(select nearest value)*

a. 15%	b. 20%	c. 25%	d. 30%	e. 35%	f. 40%
g. 45%	h. 50%	i. 55%	j. 60%	k. 65%	l. $\geq$70%

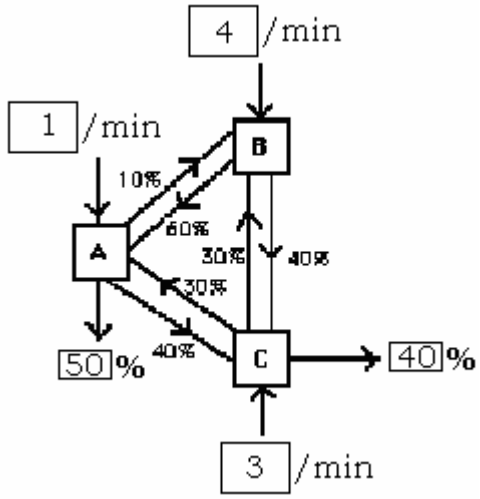
Note: Idle probability $\pi_0 = 1/2.92 = 34\%$, so $1 - \pi_0 = 66\%$ = probability loader is busy

_c_ 5. The fraction of the time that both dumpers are at the loading site is *(select nearest value)*

a. 15%	b. 20%	c. 25%	d. 30%	e. 35%	f. 40%
g. 45%	h. 50%	i. 55%	j. 60%	k. 65%	l. $\geq$70%

Note: $\pi_2 = 1.2 \times 0.6 \times \pi_0 = 1.2 \times 0.6 \times 0.34 = 24\%$

A network of three computer centers (A, B, & C) each receive messages from outside, at the rates shown. The messages may then be routed to another computer in the network for further processing, also as shown, or a reply sent to the sender of the message. Each center has two computers, processing messages at the rate of 8/minute.



Consult the RAQS model output below to answer the questions.

_c_ 1. Which is the busiest computer center?
a. A b. B c. C

_e_ 2. What is the average time to respond to a message? *(Choose nearest value)*
a. 0.2 minute b. 0.3 minute c. 0.4 minute
d. 0.5 minute e. 0.6 minute f. 0.7 minute
g. 0.8 minute h. 0.9 minute i. 1 minute
(Note: Average time spent in the Network = 0.565.)

3. Complete the three arrival rates and the two fractions of messages routed back to sender in the diagram above. *(Five boxes to fill.)*

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Input information
This model has been developed in the Basic Mode
Type of network—Open Network
Number of nodes = 3

Node      Number       Arrival      Arrival   Mean Serv    Service time
#         of servers   Rate         SCV        Time        SCV
1         2            1.000        1.000     0.125       1.00
2         2            4.000        1.000     0.125       1.00
3         2            3.000        1.000     0.125       1.00

Routing (Pij) Matrix
   0.000   0.100   0.400
   0.600   0.000   0.400
   0.300   0.300   0.000
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Output Report
This Model has been developed in the Basic Mode
Type of Network – Open Network

Network Measures
Average Number in the Network = 4.523
Average time spent in the Network = 0.565

Node Measures
Node Util    AvTIQ     VarTIQ    AvNIQ     AvTAN     VarTAN    AvNAN     VarNAN
1    0.528   0.048     0.006     0.407     0.173     0.022     1.462     1.675
2    0.480   0.037     0.004     0.287     0.162     0.020     1.247     1.259
3    0.591   0.067     0.011     0.632     0.192     0.026     1.813     2.526

Util – the Utilization at a Node
AvTIQ, VarTIQ – Mean and Variance of the waiting time in queue at a node
AvNIQ – Mean queue length at a node
AvTAN, VarTAN – Mean and Variance of time spent at a node
AvNAN, VarNAN – Mean and Variance of the number of customers at a node
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