Solutions

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57:022 Principles of Design II
Midterm Exam -- March 6, 2002

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Part	I	II	III	IV	Total
Possible	15	15	10	10	50

Tables of Student's t-distribution and Chi-square distribution are on last page of exam!

Part I. Suppose that vehicles on highway I-80 pass Iowa City at the rate of 5 per minute, and that 20% of the vehicles are trucks. At 8:00 a.m., we begin recording the type and time of each passing vehicle.

In each blank below, write the **number** *corresponding to the most appropriate probability distribution below.. Note that some distributions may apply in more than one case, while others not at all!*

binomial the number of trucks among the next ten vehicles to pass.
Gumbel the weight of the heaviest truck load during the day.
Poisson the number of vehicles passing during the next one-minute interval.
Bernouilli the value 1 if vehicle #i is a truck, else 0.
exponential the time between successive vehicles.
exponential the time between successive trucks.
geometric the number of the first passing vehicle which is a truck.
Pascal the number of the second vehicle which is a truck.
uniform the number on the license plate of the next vehicle.
Erlang-2 the time until the second truck passes.

Probability distributions:

a. Bernouilli
b. Binomial
c. Poisson
d. Lambda
e. Beta
f. Geometric
g. Exponential
h. Normal
i. Weibull
j. Triangular
k. Pascal (negative binomial)
l. Erlang-k with k>1
m. Gumbel
n. Chi-square
o. Uniform

_e_ 1 . If 10 vehicles pass, the probability that *none* of them are trucks is

a. $(0.8)^9(0.2)$ c. $(0.2)^9(0.8)$ $\boxed{\text{e. } (0.8)^{10}}$

b. $(10)(0.2)$ d. $(0.2)^{10}$ f. *None of the above*

_a__ 2. If 10 vehicles pass, the probability that *exactly two* of them are trucks is

$\boxed{\text{a. } \binom{10}{2}(0.8)^8(0.2)^2}$ c. $\binom{10}{2}(0.2)^8(0.8)^2$ e. $(0.8)^8(0.2)^2$

b. $1-\binom{10}{2}(0.8)^8(0.2)^2$ d. $1-\binom{10}{2}(0.2)^8(0.8)^2$ f. $1-(0.8)^8(0.2)^2$

g. *None of the above*

_e_ 3. The probability that a truck passes during any one-minute interval is

a. 0.2 c. 1/e $\boxed{\text{e. } 1-(1/e)}$ *<-same as #5!*

b. $(0.2)^5$ d. 1−e f. e

g. *None of the above*

d=e_ 4. The probability that *exactly* 2 trucks pass during the next two minutes is

a. $\dfrac{2^{-2}e^2}{2}$ c. $1-e^{-4}$ $\boxed{\text{e. } 2e^{-2}}$

b. $1-e^{-2}$ $\boxed{\text{d. } \frac{2^2e^{-2}}{2}}$ f. *None of the above*

_d_ 5. The probability that the time until the first truck passes is less than or equal to 1 minute is

a. e^{-1} c. $1-0.2e^{-1}$ e. $0.2e^{-1}$

b. $1-e^{-0.2}$ $\boxed{\text{d. } 1-e^{-1}}$ *<-same as #3!* f. *None of the above*

Part II.

Classify the random variables with the distributions below as discrete or continuous :

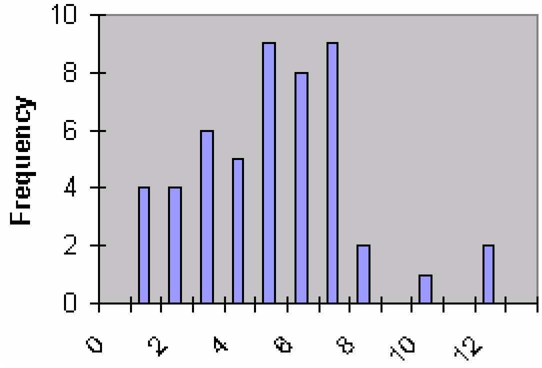
Discrete: Bernouilli, Binomial, Poisson, Geometric, Pascal

Continuous: Uniform, Students t-distribution, Exponential, Normal, Weibull, Triangular, Erlang-k, Gumbel, Chi-square

Part III: Goodness-of-fit

The number of vehicles arriving during each of the first 50 minutes were recorded, and shown at right. The sample mean and sample variance were 5.12 and 6.39, respectively.

cell	*values*	*Frequency*
1	0-2	8
2	3	6
3	4	5
4	5	9
5	6	8
6	7	9
7	>=8	5



We want to test whether the assumption that the arrival process is Poisson, with mean equal to the sample mean, is a "good fit" to the data.

i	N_i	O_i	P_i	E_i	$(E_i-O_i)^2/E_i$
1	0-2	8	0.1149	5.745	0.88512
2	3	6	0.13368	6.684	0.069996
3	4	5	0.17111	8.5555	1.4776
4	5	9	0.17522	8.761	0.00652
5	6	8	0.14952	7.476	0.036728
6	7	9	0.10936	5.468	2.2815
7	>=8	5	0.14374	7.187	0.6655
				Sum=D=	5.4229

Indicate "+" for true, "o" for false:

_+__ 1. The CDF of the distribution of interarrival times is assumed to be $F(t) = 1 - e^{-\lambda t}$ where λ=5.12/minute.

_+__ 2. The quantity E_i , the expected number of observations in interval #i, is np_i, where p_i is the probability that N_i arrivals are observed.

_o__ 3. The quantity **D** is assumed to have Student's t-distribution. *Should be Chi-square!*

_+__ 4. The number of observations, O_i, in an interval should have a binomial distribution, with parameters *(n,p)* = $(50,p_i)$.

_o__ 5. The number of degrees of freedom is reduced by 1 in this situation. *Should be reduced by 2!*

_+__ 6. The sum of several N(0,1) random variables has a normal distribution.

_+__ 7. The smaller the value of D, the better the fit for the distribution being tested.

_o__ 8. The sum of several N(0,1) random variables has approximately a chi-square distribution. *Should be sum of <u>squares</u> of N(0,1) random variables!*

_+__ 9. The assumed distribution is at least 95% likely to be a good fit of the data.

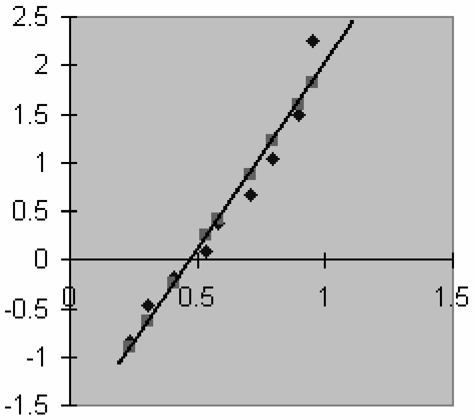
_c__ 10. If the arrival process *does* have the assumed distribution, then P{D ≥ _**?**_} = 10% *(choose nearest value)*

a. ≤7 b. 8 [✓ c. 9] d. 10 e. 11 f. 12 g. 13 h. ≥14

Part IV. An electronic device is made up of a large number of redundant components, and the device fails when the last component fails. The lifetime of each component is random, but its probability distribution is unknown. The manufacturer has provided a 90-day (or 3-month) warranty on this device.

A test of the device is performed, in which *ten* units of the device are operated simultaneously, and the time of the first *nine* failures is recorded. The failure time **T** (in **years**), and the fraction **F** of the deviceswhich have failed at time **T** are shown in the table, together with -ln(-ln F):

T	F	-ln(-ln F)
0.233	0.1	–0.8340
0.304	0.2	–0.4759
0.408	0.3	–0.1856
0.537	0.4	0.0874
0.579	0.5	0.3665
0.703	0.6	0.6717
0.792	0.7	1.0309
0.895	0.8	1.4999
0.953	0.9	2.2504



We will make the assumption that the unit's lifetime has a *Gumbel* distribution. In order to estimate the parameters α and u, a linear regression was performed, using the data in the table above, with the resulting equation:

$$\boldsymbol{(-\ln(-\ln F))} = 3.77\boldsymbol{T} - 1.77 \quad \textit{(where } -\ln(-\ln F) = \alpha T - \alpha u\textit{)}$$

_f_ 1. Based upon the above linear fit, the value of the "shape" parameter (α) of the probability distribution is approximately *(choose nearest value).*

a. 0 b. 0.5 c. 1.0 d. 2.0 e. 3.0 [✓ f. 4.0] g. 5.0

_b_ 2. Based upon the above linear fit, the value of the "location" parameter (u) of the probability distribution is approximately *(choose nearest value).*

a. 0 [✓ b. 0.5] c. 1.0 d. 2.0 e. 3.0 f. 4.0 g. 5.0

_J_ 3. The percent of the units which are expected to fail during the 90-day warranty period is *(choose nearest value):*

a. 1% b. 2% c. 3% d. 4% e. 5% f. 6%
g. 7% h. 8% i. 9% [✓ j. 10%] k. 11% l. ≥ 12%

_f_ 4. The expected lifetime of the unit (in years) is *(choose nearest value):*

a. 0.1 b. 0.2 c. 0.3 d. 0.4 e. 0.5 [✓ f. 0.6]
g. 0.7 h. 0.8 i. 0.9 j. 1.0 k. 1.1 l. ≥ 1.2

_a_ 5. To simulate values of T having this distribution, one might use…

[✓ a. Inverse transformation method] b. Rejection method
c. *Both* a & b are correct d. *Neither* a *nor* b are correct.

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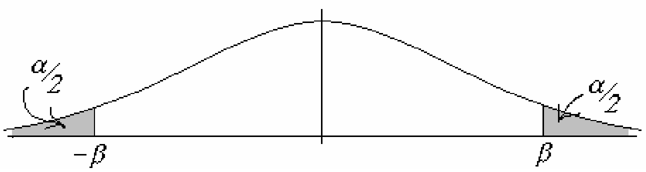
Student's t-distribution: β such that total shaded area = α

Degrees of freedom	α=10%	α=5%	α=1%
1	6.314	12.706	63.657
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499



Chi-square distribution: χ^2 such that P{**D**≥ χ^2} = α

Degrees of freedom	α= 10%	α= 5%	α= 1%
1	2.706	3.841	6.635
2	4.605	5.991	9.210
3	6.251	7.815	11.345
4	7.779	9.488	13.277
5	9.236	11.070	15.086
6	10.645	12.592	16.812
7	12.017	14.067	18.475