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57:022 Principles of Design II

Midterm Exam Solutions -- March 19, 1997

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Part	I	II	III	IV	V	VI	VII	Total
Possible	14	16	19	12	11	8	20	100

Part I. Miscellaneous Multiple Choice

_c_ 1. In simulating a Poisson arrival process with an average of 4 arrivals every minute, an inter-arrival time T (in minutes) can be randomly generated by first obtaining a uniformly-generated random variable X and then computing

a. $T = -\frac{\ln(1-X)}{4}$ d. $T = -\frac{\ln X}{4}$

b. $T = 1 - e^{-4X}$ e. $T = e^{-4X}$

c. Both (a) & (d) are correct f. Both (b) & (e) are correct

g. *None of the above is correct*

_a_ 2. The CDF of the distribution in (1) above, i.e., the inter-arrival time, is F(t) =

a. $1 - e^{-4t}$ b. $4e^{-4t}$

c. $1 - 4e^{-4t}$ d. $4 - e^{-4t}$

e. e^{-4t} f. *None of the above*

Note: The original exam contained a typographical error here, giving ⟨a. $1 - e^{-4X}$⟩ *with "X" instead of "t".*

3. The exponential distribution is a special case of *(check all that apply)*

x_ a. Weibull distribution __ d. Gumbel distribution

__ b. Poisson distribution. x_ e. Erlang distribution

__ c. Uniform distribution __ f. *None of the above*

_c_ 4. If you use the Cricket Graph program to fit a line to n data points (x_i,y_i), i=1,2,...n, it will find the coefficients a & b of the straight line y=ax+b which

a. minimizes $\sum_{i=1}^{n} |y_i - ax_i - b|$ d. maximizes the # of points such that $y_i = ax_i + b$

b. minimizes $\max_i \{ax_i + b - y_i\}$ e. minimizes $\sum_{i=1}^{n} |ax_i + b - y_i|$

c. minimizes $\sum_{i=1}^{n} (y_i - ax_i - b)^2$ f. *None of the above*

_d_ 5. In a Poisson arrival process, the time between arrivals has a/an

a. Poisson distribution. b. Erlang distribution

c. Binomial distribution d. Exponential distribution

e. Uniform distribution f. *None of the above*

_e_ 6. If F(t) is the CDF of the interarrival time for a Poisson process, the expected fraction of arrivals which fall in the time interval $[t_{i-1},t_i]$ is

a. $f(t_i) \times (t_i - t_{i-1})$ d. $F(t_i) \times (t_i - t_{i-1})$

b. $F(t_i - t_{i-1})$ e. $F(t_i) - F(t_{i-1})$

c. $\frac{F(t_i - t_{i-1})}{(t_i - t_{i-1})}$ f. *None of the above*

_f_ 7. The "Cumulative Distribution Function" (CDF) of any random variable X is defined as

a. F(x) = P{X=x} d. f(x) = P{X|x}

b. $f(x) = P\{x\}$ e. $F(x) = P\{X \geq x\}$
c. $f(x) = P\{x \mid X\}$ f. $F(x) = P\{X \leq x\}$

Part II. *Consider the vehicles passing a certain point on the freeway to be a Poisson process. Ten percent of these vehicles are trucks, and the remainder are cars. Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all! When appropriate, you may answer NOTA (None of the Above).*

_E_ 1. time between arrival of vehicle #1 and vehicle #2
_K_ 2. the vehicle# of the second vehicle which is *not* a car.

_J_ 3. number of vehicles arriving during the first 5 minutes
_H_ 4. vehicle# of the first vehicle which is *not* a car.
_E_ 5. time of arrival of first vehicle
_G_ 6. time of arrival of vehicle #2
_D_ 7. the number of trucks among the first 10 vehicles to arrive
_A_ 8. an indicator for vehicle #n which is 1 if a car, 0 otherwise.

Some common probability distributions:

A. Bernouilli	E. Exponential	I. Uniform
B. Normal	F. Beta	J. Poisson
C. Lambda	G. Erlang	K. Pascal
D. Binomial	H. Geometric	L. Random

Part III. *Suppose that 500 light bulbs are tested by simultaneously lighting them and recording the number of failures every 100 hours. The test is interrupted at the end of 1000 hours, when 291 bulbs have failed. A Weibull probability model is then "fit" to the data.*

For each statement, indicate "+" for true, "o" for false:

_+_ 1. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

_o_ 2. We assume that the number of survivors at time t , $N_S(t)$, has a Weibull distribution.

_+_ 3. The Weibull CDF, i.e., F(t), gives, for each bulb, the probability that at time t it has already failed.

_o_ 4. The method used in this situation to estimate the Weibull parameters u & k requires that you first compute the mean and standard deviation of the 291 bulbs which have failed.

_+_ 5. Given a coefficient of variation for the Weibull distribution (the ratio σ/μ), the Weibull shape parameter k can be computed.

_+_ 6. The sum of the CDF (cumulative distribution function) F(t) and the Reliability function R(t), i.e. F(t) + R(t), is always equal to 1 if the Weibull probability model is assumed.

_+_ 7. The exponential distribution is a special case of the Weibull distribution, with failure rate constant, i.e. neither increasing or decreasing over time.

_o_ 8. A value of ln k greater than 1 indicates an increasing failure rate, and a value of ln k less than 1 indicates a decreasing failure rate.

_+_ 9. If each bulb's lifetime has an exponential distribution, the time of the 10^{th} failure has Erlang-10 distribution.

_o_ 10. If 6 bulbs are installed in an office's light fixtures, the number still functioning after 1000 hours has a Weibull distribution.

_o_ 11. If k=1, then $\Gamma\left(1+\frac{1}{k}\right) = 2$. **Note:** $\Gamma(1+x) = x!$, or $\Gamma(x) = (x-1)!$, so

$$\Gamma\left(1+\frac{1}{1}\right) = \Gamma(2) = 1! = 1$$

Select the letter ("A" through "X") below which indicates each correct answer: When preparing a plot so as to estimate the Weibull parameters, ...

_W_ 12. The label on the vertical axis should be ...
_L_ 13. The label on the horizontal axis should be ...
_A_ 14. The slope of the line fit by Cricket Graph should be approximately ...
_C_ 15. The vertical intercept of the line fit by Cricker Graph should be approximately ...

A shape parameter k	I. scale parameter u	Q. coefficient of variation σ/μ
B. +k ln u	J. + u ln k	R. +ln k
C. - k ln u	K. - u ln k	S. -ln k
D. t	L. ln t	T. -ln t
E. ln $^1/_t$	M. ln ln t	U. ln ln $^1/_t$
F R_t	N. ln R_t	V. - ln R_t
G. ln $^1/_{R_t}$	O. ln ln R_t	W. ln ln $^1/_{R_t}$
H. mean value μ	P. standard deviation σ	X. *None of the above*

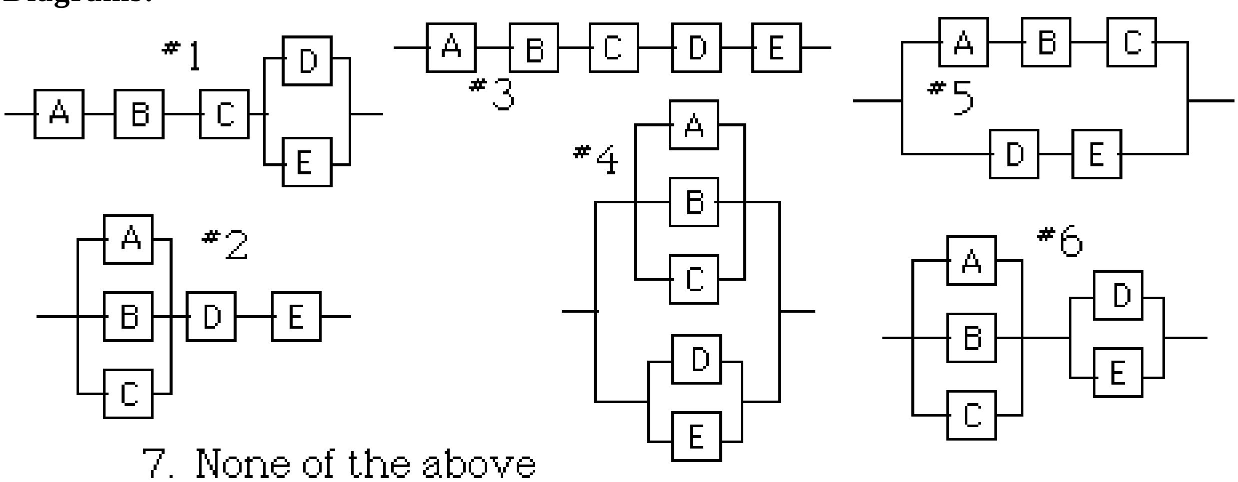
Part IV: A system consists of five components (A,B,C,D, &E). The probability that each component *fails during the first year* of operation is 10% for A, B, and C, and 20% for D and E. For each alternative of (a) through (e), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)

Diagram	Reliability	
_1_	_3_	a. The system requires that all of A, B, & C function, and that either D or E function.
_2_	_7_	b. The system will fail if all of A, B, and C fail or if either D or E fail.

Note: the original exam had and instead of or in system b, and so the answers would be "None of the above" for both diagram and reliability calculation!

Diagrams:



Reliabilities:

1. $(0.9)^3(0.8)^2 = 46.6\%$
2. $1- [1-(0.9)^3]\ [1-(0.8)^2] = 90.2\%$
3. $(0.9)^3[1-(0.2)^2] = 69.9\%$
4. $1 - (0.1)^3(0.2)^2 = 99.9\%$

5. $[1-(0.1)^3]\ [1-(0.2)^2] = 95.9\%$

6. $[1-(0.1)^3]\ [1- (0.2)^2] = 95.9\%$

7. $[1-(0.1)^3]\ (0.8)^2 = 63.9\%$

8. *None of the above*

Part V. Project Scheduling. Indicate true by "+" and false by "o":

_+_1. The quantity LT(i) [i.e. latest time] for each node i is determined by a backward pass through the AOA network.

_+_2. If an activity is represented by an arrow from node i to node j, then LF (latest finish time) for that activity is LT(j).

_+_3. An activity is critical if and only if its total float ("slack") is zero.

_o_4. If an activity is represented by an arrow from node i to node j, then EF (early finish time) for that activity is ET(j).

_o_5. The *MacProject* software requires that you enter the project network in "Activity on Arrow" (AOA) form.

_o_6. If an activity is represented by an arrow from node i to node j, then that activity has zero "float" or "slack" if and only if ET(i)=LT(j).

_o_7. A "dummy" activity cannot be critical.

_+_8. PERT assumes that each activity's duration has a Beta distribution.

_o_9. PERT assumes that the project duration has a Beta distribution.

_+_10. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.

_o_11. PERT estimates the standard deviation of the project duration by summing the standard deviations of the durations of activities on a critical path.

Part VI: Goodness-of-fit The time *between* arrivals of forty vehicles are measured. The number of observations O_i falling within each half-minute interval is shown in the table below. The average is computed by weighting the midpoint of each interval by its number of observations: 0.25x9 + 0.75x4 + 1.25x5 + ... = 2.225 minutes. We wish to test the "goodness of fit" of the exponential distribution having mean 2.225 minutes.

i	Interval	O_i	p_i	E_i	$(E_i-O_i)^2/E_i$
1	0.0 - 0.5	9	0.2015	8.0594	0.1098
2	0.5 - 1.0	4	0.1609	6.4355	0.9217
3	1.0 - 1.5	5	0.1285	5.1389	0.0038
4	1.5 - 2.0	3	0.1026	4.1035	0.2967
5	2.0 - 2.5	7	0.0819	3.2767	4.2308
6	2.5 - 3.0	3	0.0654	2.6165	0.0562
7	> 3.0	9	0.2592	10.3696	0.1809

The sum of the values in the last column is D = 5.8.

Indicate "+" for true, "o" for false:

__o_ 1. The CDF of the distribution of interarrival times is assumed to be $F(t) = 1 - \lambda e^{-\lambda t}$

__+_ 2. The quantity E_i , the expected number of observations in interval #i, is np_i, where p_i is the probability that the interarrival time falls within interval #i.

__o_ 3. The quantity D is assumed to have the chi-square distribution with 7 degrees of freedom.

__+_ 4. The number of observations, O_i, in an interval should have a binomial distribution, with parameters $(n,p)= (40,p_i)$.

__+_ 5. The degrees of freedom is reduced by 2 because (i) the total number of observations is fixed, and (ii) the data was used to estimate one parameter for the distribution being tested.

__o_ 6. The bigger the value of D, the better the fit for the distribution being tested.

__+_ 7. The sum of squares of several N(0,1) random variables has a chi-square distribution.

__+_ 8. The probability p_i that an interarrival time falls in interval #i: $[t_{i-1}, t_i]$, is $F(t_i) - F(t_{i-1})$.

Part VII. (s,S) Inventory System: Consider the following inventory system for a certain spare part for a company's 2 production lines. A maximum of four parts may be kept on the shelf. At the end of each day, the parts in use are inspected and, if worn, replaced with one off the shelf. The probability distribution of the number replaced each day is:

n=	0	1	2
P{n}=	0.3	0.5	0.2

To avoid shortages, the current policy is to **restock** the shelf at the end of each day (after spare parts have been removed) so that the shelf is again filled to its limit (i.e., 4) **if** there is 1 or fewer parts on the shelf.

The inventory system has been modeled as a Markov chain, with the state of the system defined as the end-of-day inventory level (before restocking). Refer to the computer output which follows to answer the following questions: **Note that in the computer output, state #1 is inventory level 0, state #2 is inventory level 1, etc.**

$$P=\begin{bmatrix} 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \\ 0.2 & 0.5 & \boxed{A} & 0 & 0 \\ 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0.2 & 0.5 & 0.3 \end{bmatrix},\ P^2=\begin{bmatrix} 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \\ 0.06 & 0.15 & 0.23 & 0.35 & 0.21 \\ 0.1 & 0.31 & 0.34 & 0.19 & 0.06 \\ 0.04 & 0.2 & 0.37 & 0.3 & 0.09 \end{bmatrix}$$

$$P^3=\begin{bmatrix} 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \\ 0.046 & 0.185 & 0.328 & 0.315 & 0.126 \\ 0.068 & 0.208 & 0.291 & 0.292 & 0.141 \\ 0.074 & 0.245 & 0.327 & 0.255 & 0.099 \end{bmatrix},\ P^4=\begin{bmatrix} 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \\ 0.065 & 0.227 & 0.327 & 0.273 & 0.107 \\ 0.058 & 0.203 & 0.316 & 0.296 & 0.125 \\ 0.065 & 0.214 & 0.309 & 0.285 & 0.125 \end{bmatrix}$$

$$\sum_{n=1}^{4} P^n=\begin{bmatrix} 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \\ 0.3716 & 1.062 & 1.1853 & 0.938 & 0.4431 \\ 0.2262 & 0.9219 & 1.4477 & 1.0781 & 0.3261 \\ 0.1794 & 0.6595 & 1.2062 & 1.3405 & 0.6144 \end{bmatrix}$$

$$M=\begin{bmatrix} 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 15.769 & 4.641 & 3.076 & 2.571 & 8.367 \\ 12.692 & 2.754 & 3.153 & 4 & 9.795 \\ 15 & 3.396 & 2.307 & 3.514 & 10.816 \\ 15.769 & 4.641 & 3.076 & 2.571 & \boxed{B} \end{bmatrix}$$

i	π_i
1	0.063
2	0.215
3	0.317
4	0.284
5	0.119

__c_ 1. The value $P_{2,3}$ is
a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. None of the above

__c_ 2. The value $P_{3,1}$ is
a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. None of the above

__b_ 3. The value $P_{5,4}$ is
a. P{demand=0} b. P{demand=1} c. P{demand=2}
d. P{demand≤1} e. P{demand≥1} f. None of the above

__d_ 4. The numerical value **A** in the matrix P above is
a. 0 b. 0.1 c. 0.2
d. 0.3 e. 0.4 f. 0.5

__f_ 5. The numerical value **B** in the mean-first-passage time matrix (M) above is *(select nearest value)*

a. 1 b. 2 c. 3

d. 4 e. 6 f. 8 (m = $^1/\pi_5$ = 8.40)

g. 10 h. 12 i. >12

__d_ 6. If the shelf is full Sunday evening after restocking, and therefore Monday morning as well, the expected number of days until a stockout (empty shelf) occurs is *(select nearest value):*

a. 2 days b. 5 days c. 10 days

d. 15 days (m_{51}=15.769) e. 20 days f. more than 20 days

__d_ ____ 7. If the shelf is restocked Sunday p.m. so that it is full Monday a.m., the probability that the shelf is full Wednesday night is *(select nearest value):*

a. 7% b. 8% c. 9%

d. 10% ($p_{55}^{(3)}$ = 9.9%) e. 11% f. more than 12%

__a_ 8. The number of *transient* states in this Markov chain model is

a. zero b. 1 c. 2

d. 3 e. 5 f. None of the above

__a_ 9. The number of *absorbing* states in this Markov chain model is

a. zero b. 1 c. 2

d. 3 e. 5 f. None of the above

10. Mark one or more of the following equations which are among those might be used to compute the steady state probability distribution?

____ a. $\pi_1 = 0.2\pi_3 + 0.5\pi_4 + 0.3\pi_5$

__x_ b. $\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1$

__x_ c. $\pi_1 = 0.2\pi_3$

____ d. $\pi_4 = 0.2\pi_2 + 0.5\pi_3 + 0.3\pi_4$

__x_ e. $\pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.3\pi_3 + 0.5\pi_4 + 0.2\pi_5$