57:022 Principles of Design II
Midterm Exam Solutions - Spring 1994

Part:	I	II	III	IV	Total
Possible Pts:	12	24	14	24	74

PART I

We wish to simulate persons arriving at an elevator on the first floor of the Engineering Building randomly at the rate of 5/minute (forming a memoryless process). Eighty percent of the persons are engineering students.

Write the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!

Gumbel	1. the weight of the heaviest passenger when the elevator is full
Poisson	2. the number of persons arriving during the first minute
Exponential	3. the time of arrival of first person
Geometric	4. the sequence number of the first non-engineering student.
Exponential	5. the time between arrival of first and second persons
Normal	6. the total weight of the passengers when the elevator is full
Binomial	7. the number of engineers among the first 10 persons to arrive
Erlang	8. the time of arrival of the fourth passenger

Write the numerical value of the following probabilities:

$1 - e^{-0.5}$	9. probability that the first passenger has already arrived at time t=0.1
$\frac{5^5}{5!} e^{-5}$	10. probability that exactly 5 passengers arrive during the first minute.
$\binom{5}{4}\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	11. probability that four of the first five passengers are engineers.
$\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	12. probability that the first non-engineer is the fifth person to arrive.

PART II

A system consists of five components (A,B,C,D, & E). The probability that each component *survives* the first year of operation is 70% for A, B, & C, and 80% for D & E. For each alternative of (1) through (4), indicate:

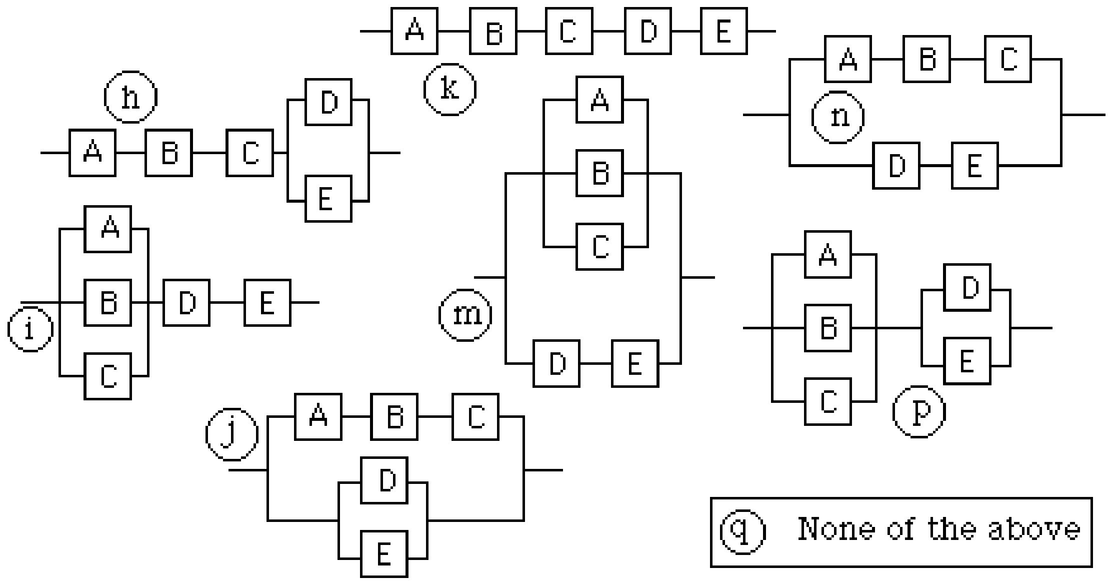
(i) the letter of the reliability diagram below which represents the system
(ii) the letter of the SLAM network model which represents the system
(iii) the letter with the computation of the 1-year reliability (i.e., survival probability)

i.	ii.	iii.	
_m_	_z_	_c_	1. The system requires at least one of A ,B, & C, or both of D & E .
_i_	_t_	_d_	2. The system requires at least one of A, B, & C, and both of D & E.
_q_	_y_	_g_	3. Component E is a back-up unit for D, and is switched on automatically when D fails; the system fails when both D & E have failed, or all three of A, B, & C.
_h_	_s_	_b_	4. The system requires all of A, B, & C, and at least one of D & E.

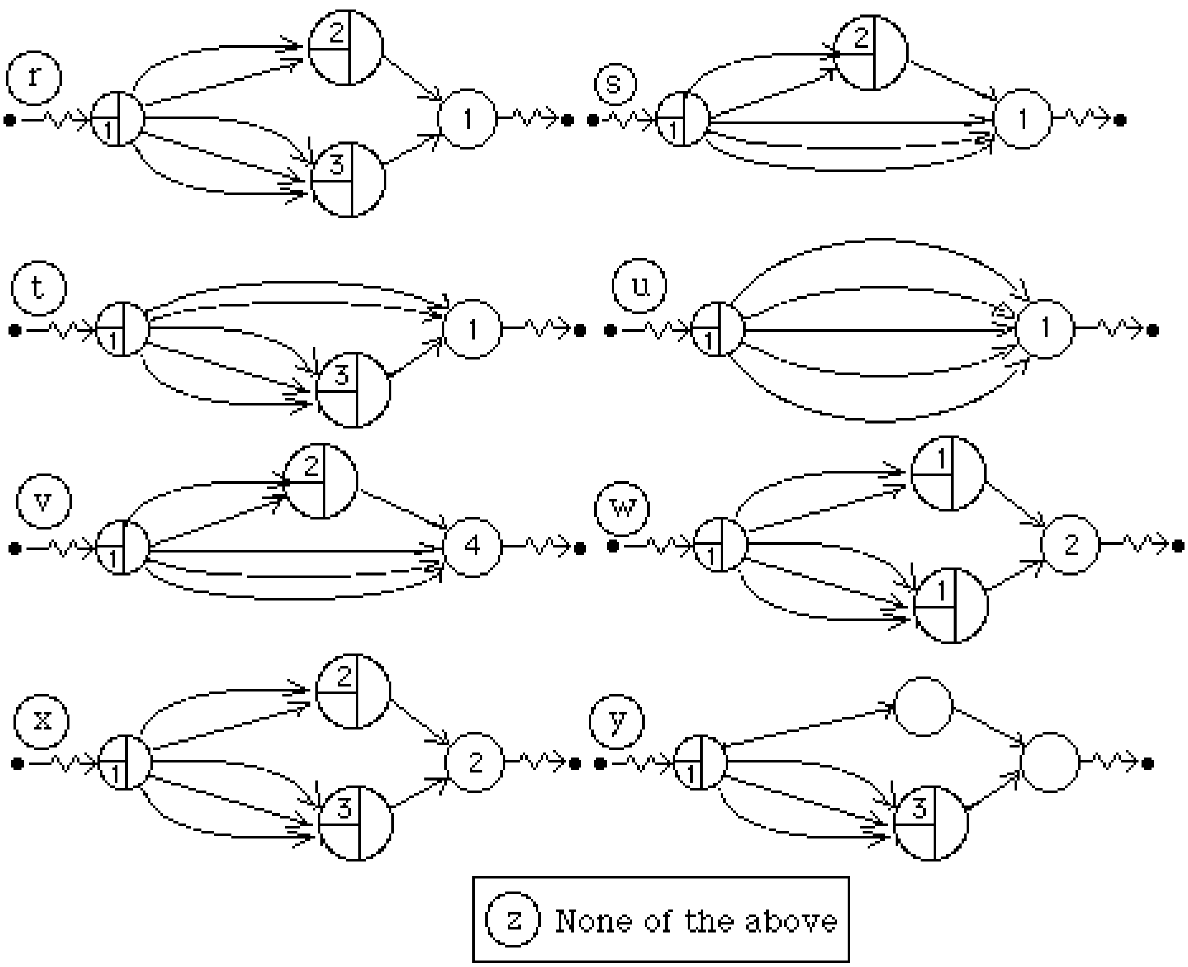
Reliabilities:

a. $1 - (0.3)^3(0.2)^2 = 0.99892$
b. $(0.7)^3[1-(0.2)^2] = 0.32928$
c. $1-(0.3)^3[1- (0.8)^2] = 0.99028$
d. $[1-(0.3)^3] (0.8)^2 = 0.62272$
e. $[1-(0.3)^3] [1-(0.2)^2] = 0.93408$
f. $1- [1-(0.7)^3](0.2)^2 = 0.97372$
g. None of the above

Diagrams:



SLAM networks:

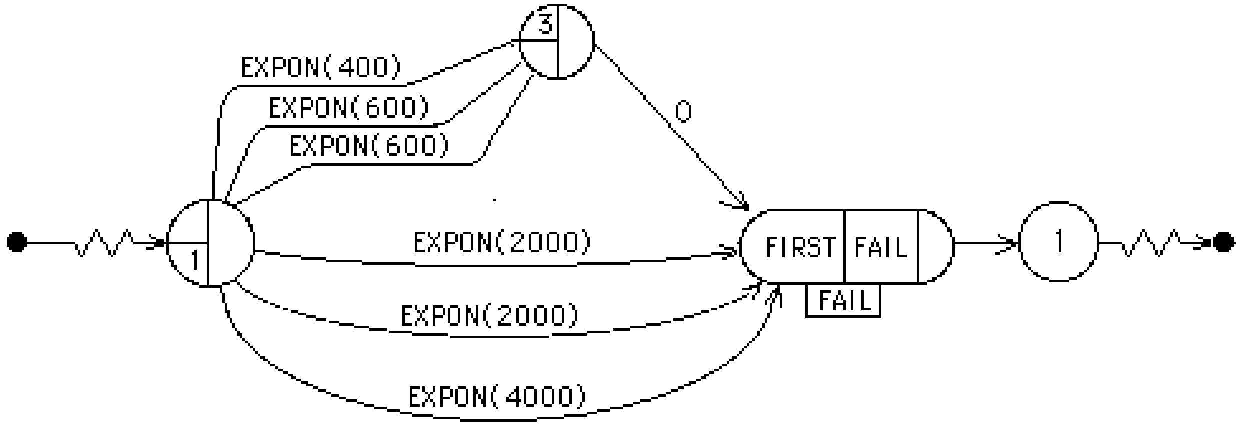


PART III

The following SLAM network was used to simulate a system consisting of six components (where the time units are days). Refer to the output for five hundred runs to find (or estimate) the quantities below:

489 days 1. The average lifetime of the system.
0.209 2. The time which the first failure occurred.

93.8% 3. The reliability of the system if its designed lifetime is specified to be 50 days.
3% 4. The probability that the system survives 1400 days.

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**STATISTICS FOR VARIABLES BASED ON OBSERVATION**

                  MEAN     STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                  VALUE    DEVIATION VARIATION   VALUE     VALUE    OBS
FAIL TIME      0.489E+03 0.381E+03 0.780E+00 0.209E+00 0.217E+04   500

                                FAIL TIME
 OBS  RELA   UPPER
 FREQ FREQ  CELL LIM 0         20        40        60        80       100
                     +    +    +    +    +    +    +    +    +    +    +
  32 0.064 0.500E+02 +***                                              +
  34 0.068 0.125E+03 +***   C                                          +
  65 0.130 0.200E+03 +*******     C                                    +
  44 0.088 0.275E+03 +****             C                               +
  41 0.082 0.350E+03 +****                 C                           +
  44 0.088 0.425E+03 +****                     C                       +
  49 0.098 0.500E+03 +*****                         C                  +
  30 0.060 0.575E+03 +***                              C               +
  30 0.060 0.650E+03 +***                                 C            +
  20 0.040 0.725E+03 +**                                    C          +
  19 0.038 0.800E+03 +**                                     C         +
  12 0.024 0.875E+03 +*                                       C        +
  15 0.030 0.950E+03 +**                                       C       +
  14 0.028 0.103E+04 +*                                          C     +
   7 0.014 0.110E+04 +*                                           C    +
   8 0.016 0.118E+04 +*                                           C    +
   7 0.014 0.125E+04 +*                                            C   +
   9 0.018 0.133E+04 +*                                             C  +
   5 0.010 0.140E+04 +*                                             C  +
   4 0.008 0.148E+04 +                                               C+
   3 0.006 0.155E+04 +                                               C+
   8 0.016    INF    +*                                                C
 ---                 +    +    +    +    +    +    +    +    +    +    +
 500                 0         20        40        60        80       100
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Using the mean and standard deviation from the simulation output, the Weibull parameters U=528.961 and k=1.29394 are determined.

5. According to this result, is the failure rate increasing or decreasing? INcreasing

Based upon the cumulative probability function F(t) for this Weibull distribution, the following probabilities and expected values for each cell were calculated, where t_i is the upper limit of the cell. The cells at the upper end were grouped, as indicated by the horizontal lines, so as to obtain a more even distribution of observations. Next we calculate for each cell (or group of cells) the square of the deviation of O from E, and divide by E, and then sum to obtain the chi-square statistic in the table on the right:

i	t_i	p_i	E_i	O_i
1	50	0.04615	23.07647	32
2	125	0.09713	48.56294	34
3	200	0.10402	52.00927	65
4	275	0.10151	50.75408	44
5	350	0.09467	47.33290	41
6	425	0.08577	42.88484	44
7	500	0.07610	38.05003	49
8	575	0.06643	33.21305	30
9	650	0.05721	28.60495	30
10	725	0.04871	24.35746	20
11	800	0.04107	20.53625	19
12	875	0.03433	17.16303	12
13	950	0.02846	14.23084	15
14	1025	0.02343	11.71482	14
15	1100	0.01916	9.57989	7
16	1175	0.01557	7.78599	8
17	1250	0.01258	6.29178	7
18	1325	0.01011	5.05699	9
19	1400	0.00809	4.04392	5
20	1475	0.00644	3.21825	4
21	1550	0.00510	2.54946	3

t	E	O	D
50	23.07647	32	3.45068
125	48.56294	34	4.36710
200	52.00927	65	3.24479
275	50.75408	44	0.89880
350	47.33290	41	0.84731
425	42.88484	44	0.02900
500	38.05003	49	3.15116
575	33.21305	30	0.31083
650	28.60495	30	0.06804
800	44.89371	39	0.77373
1025	43.10869	41	0.10315
∞	47.50906	51	0.25651
SUM	500	500	17.5011

6. What is the number of degrees of freedom for the chi-square goodness-of-fit test? 9 (=12-1-2), since there are 12 cells (after combining), and two parameters were estimated from the data.
The chi-square probability table indicates that with this # of degrees of freedom , if the system lifetime does have the Weibull distribution with the parameters above, P{D>16.919} is α=5%.
7. Based upon this value, should we accept for the system lifetime the Weibull distribution model with the parameters U=528.961 and k=1.29394? __NO__

ggggg PART IV ggggg

*For each system described below, indicate the appropriate SLAM network setment (A through R) which might best model it. If no network segment shown could be used, indicate "**S**".*

__B__ 1. Jobs are to be processed first by server #1 and then by #2. When the queue for #2 is filled and #1 completes a job, the job must wait until space becomes available before server #1 is free to begin the next job.

__D_ 2. Customers who arrive at a shoe repair shop are equally likely to have one or both shoes to be repaired. A single repairman works on the shoes one at a time.

__N_ 3. Two workers, who differ in the speed with which they work, select their next job from queue #1 if any are waiting there, and queue #2 otherwise.

__F_ 4. A production shop has ten machine operators and twelve machines. The extra two machines are used for back-up when a machine fails. One repairman is available to repair a failed machine, which is then available as a backup for the next machine that fails.

__M_ 5. Customers select the check-out lane at the grocery store which has the shortest queue (and cannot change queues once they've entered it.)

__Q_ 6. Two (identical) servers select their next job from the longer of two queues.

__C_ 7. Customers arrive at a bank's drive-up window; if the waiting line is filled, they drive around the block and then try again.

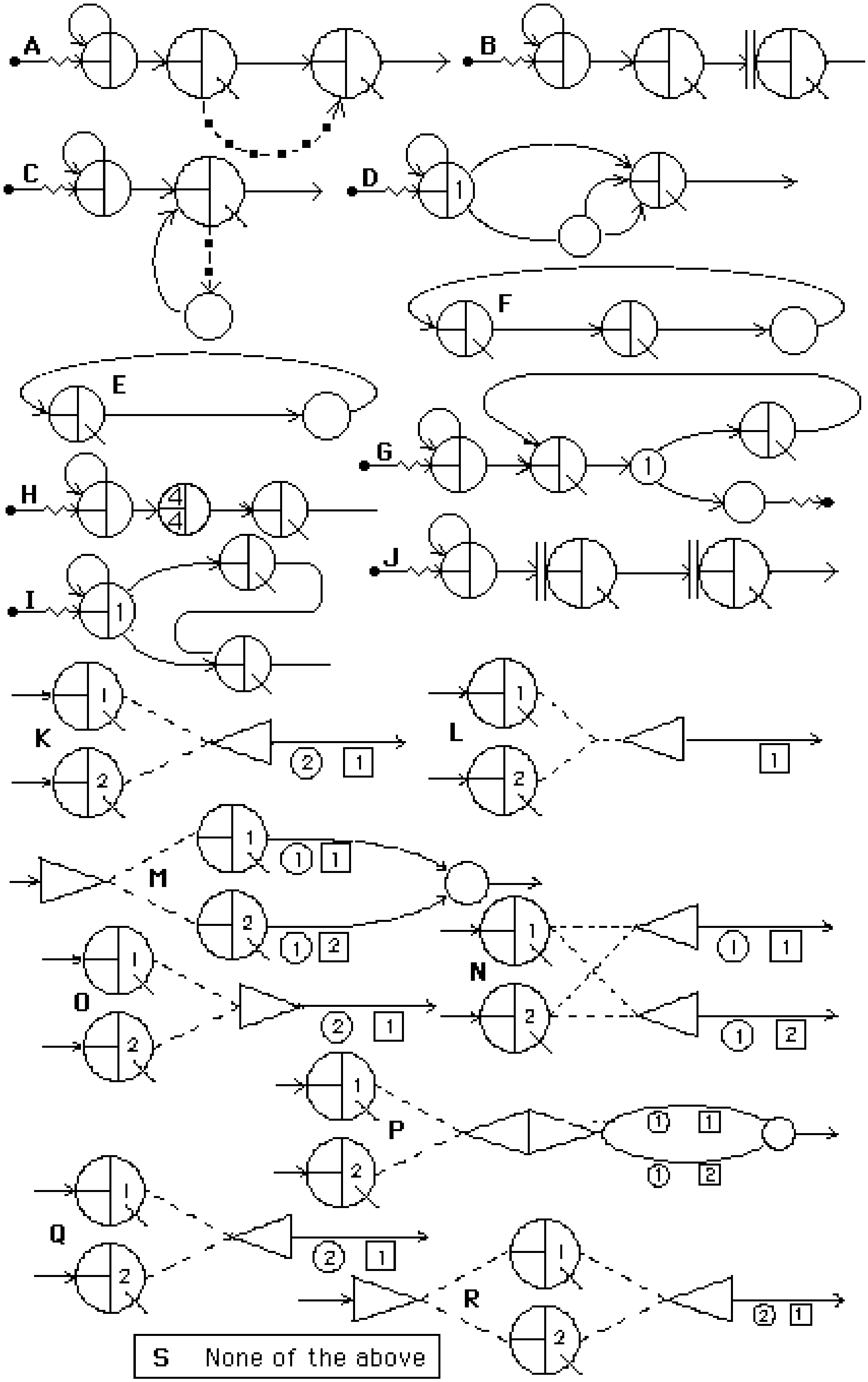
__A_ 8. Customers usually are served by server #1 and then by server #2; however, if the queue for #1 has no space, the customer proceeds directly to the queue for #2.

__H_ 9. Widgets arrive one at a time on a conveyor, drop into a box, and when four widgets have arrived, the box is put into a queue to be prepared for shipping.

__G_ 10. TVs arrive at an inspection station, where they wait to be inspected by a single inspector. An average of five percent require adjusting; one person is available for this task, who then sends to TV back to be re-inspected.

__S_ 11. Widgets and boxes arrive on two separate conveyors at the final station on an assembly line. If there is both a widget and a box waiting, a worker packs the widget into the box and seals it. *The correct SLAM network would appear as "L", except that the point of the SELECT node is in the wrong direction!*

__P_ 12. Customers arrive at two queues to wait for service by either of two clerks. If both clerks are idle, customers prefer clerk #1. If a clerk finishes serving a customer and both queues have persons waiting, he selects the customer at the head of the longest queue.


A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S None of the above