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57:022 Principles of Design II

Midterm Exam Solutions -- Fall 1996

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Part	I	II	III	IV	V	VI	Total
Possible	10	16	8	16	15	10	75

«»«»«»«»«»«»«» PART I «»«»«»«»«»«»«»

Indicate the SLAM network model ("A" through "I") for each system described below. If no SLAM model is given, indicate "X" for "none".

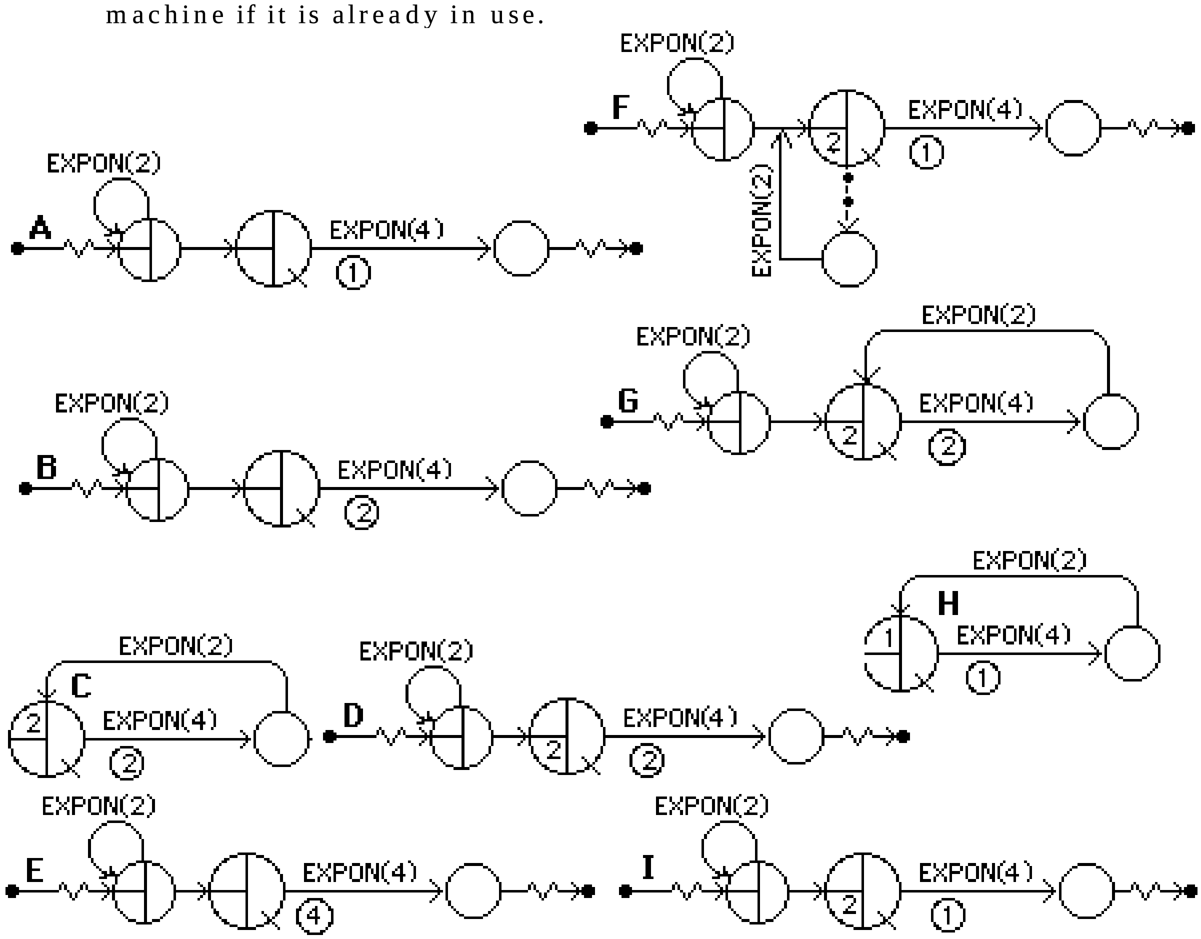
_B_ 1. Customers arriving at the post office wait in a single queue; each of the two postal workers serve the next customer at the head of the queue.

_F_ 2. Vehicles arrive at a bank with a single teller window, with space for two additional waiting vehicles. When no waiting space is available, an arriving vehicle circles the block and tries again to enter the queue.

_I_ 3. Vehicles arrive at a bank with a single teller window, with space for two additional waiting vehicles. When no waiting space is available, no vehicle enters the system.

_D_ 4. Vehicles arrive at a bank with two teller windows, with a single queue having space for two additional waiting vehicles. When no waiting space is available, an arriving vehicle leaves instead of entering the system.

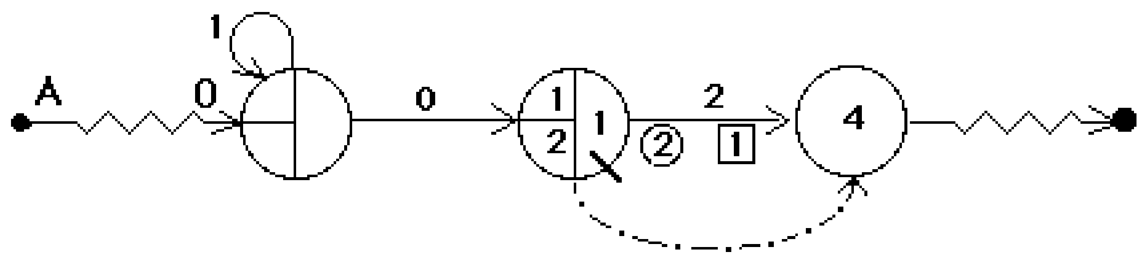
_H_ 5. Two workers each individually prepare parts to be painted; a single spray painting machine is used by both workers, with a worker waiting for the machine if it is already in use.



«»«»«»«»«»«» PART II «»«»«»«»«»«»

Note that

- all activity durations in the SLAM networks below are constants -- none are random!
- first entity is created at time=0



_d_ 1. In Network A, before the first entity, there are how many entities already in the network?

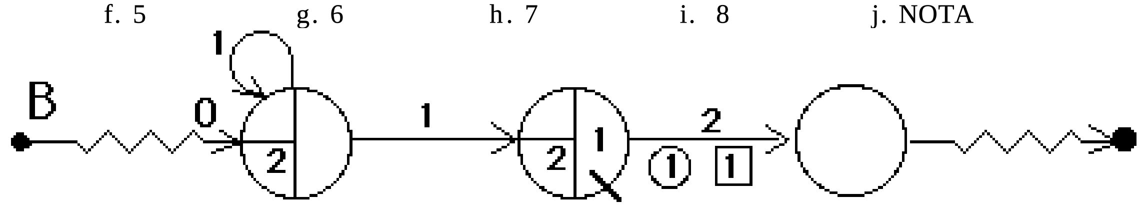
a. none	b. one	c. two	d. three
e. four	f. five	g. can't be determined	h. NOTA

_b_ 2. In Network A, the first entity to leave the system (& is terminated) leaves at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. NOTA

_c_ 3. In Network A, the first created entity begins being served at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. NOTA

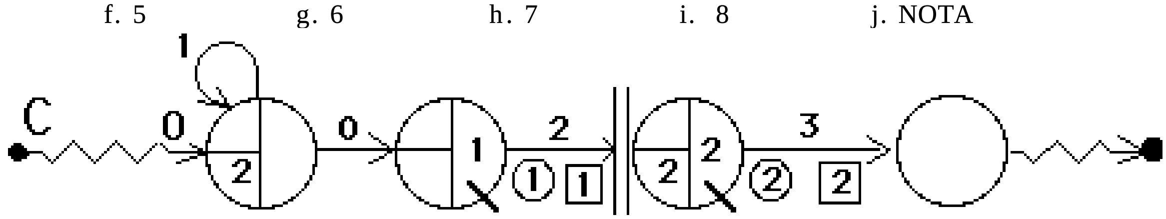


_d_ 4. In Network B, the first entity completes being served at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. NOTA

_c_ 5. In Network B, the second entity will arrive at the queue at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. NOTA



_a_ 6. In Network C, the first entity enters the first queue at time =

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. NOTA

_d_ 7. In Network C, the total number of servers is

a. 0	b. 1	c. 2	d. 3	e. 4
f. 5	g. 6	h. 7	i. 8	j. NOTA

_c_ 8. Of the three SLAM networks, the network in which "blocking" may occur is

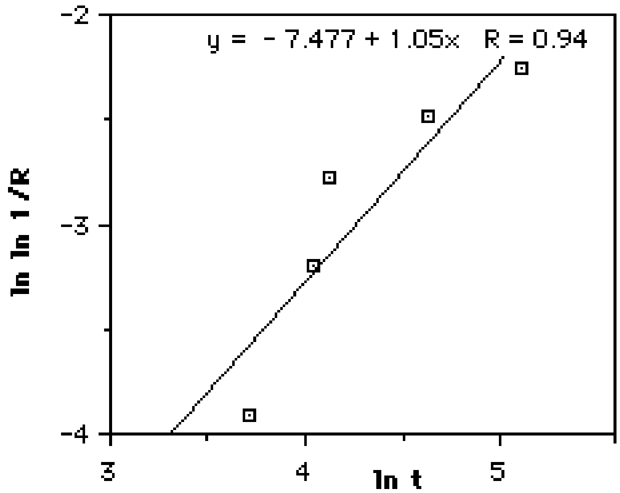
a. A only	b. B only	c. C only	d. both A & C
e. both A & B	f. both B & C	g. NOTA	

«»«»«»«»«»«» PART III «»«»«»«»«»«»

An electronic device is made up of a large number of components. Every component is essential, so that the device will fail when the first component fails. The lifetime of each component is random, but its probability distribution is unknown. The manufacturer has provided a 270-day (approx. 9 month) warranty on this device.

A test of the device is performed, in which fifty units of the device are operated simultaneously, and the time of the first six failures is noted, namely 41, 57, 62, 102, 165, and 185 days. (The test was then terminated at 185 days.) Letting R be the fraction of the devices surviving, "Cricket Graph" was used to prepare the following table and plot, with line fit:

t	R	ln t	1/R	ln ln 1/R
41	.98	3.714	1.020	-3.912
57	.96	4.043	1.042	-3.194
62	.94	4.127	1.064	-2.781
102	.92	4.625	1.087	-2.489
165	.90	5.106	1.111	-2.254
185	.88	5.220	1.136	-2.056



We will make the assumption that the unit's lifetime has a Weibull distribution. Let ε_i denote the "error", i.e., the vertical distance between data point #i and the line determined by Cricket Graph. (Use the table of the Gamma function below, interpolating as necessary).

_c_ 1. The Cricket Graph program fits a line through the data points which minimizes

a. $\sum_{i=1}^{6} \varepsilon_i$ b. $\sum_{i=1}^{6} |\varepsilon_i|$ c. $\sum_{i=1}^{6} (\varepsilon_i)^2$

d. $\max_i \{ \varepsilon_i \}$ e. $\max_i \{ |\varepsilon_i| \}$ f. none of the above

2. Based upon the above plot, the value of the "shape" parameter (k) of the probability dist'n is approximately <u>1.05</u> .

3. Based upon the above plot, the value of the "location" parameter (u) of the probability dist'n is approximately <u>1237</u> . (= exp(7.477/1.05)

_a_ 4. For the distribution with the parameters you specified in (1) & (2), the failure rate is

a. increasing b. decreasing c. constant d. cannot be determined

$$\Gamma\left(1+\frac{1}{k}\right)$$

k	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	3628800	120	9.2605	3.3234	2.0000	1.5046	1.2658	1.1330	1.0522
1	1.0000	0.9649	0.9407	0.9236	0.9114	0.9027	0.8966	0.8922	0.8893	0.8874
2	0.8862	0.8857	0.8856	0.8859	0.8865	0.8873	0.8882	0.8893	0.8905	0.8917
3	0.8930	0.8943	0.8957	0.8970	0.8984	0.8997	0.9011	0.9025	0.9038	0.9051

«»«»«»«»«»«» PART IV «»«»«»«»«»«»

The times T_1, ... T_{50} (in seconds) between arrivals of the first fifty vehicles at an intersection are recorded (the table on the left below):

Observed interarrival times

0.0226392	0.768035	1.65885	3.08626	6.29563
0.0485026	0.790591	1.66189	3.64492	7.04469
0.236294	1.14222	1.68663	3.70833	7.58034
0.412293	1.17618	1.98549	4.06761	7.97349
0.44836	1.20924	2.03548	4.87876	7.98124
0.477881	1.30452	2.07311	4.98918	9.20103
0.480905	1.33905	2.12645	5.07361	10.5373
0.514895	1.3464	2.15331	5.16394	13.7621
0.603458	1.56215	2.62304	5.2581	14.9808
0.716652	1.64656	3.0584	5.70407	16.0848

t	$P\{T \le t\}$
1	0.23758100
2	0.41871726
3	0.55681899
4	0.66211038
5	0.74238653
6	0.80359060
7	0.85025374
8	0.88583060
9	0.91295508
10	0.93363530
11	0.94940229
12	0.96142335
13	0.97058843
14	0.97757606
15	0.98290356

The average of these interarrival times is 3.6865 seconds. We believe that the arrival process is Poisson. Based upon the computed average interarrival time above, the table on the right above is computed. The number of interarrival times are grouped into seven "cells":

i	interval	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
1	0-1	12	11.879	0.00123
2	1-2	12	9.05681	0.95644
3	2-3	5	6.90509	0.52560
4	3-4	4		
5	4-6	7	7.07401	0.00077
6	6-8	5	4.112	0.19176
7	8-∞	5	5.70848	0.08792

The total of the numbers in the last column is D = 2.06751.

Indicate, for each statement, whether true ("+") or false ("o"):

__o_ 1. The value of E_4 (blanked in the table above) is between 6 and 7. (E_4=5.26)

__+_ 2. The probability p_3 that a car arrives in an interval [2,3], is F(3) - F(2)

__o_ 3. The CDF of a random variable T is $F(t) = P\{T \ge t\}$

__o_ 4. The CDF of the distribution is assumed to be $F(t) = 1 - \lambda e^{-\lambda t}$ where λ = 3.6865 sec.

__+_ 5. The number of observations, O_i, in interval #i should have the binomial distribution.

__+_ 6. The quantity D is assumed to have the chi-square distribution.
__+_ 7. The chi-square distribution for this test will have 5 "degrees of freedom".
__o_ 8. The quantity $(E_i-O_i)^2/E_i$ is assumed to have the normal N(0,1) distribution.
__o_ 9. The number of observations, O_i, in interval #i should have the Poisson distribution.
__+_ 10. The sum of the squares of several N(0,1) random variables has chi-square distribution.
__o_ 11. <u>If</u> T actually has a mean value of 3.6865 seconds, the probability that D exceeds the observed value 2.0675 is less than 10%.
__+_ 12. The exponential distribution with mean 3.6865 seconds should be accepted as a model for the interarrival times of the vehicles.
__+_ 13. The quantity E_i is the expected number of observations in interval #i
__o_ 14. The chi-square distribution for this test will have 6 "degrees of freedom".
__o_ 15. The quantity D is assumed to have approximately a Normal distribution.
__+_ 16. The smaller the value of D, the better the fit for the distribution being tested.

deg.of	Chi-square Dist'n $P\{D \geq \chi^2\}$					
freedom	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

«»«»«»«»«»«» Part V «»«»«»«»«»«»

Five components (A,B,C,D, & E) are available for constructing a system. The probability that each component survives the <u>first</u> year of operation is 70% for A, B, & C, and 80% for D & E. For each system ((1) through (5) below, indicate:

(i) the letter of the SLAM network model which represents the system

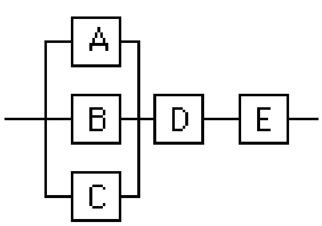
(ii) for each of the three scenarios (a,b,c) below, whether the system will Fail or Survive (circle "F" or "S"):

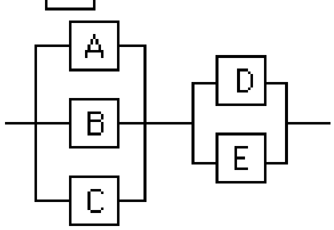
(a) components A and C fail.

(b) components B and D fail.

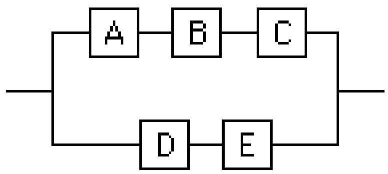
(c) components C, D, & E fail.

(iii) the letter with the computation of the 1-year reliability (i.e., survival probability)

1. 

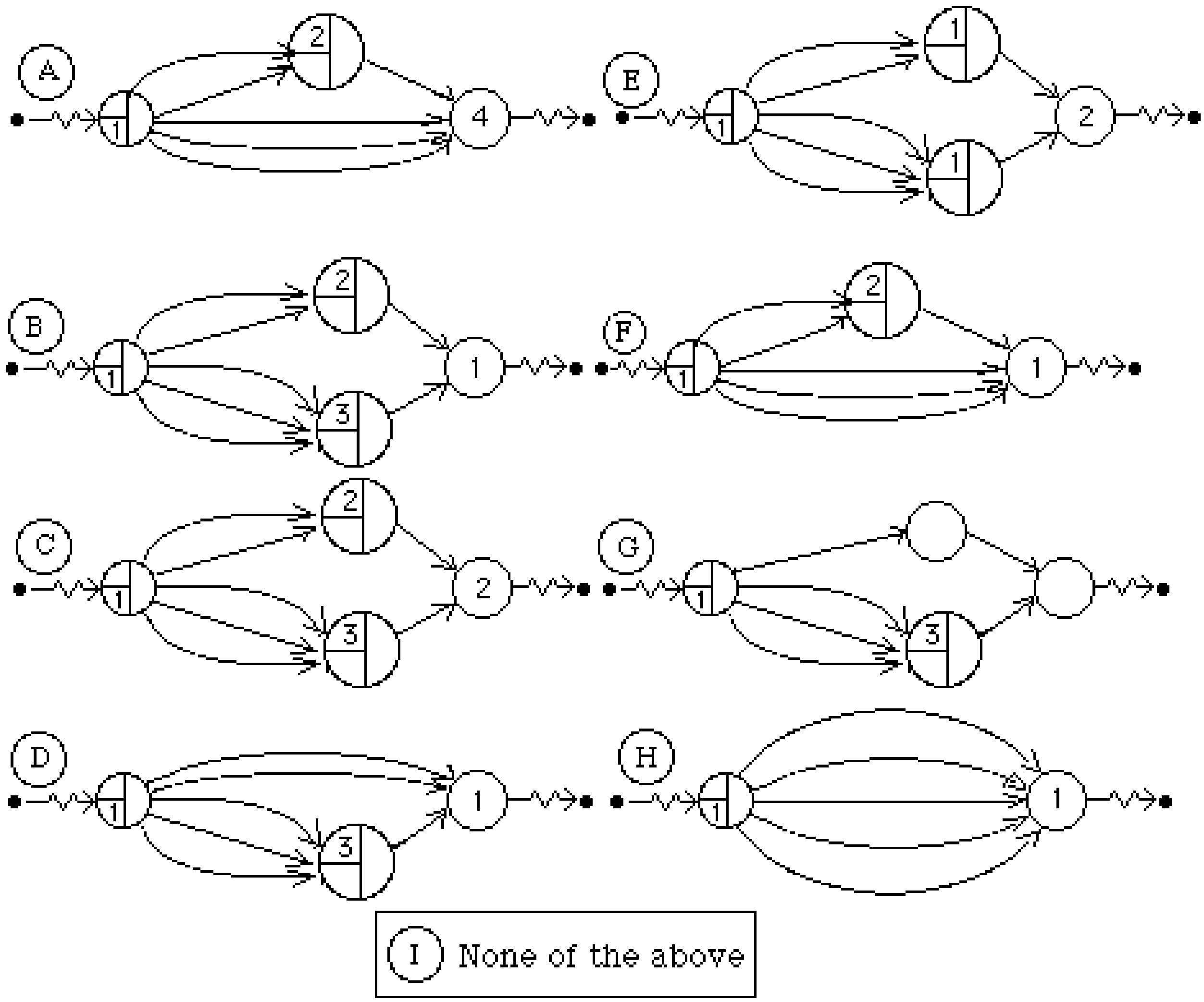
2. 

	SLAM network	Scenario (a)	Scenario (b)	Scenario (c)	Reliability
1.	D	F (S)	(F) S	(F) S	P
2.	B	F (S)	F (S)	(F) S	L

3. 

E | F (S) (F) S (F) S | K

SLAM network models:



Reliabilities:

J. $1 - (0.3)^3(0.2)^2 = 0.99892$

K. $1 - [1-(0.7)^3][1 - (0.8)^2] = 0.76348$

L. $[1 - (0.3)^3] [1 - (0.2)^2] = 0.93408$

M. $1 - (0.7)^3(0.8)^2 = 0.78048$

N. $1- (0.7)^3[1- (0.2)^2] = 0.67072$

O. $1 - (0.3)^3[1- (0.8)^2] = 0.99028$

P. $[1 - (0.3)^3] (0.8)^2 =0.62272$

Q. None of the above

«»«»«»«»«»«» Part VI «»«»«»«»«»«»

Consider again the drive-up bank teller window system described repeatedly in class and your homework assignments.

```
GEN,BRICKER,BANKTELLERS,2/11/1993,,,,,,,72;
LIM,2,1,50;
```

```
INIT,0,480;
NETWORK;
      CREATE,EXPON(5.0),,1;
      QUE(1),0,4,BALK(OVFLO);
      ACT(1)/1,EXPON(2.0);
      COLCT,INTVL(1),CUSTOMER_TIME,20/.5/.5;
      TERM;
OVFLO COLCT,FIRST;
      TERM,1;
      END;
FIN;
```

S L A M   I I   S U M M A R Y   R E P O R T

CURRENT TIME 0.4081E+03
STATISTICAL ARRAYS CLEARED AT TIME 0.0000E+00

STATISTICS FOR VARIABLES BASED ON OBSERVATION

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
CUSTOMER_TIME	0.303E+01	0.286E+01	0.944E+00	0.345E-01	0.110E+02	88
	0.408E+03	0.000E+00	0.000E+00	0.408E+03	0.408E+03	1

FILE STATISTICS

FILE NUMBER	LABEL/TYPE	AVERAGE LENGTH	STANDARD DEVIATION	MAXIMUM LENGTH	CURRENT LENGTH	AVERAGE WAIT TIME
1	QUEUE	0.300	0.724	4	4	1.317
2	CALENDAR	1.439	0.496	3	2	2.669

SERVICE ACTIVITY STATISTICS

ACT NUM	ACT LABEL OR START NODE	SER CAP	AVERAGE UTIL	STD DEV	CUR UTIL	AVERAGE BLOCK	MAX IDL TME/SER	MAX BSY TME/SER	ENT CNT
1	QUEUE	1	0.439	0.50	1	0.00	17.35	29.23	88

```
                              **HISTOGRAM NUMBER 1**
                                  CUSTOMER_TIME

 OBS  RELA   UPPER
 FREQ FREQ  CELL LIM 0         20        40        60        80        100
                     +    +    +    +    +    +    +    +    +    +    +
  10 0.114 0.500E+00 +******                                            +
   9 0.102 0.100E+01 +*****     C                                       +
  16 0.182 0.150E+01 +*********          C                              +
  13 0.148 0.200E+01 +*******                   C                       +
   4 0.045 0.250E+01 +**                           C                    +
   7 0.080 0.300E+01 +****                             C                +
   5 0.057 0.350E+01 +***                                C              +
   1 0.011 0.400E+01 +*                                   C             +
   3 0.034 0.450E+01 +**                                    C           +
   0 0.000 0.500E+01 +                                      C           +
   2 0.023 0.550E+01 +*                                      C          +
   1 0.011 0.600E+01 +*                                      C          +
   3 0.034 0.650E+01 +**                                       C        +
   0 0.000 0.700E+01 +                                         C        +
   4 0.045 0.750E+01 +**                                         C      +
   1 0.011 0.800E+01 +*                                           C     +
   2 0.023 0.850E+01 +*                                            C    +
   2 0.023 0.900E+01 +*                                             C   +
   2 0.023 0.950E+01 +*                                              C  +
   2 0.023 0.100E+02 +*                                               C+
   0 0.000 0.105E+02 +                                                C+
   1 0.011    INF    +*                                                 C
 ---                 +    +    +    +    +    +    +    +    +    +    +
  88                 0         20        40        60        80        100

              **STATISTICS FOR VARIABLES BASED ON OBSERVATION**

                     MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                     VALUE   DEVIATION VARIATION   VALUE     VALUE    OBS

 CUSTOMER_TIME   0.303E+01 0.286E+01 0.944E+00 0.345E-01 0.110E+02    88
Fortran STOP
```

1. Estimate the mean (average) time in the system. 3.03 min.
2. What fraction of the customers spend more than 5 minutes (total of both waiting and being served) at the bank? 20 %
3. What fraction of the time was the teller idle? 56.1 %
4. What is the maximum time that any customer spent in the system? 11 min.
5. What is the average time that a customer spent in the waiting line before being served? 1.317 min.