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57:022 Principles of Design II

Midterm Exam -- March 10, 1999 *revised*

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Part	I	II	III	IV	V	Total
Your score:						
Possible	14	12	10	12	10	58

Part I. Miscellaneous Multiple Choice

___ 1. In simulating a Poisson arrival process with an average of 2 arrivals every minute, an inter-arrival time T (in minutes) can be randomly generated by first obtaining a uniformly-generated random variable X and then computing

a. $T = 1 - e^{-2X}$

b. $T = -\frac{\ln X}{2}$

c. Both (a) & (d) are correct

d. $T = e^{-2X}$

e. $T = -\frac{\ln(1-X)}{2}$

f. Both (b) & (e) are correct

g. *None of the above is correct*

___ 2. The CDF of the distribution in (1) above, i.e., the inter-arrival time, is F(t) =

a. $1 - 2e^{-2X}$

b. $1 - e^{-2X}$

c. $2e^{-2X}$

d. e^{-2X}

e. $1 - e^{-X/2}$

f. $1 - \frac{1}{2}e^{-X/2}$

g. $e^{-X/2}$

h. $\frac{1}{2}e^{-X/2}$

i. *None of the above*

3. The exponential distribution is a special case of *(check all that apply)*

__ a. Weibull distribution

__ b. Poisson distribution.

__ c. Uniform distribution

__ d. Gumbel distribution

__ e. Erlang distribution

__ f. *None of the above*

___ 4. If you use the Minitab program to fit a line to n data points (x_i,y_i), i=1,2,...n, it will find the coefficients a & b of the straight line y=ax+b which

a. minimizes $\sum_{i=1}^{n} \left| y_i - ax_i - b \right|$

b. minimizes $\max_i \left\{ y_i - ax_i - b \right\}$

c. minimizes $\sum_{i=1}^{n} \left(y_i - ax_i - b \right)^2$

d. maximizes the # of points such that $y_i = ax_i + b$

e. minimizes $\sum_{i=1}^{n} \left| ax_i + b - y_i \right|$

f. *None of the above*

___ 5. In a Poisson arrival process, the time between arrivals has a/an

a. Poisson distribution.

b. Erlang distribution (k>1)

c. Binomial distribution

d. Exponential distribution

e. Uniform distribution

f. *None of the above*

___ 6. If F(t) is the CDF of the interarrival time for a Poisson process, the expected fraction of arrivals which fall in the time interval $[t_{i-1},t_i]$ is

a. $\frac{F(t_i - t_{i-1})}{t_i - t_{i-1}}$

b. $F(t_i) - F(t_{i-1})$

c. $f(t_i) \times (t_i - t_{i-1})$

d. $\frac{F(t_i)}{t_i - t_{i-1}}$

e. $F(t_i) \times (t_i - t_{i-1})$

f. $F(t_i - t_{i-1})$

g. $F(t_{i-1}) - F(t_i)$

h. *None of the above*

___ 7. The "Cumulative Distribution Function" (CDF) of any random variable X is defined as

a. F(t) = P{X=t}

b. f(t) = P{t}

c. f(t) = P{t | X}

d. f(t) = P{X|t}

e. F(t) = P{X≤t}

f. F(t) = P{X≥t}

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Part II. *In each blank below, write the* ***number*** *corresponding to the most appropriate probability distribution.. Note that some distributions may apply in more than one case, while others not at all!*

_____ a. the number of cars passing through an intersection during a 1-minute green light.
_____ b. the number of trucks among the first 10 vehicles to arrive at an intersection during a red light
_____ c. the sum of ten N(0,1) random variables
_____ d. the time until the arrival of the <u>second</u> car at an intersection after a traffic light has turned red.
_____ e. the total weight of the passengers on a full elevator.
_____ f. the maximum height of the passengers on a full elevator.
_____ g. the time between the arrivals of the first and second vehicle during a red light.
_____ h. the magnitude of the highest rate of flow into the Coralville Reservoir next year
_____ i. the number of heads obtained by tossing a single coin once.
_____ j. number of defective items found when testing a batch of size 10
_____ k. The sum of the squares of ten N(0,1) random variables
_____ l. the # of items produced in order to get 4 acceptable items, if each is tested before producing the next

<u>Probability distributions:</u>

1. Bernouilli
2. Binomial
3. Poisson
4. Lambda
5. Beta
6. Geometric
7. Exponential
8. Normal
9. Weibull
10. Triangular
11. Pascal (negative binomial)
12. Erlang-k with k>1
13. Gumbel
14. Chi-square
15. Uniform

Part III. *Suppose that 500 light bulbs are tested by simultaneously lighting them and recording the number of failures every 100 hours. The test is interrupted at the end of 1000 hours, when 291 bulbs have failed. A Weibull probability model is then "fit" to the data.*

For each statement, indicate "+" for true, "o" for false:

___ 1. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.
___ 2. We assume that the number of survivors at time t , $N_S(t)$, has a Weibull distribution.
___ 3. The Weibull CDF, i.e., F(t), gives, for each bulb, the probability that at time t it has already failed.
___ 4. The method used in this situation to estimate the Weibull parameters u & k requires that you first compute the mean and standard deviation of the 291 bulbs which have failed.
___ 5. Given a coefficient of variation for the Weibull distribution (the ratio σ/μ), the Weibull shape parameter k can be computed.
___ 6. The sum of the CDF (cumulative distribution function) F(t) and the Reliability function R(t), i.e. F(t) + R(t), is always equal to 1 if the Weibull probability model is assumed.
___ 7. The exponential distribution is a special case of the Weibull distribution, with failure rate constant, i.e. neither increasing or decreasing over time.
___ 8. A positive value of ln k indicates an increasing failure rate, and negative ln k indicates a decreasing failure rate.
___ 9. If each bulb's lifetime has an exponential distribution, the time of the 10^{th} failure has Erlang-10 distribution.
___ 10. If 6 bulbs are installed in an office's light fixtures, the number still functioning after 1000 hours has a Weibull distribution.

Part IV: A system consists of five components (A,B,C,D, &E). The probability that each component *fails during the first year* of operation is 30% for A, B, and C, and 40% for D and E. For each alternative of (a) through (e), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)

<u>Diagram</u> <u>Reliability</u>

___ ___ a. The system requires that at least one of A, B, & C function, <u>and</u> that either D or E function.
___ ___ b. The system will fail if any one of the five components fails.

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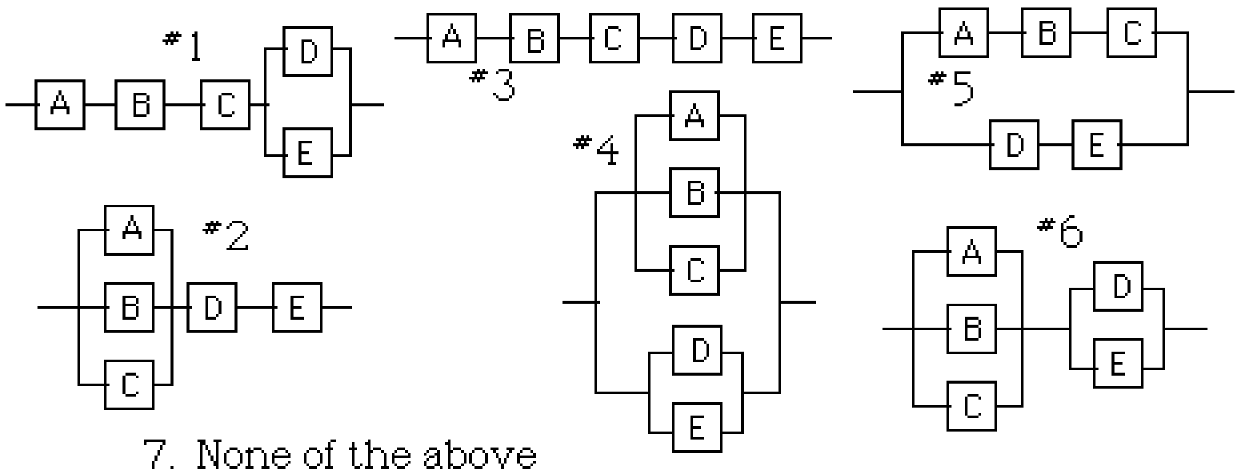
Diagrams:



Reliabilities:

1. $(0.7)^3(0.6)^2$ = 12.3%
2. 1- $[1-(0.7)^3]$ $[1-(0.6)^2]$ = 57.9%
3. $(.7)^3(1-[.4]^2)$ = 28.8%
4. 1 - $(0.3)^3(0.4)^2$ = 94.5%
5. $[1-(0.3)^3]$ $[1-(0.4)^2]$ = 81.7%
6. 1 - $[1-(0.3)^3]$ $[1-(0.4)^2]$ = 18.3%
7. $[1-(0.3)^3]$ $(0.6)^2$ = 35.0%
8. *None of the above*

Part V. An electronic device is made up of a large number of components. Every component is essential, so that the device will fail when the first component fails. The lifetime of each component is random, but its probability distribution is unknown. The manufacturer has provided a 90-day warranty on this device.

A test of the device is performed, in which *fifty* units of the device are operated simultaneously, and the time of the first nine failures is recorded. (The test was then terminated at 250 days.) Let T be the failure time (days), and let R be the fraction of the devices surviving.

i	Ti	R(Ti)	ln Ti	ln ln 1/R(Ti)
1	44.381	0.980	3.793	-3.902
2	77.631	0.960	4.352	-3.199
3	114.954	0.940	4.745	-2.783
4	130.303	0.920	4.870	-2.484
5	150.205	0.900	5.012	-2.250
6	161.517	0.880	5.085	-2.057
7	192.347	0.860	5.259	-1.892
8	201.954	0.840	5.308	-1.747
9	244.001	0.820	5.497	-1.617

A linear regression was performed, using the data in the table below, with the resulting equation:

$$(\ln \ln 1/R) = 1.4034 (\ln T) - 9.285$$

We will make the assumption that the unit's lifetime has a Weibull distribution. *(Use the tables of the Gamma function below, interpolating as necessary).*

___ 1. Based upon the above plot, the value of the "shape" parameter (k) of the probability dist'n is approximately *(choose nearest value).*
a. 1 b. 10 c. 100 d. 1000

___ 2. Based upon the above plot, the value of the "location" parameter (u) of the probability dist'n is approximately *(choose nearest value).*
a. 1 b. 10 c. 100 d. 1000

___ 3. The failure rate is
a. increasing b. decreasing c. constant d. cannot be determined

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___ 4. The percent of the units which are expected to fail during the 90-day warranty period is *(choose nearest value):*

a. 1% b. 2% c. 3% d. 4%
e. 5% f. 6% g. 7% h. 8%

___ 5. The expected lifetime of the unit is *(choose nearest value):*

a. 100 b. 500 c. 1000 d. 1500
e. 2000 f. 2500 g. 3000 h. 4000

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Table 1: $\Gamma\left(1+\dfrac{1}{k}\right)$

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	362880.	120.000	9.26053	3.32335	2.00000	1.50458	1.26582	1.13300	1.05218
1	1.00000	0.96491	0.94066	0.92358	0.91142	0.90275	0.89657	0.89224	0.88929	0.88736
2	0.88623	0.88569	0.88562	0.88591	0.88648	0.88726	0.88821	0.88928	0.89045	0.89169
3	0.89298	0.89431	0.89565	0.89702	0.89838	0.89975	0.90111	0.90245	0.90379	0.90510
4	0.90640	0.90768	0.90894	0.91017	0.91138	0.91257	0.91374	0.91488	0.91600	0.91710
5	0.91817	0.91922	0.92025	0.92125	0.92224	0.92320	0.92414	0.92507	0.92597	0.92685

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Table 2: Coefficient of variation $\frac{\sigma}{\mu}$ of the Weibull distribution, as a function of k alone:

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	---	---	15.84298	5.40769	3.14086	2.23607	1.75807	1.46242	1.26051	1.11303
1	1.00000	0.91022	0.83690	0.77572	0.72375	0.67897	0.63991	0.60548	0.57487	0.54745
2	0.52272	0.50029	0.47983	0.46108	0.44384	0.42791	0.41314	0.39942	0.38662	0.37466
3	0.36345	0.35292	0.34300	0.33365	0.32482	0.31646	0.30853	0.30101	0.29385	0.28704
4	0.28054	0.27435	0.26842	0.26276	0.25733	0.25213	0.24714	0.24235	0.23775	0.23332
5	0.22905	0.22495	0.22099	0.21717	0.21348	0.20991	0.20647	0.20314	0.19992	0.19680

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