# *57:022 Principles of Design II*
## *Midterm Exam - Spring 1994*

| Part: | I | II | III | IV | Total |
|---|---|---|---|---|---|
| Possible Pts: | 12 | 24 | 14 | 24 | 74 |
| Your score: | ______ | ______ | ______ | ______ | ______ |

## PART I

We wish to simulate persons arriving at an elevator on the first floor of the Engineering Building randomly at the rate of 5/minute (forming a memoryless process). Eighty percent of the persons are engineering students.

*Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!*

___ 1. the weight of the heaviest passenger when the elevator is full
___ 2. the number of persons arriving during the first minute
___ 3. the time of arrival of first person
___ 4. the sequence number of the first non-engineering student.
___ 5. the time between arrival of first and second persons
___ 6. the total weight of the passengers when the elevator is full
___ 7. the number of engineers among the first 10 persons to arrive
___ 8. the time of arrival of the fourth passenger

| | | | |
|---|---|---|---|
| A. uniform | B. geometric | C. Bernouilli | D. binomial |
| E. normal | F. Weibull | G. Gumbel | H. exponential |
| I. Poisson | J. Erlang | K. chi-square | L. Pascal |
| | M. none of the above | | |

*Write the alphabetic letter below corresponding to the numerical value of the following probabilities:*

___ 9. probability that the first passenger has already arrived at time t=0.1
___ 10. probability that exactly 5 passengers arrive during the first minute.
___ 11. probability that four of the first five passengers are engineers.
___ 12. probability that the first non-engineer is the fifth person to arrive.

| | | | |
|---|---|---|---|
| N. $\frac{(0.8)^5}{4!} e^{-5}$ | Q. $1 - e^{-0.5}$ | T. $1 - e^{-5}$ | W. $e^{-5}$ |
| O. $\binom{5}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$ | R. $\frac{5^5}{5!} e^{-5}$ | U. $e^{-0.5}$ | X. none of the above |
| P. $\binom{5}{4}\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ | S. $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$ | V. $\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$ | |

## PART II

A system consists of five components (A,B,C,D, & E). The probability that each component *survives* the first year of operation is 70% for A, B, & C, and 80% for D & E. For each alternative of (1) through (4), indicate:

(i) the letter of the reliability diagram below which represents the system
(ii) the letter of the SLAM network model which represents the system
(iii) the letter with the computation of the 1-year reliability (i.e., survival probability)

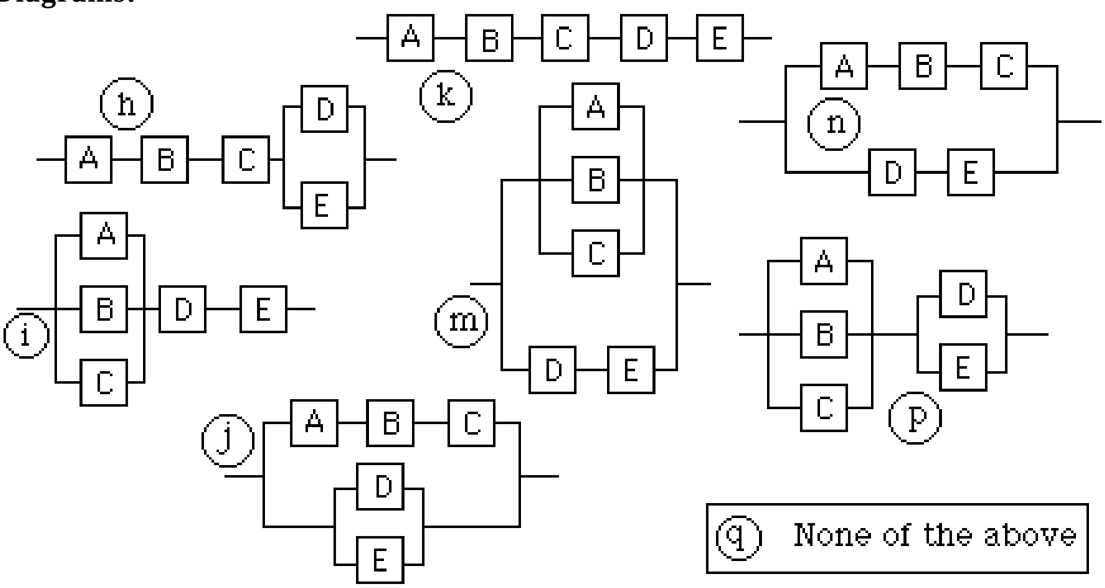
i. ii. iii.

___ | ___ | ___ 1. The system requires at least one of A ,B, & C, or both of D & E .
___ | ___ | ___ 2. The system requires at least one of A, B, & C, and both of D & E.
___ | ___ | ___ 3. Component E is a back-up unit for D, and is switched on automatically when D fails; the system fails when both D & E have failed, or all three of A, B, & C.
___ | ___ | ___ 4. The system requires all of A, B, & C, and at least one of D & E.

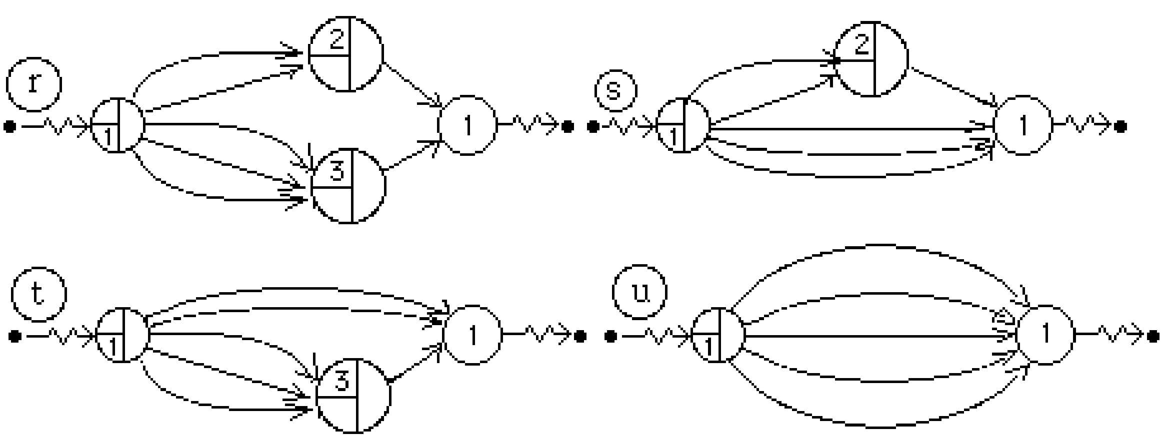
**Reliabilities:**

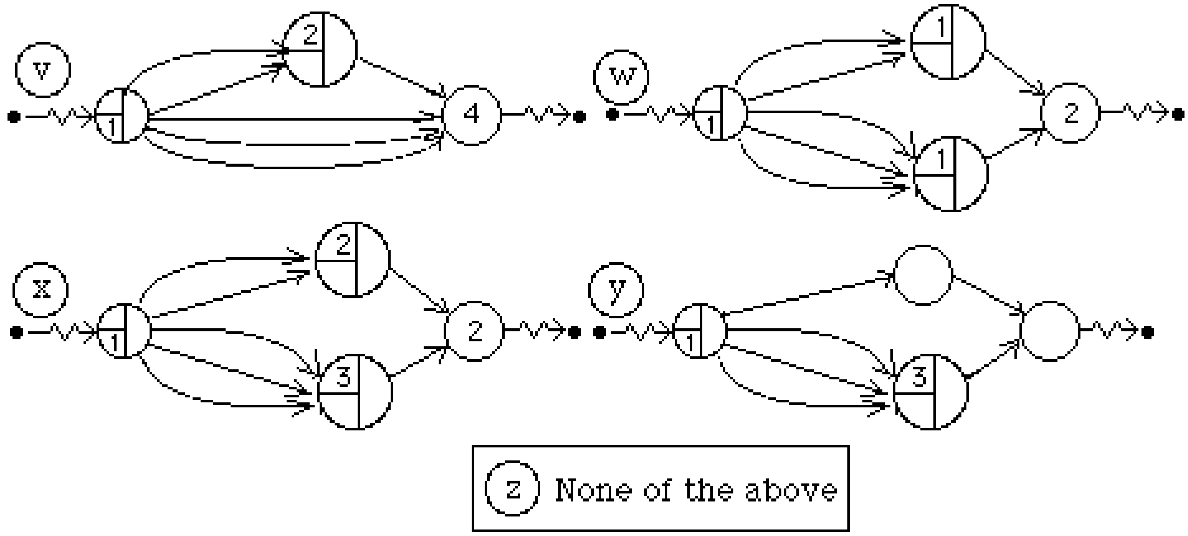
a. $1 - (0.3)^3(0.2)^2 = 0.99892$
b. $(0.7)^3[1-(0.2)^2] = 0.32928$
c. $1-(0.3)^3[1- (0.8)^2] = 0.99028$
d. $[1-(0.3)^3] (0.8)^2 = 0.62272$
e. $[1-(0.3)^3] [1-(0.2)^2] = 0.93408$
f. $1- [1-(0.7)^3](0.2)^2 = 0.97372$
g. None of the above

**Diagrams:**




**SLAM networks:**

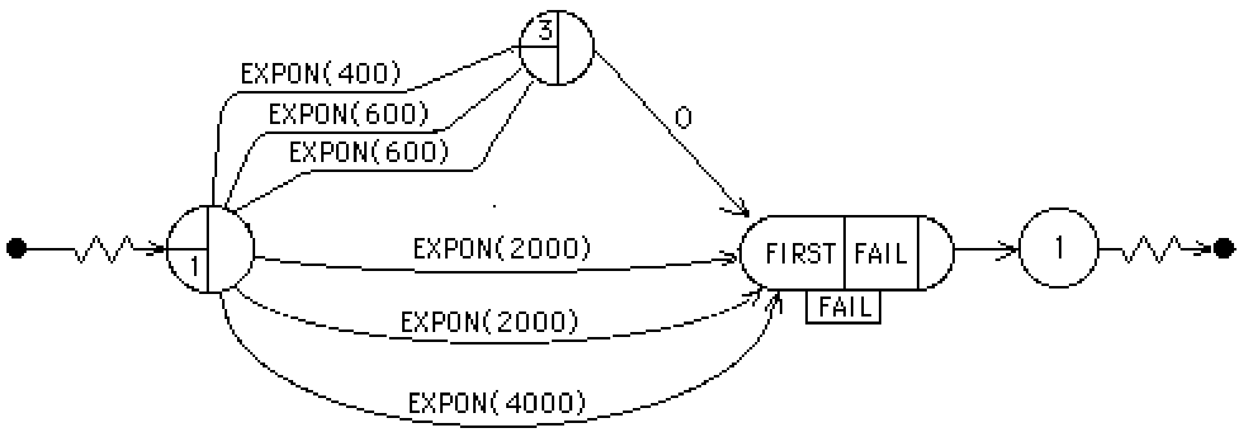





gggg PART III gggg

The following SLAM network was used to simulate a system consisting of six components (where the time units are days). Refer to the output for five hundred runs to find (or estimate) the quantities below:

____ 1. The average lifetime of the system.
____ 2. The time which the first failure occurred.
____ 3. The reliability of the system if its designed lifetime is specified to be 50 days.
____ 4. The probability that the system survives 1400 days.

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**STATISTICS FOR VARIABLES BASED ON OBSERVATION**

                   MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                   VALUE   DEVIATION VARIATION   VALUE     VALUE    OBS
FAIL TIME       0.489E+03 0.381E+03 0.780E+00 0.209E+00 0.217E+04   500

                                  FAIL TIME
 OBS  RELA   UPPER
 FREQ FREQ  CELL LIM 0        20        40        60        80       100
                     +    +    +    +    +    +    +    +    +    +    +
  32 0.064 0.500E+02 +***                                              +
  34 0.068 0.125E+03 +***   C                                          +
  65 0.130 0.200E+03 +*******     C                                    +
  44 0.088 0.275E+03 +****             C                               +
  41 0.082 0.350E+03 +****                 C                           +
  44 0.088 0.425E+03 +****                     C                       +
  49 0.098 0.500E+03 +*****                         C                  +
  30 0.060 0.575E+03 +***                              C               +
  30 0.060 0.650E+03 +***                                 C            +
  20 0.040 0.725E+03 +**                                    C          +
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 19 0.038 0.800E+03 +**                                        C        +
 12 0.024 0.875E+03 +*                                          C       +
 15 0.030 0.950E+03 +**                                          C      +
 14 0.028 0.103E+04 +*                                             C    +
  7 0.014 0.110E+04 +*                                              C   +
  8 0.016 0.118E+04 +*                                              C   +
  7 0.014 0.125E+04 +*                                               C  +
  9 0.018 0.133E+04 +*                                                C +
  5 0.010 0.140E+04 +*                                                C +
  4 0.008 0.148E+04 +                                                  C+
  3 0.006 0.155E+04 +                                                  C+
  8 0.016    INF    +*                                                  C
---                 +    +    +    +    +    +    +    +    +    +    +
500                 0         20        40        60        80        100
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Using the mean and standard deviation from the simulation output, the Weibull parameters U=528.961 and k=1.29394 are determined.

5. According to this result, is the failure rate increasing or decreasing? ___________

Based upon the cumulative probability function F(t) for this Weibull distribution, the following probabilities and expected values for each cell were calculated, where $t_i$ is the upper limit of the cell. The cells at the upper end were grouped, as indicated by the horizontal lines, so as to obtain a more even distribution of observations. Next we calculate for each cell (or group of cells) the square of the deviation of O from E, and divide by E, and then sum to obtain the chi-square statistic in the table on the right:

| $i$ | $t_i$ | $p_i$ | $E_i$ | $O_i$ |
|---|---|---|---|---|
| 1 | 50 | 0.04615 | 23.07647 | 32 |
| 2 | 125 | 0.09713 | 48.56294 | 34 |
| 3 | 200 | 0.10402 | 52.00927 | 65 |
| 4 | 275 | 0.10151 | 50.75408 | 44 |
| 5 | 350 | 0.09467 | 47.33290 | 41 |
| 6 | 425 | 0.08577 | 42.88484 | 44 |
| 7 | 500 | 0.07610 | 38.05003 | 49 |
| 8 | 575 | 0.06643 | 33.21305 | 30 |
| 9 | 650 | 0.05721 | 28.60495 | 30 |
| 10 | 725 | 0.04871 | 24.35746 | 20 |
| 11 | 800 | 0.04107 | 20.53625 | 19 |
| 12 | 875 | 0.03433 | 17.16303 | 12 |
| 13 | 950 | 0.02846 | 14.23084 | 15 |
| 14 | 1025 | 0.02343 | 11.71482 | 14 |
| 15 | 1100 | 0.01916 | 9.57989 | 7 |
| 16 | 1175 | 0.01557 | 7.78599 | 8 |
| 17 | 1250 | 0.01258 | 6.29178 | 7 |
| 18 | 1325 | 0.01011 | 5.05699 | 9 |
| 19 | 1400 | 0.00809 | 4.04392 | 5 |
| 20 | 1475 | 0.00644 | 3.21825 | 4 |
| 21 | 1550 | 0.00510 | 2.54946 | 3 |

| t | E | O | D |
|---|---|---|---|
| 50 | 23.07647 | 32 | 3.45068 |
| 125 | 48.56294 | 34 | 4.36710 |
| 200 | 52.00927 | 65 | 3.24479 |
| 275 | 50.75408 | 44 | 0.89880 |
| 350 | 47.33290 | 41 | 0.84731 |
| 425 | 42.88484 | 44 | 0.02900 |
| 500 | 38.05003 | 49 | 3.15116 |
| 575 | 33.21305 | 30 | 0.31083 |
| 650 | 28.60495 | 30 | 0.06804 |
| 800 | 44.89371 | 39 | 0.77373 |
| 1025 | 43.10869 | 41 | 0.10315 |
| $\infty$ | 47.50906 | 51 | 0.25651 |
| SUM | 500 | 500 | 17.5011 |

6. What is the number of degrees of freedom for the chi-square goodness-of-fit test? _____

The chi-square probability table indicates that with this # of degrees of freedom , if the system lifetime does have the Weibull distribution with the parameters above, P{D>16.919} is $\alpha$=5%.

7. Based upon this value, should we accept for the system lifetime the Weibull distribution model with the parameters U=528.961 and k=1.29394? _____

## PART IV

*For each system described below, indicate the appropriate SLAM network setment (A through R) which might best model it. If no network segment shown could be used, indicate "**S**".*

_____ 1. Jobs are to be processed first by server #1 and then by #2. When the queue for #2 is filled and #1 completes a job, the job must wait until space becomes available before server #1 is free to begin the next job.

____ 2. Customers who arrive at a shoe repair shop are equally likely to have one or both shoes to be repaired. A single repairman works on the shoes one at a time.

____ 3. Two workers, who differ in the speed with which they work, select their next job from queue #1 if any are waiting there, and queue #2 otherwise.

____ 4. A production shop has ten machine operators and twelve machines. The extra two machines are used for back-up when a machine fails. One repairman is available to repair a failed machine, which is then available as a backup for the next machine that fails.

____ 5. Customers select the check-out lane at the grocery store which has the shortest queue (and cannot change queues once they've entered it.)

____ 6. Two (identical) servers select their next job from the longer of two queues.

____ 7. Customers arrive at a bank's drive-up window; if the waiting line is filled, they drive around the block and then try again.

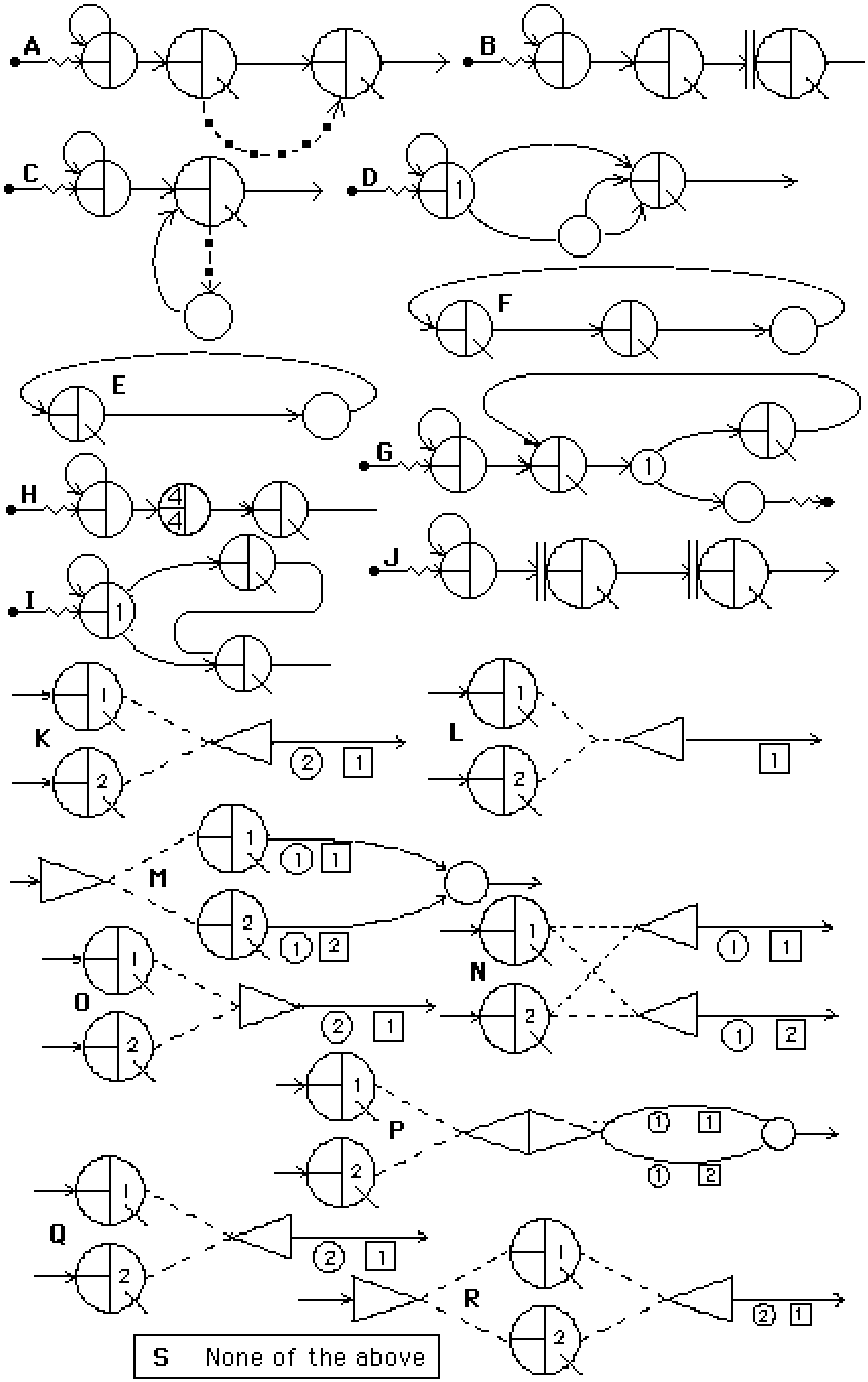
____ 8. Customers usually are served by server #1 and then by server #2; however, if the queue for #1 has no space, the customer proceeds directly to the queue for #2.

____ 9. Widgets arrive one at a time on a conveyor, drop into a box, and when four widgets have arrived, the box is put into a queue to be prepared for shipping.

____ 10. TVs arrive at an inspection station, where they wait to be inspected by a single inspector. An average of five percent require adjusting; one person is available for this task, who then sends to TV back to be re-inspected.

____ 11. Widgets and boxes arrive on two separate conveyors at the final station on an assembly line. If there is both a widget and a box waiting, a worker packs the widget into the box and seals it.

____ 12. Customers arrive at two queues to wait for service by either of two clerks. If both clerks are idle, customers prefer clerk #1. If a clerk finishes serving a customer and both queues have persons waiting, he selects the customer at the head of the longest queue.


A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S None of the above