57:022 Midterm Exam Fall 1995

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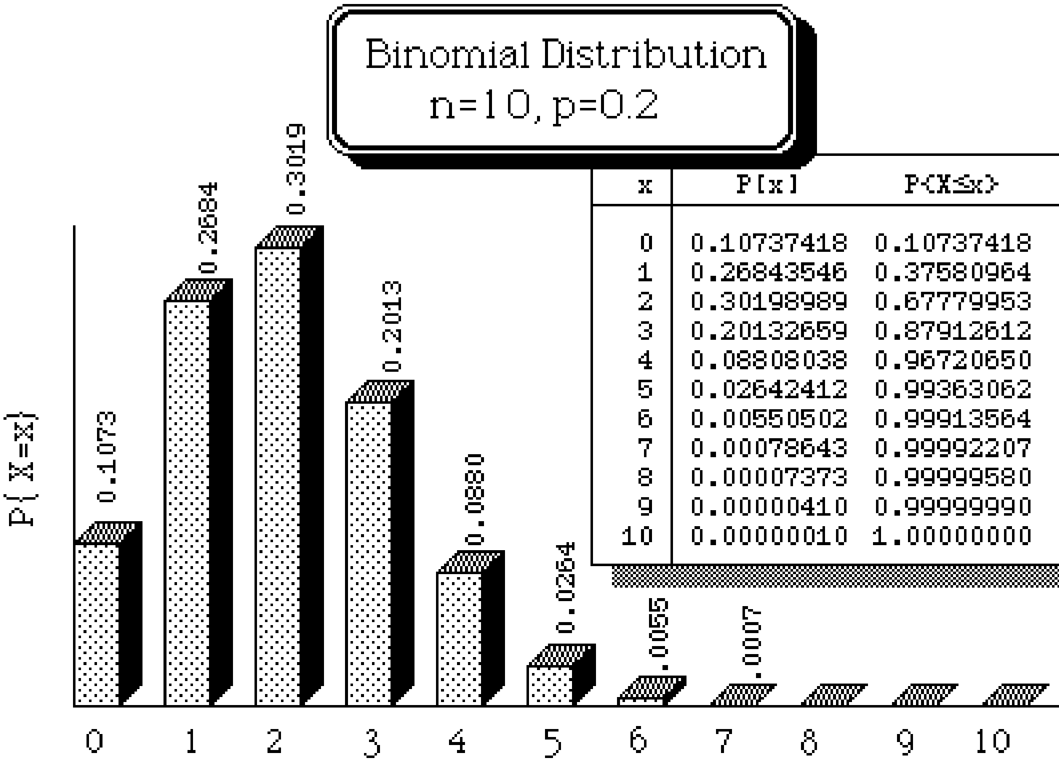
@@@@@@@@@@ Part 1 @@@@@@@@@@

Choose the answers to the questions below from the list of distributions, or the table of probabilities. (State "NONE" if the answer does not appear in either place!



x	P[x]	P{X≤x}	P{X>x}
0	0.10737418	0.10737418	0.89262582
1	0.26843546	0.37580964	0.62419036
2	0.30198989	0.67779953	0.32220047
3	0.20132659	0.87912612	0.12087388
4	0.08808038	0.96720650	0.03279350
5	0.02642412	0.99363062	0.00636938
6	0.00550502	0.99913564	0.00086436
7	0.00078643	0.99992207	0.00007793
8	0.00007373	0.99999580	0.00000420
9	0.00000410	0.99999990	0.00000010
10	0.00000010	1.00000000	0.00000000

We wish to simulate the vehicles arriving at a toll booth on the freeway at the average rate of 2/minute in a completely random fashion. The vehicles are counted and arrival times are recorded, beginning at 12:00 noon. 75 percent of the vehicles are cars, while the remainder are trucks, buses, etc.

Write the alphabetic letter ("A" through "O") corresponding to the *name* of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all! When appropriate, you may answer *NOTA (None of the Above).*

___ 1. number of vehicles arriving during the first 4 minutes
___ 2. time between arrival of vehicle #1 and vehicle #2
___ 3. vehicle# of the first vehicle which is *not* a car.
___ 4. an indicator for vehicle #n which is 1 if a car, 0 otherwise.
___ 5. the number of cars among the first 4 vehicles to arrive
___ 6. time of arrival of first vehicle
___ 7. time of arrival of vehicle #2
___ 8. the vehicle# of the second vehicle which is *not* a car.

The foreman of a casting section in a certain factory finds that, on the average, 1 in every 5 castings made is defective.

______9. What is the *probability* that 2 or fewer defective castings are made in one day?
______10. What's the *name* of the probability dist'n of the quality of casting #5 (either defective or OK).
______11. If the section makes 10 castings a day, what is the *probability* that *exactly* 2 of these will be defective?

Advertising states that, for a certain lottery ticket, "every tenth ticket carries a prize". If you buy ten tickets, what is...
______12. the *probability* that you get *at least* one winning ticket?
______13. the *probability* that you get *exactly* one winning ticket?

If, instead of deciding in advance how many tickets to buy, you continue buying tickets until you have a winning ticket...
______14. what's the *name* of the probability distribution of the number of tickets you buy ?

If you continue buying tickets until you have *two* winning tickets...
______15. what's the *name* of the probability distribution of the number of tickets you buy ?

The arrival of parts to be processed by a machine is a Poisson process, with the rate 4/hour. What is...
______16. the *name* of the probability distribution of the number of parts which arrive during the first hour?
______17. the name of the probability distribution of the time between arrivals of parts?

Some common probability distributions:

A. Bernouilli	F. Exponential	K. Uniform
B. Normal	G. Beta	L. Poisson
C. Lambda	H. Erlang	M Pascal
D Binomial	I. Geometric	N. Random
E. Chi-square	J. Weibull	O. Gumbel

Multiple Choice:

___ 18. In simulating the arrival process in (16) & (17) above, an inter-arrival time T can be randomly generated by obtaining a uniformly-generated random variable X and computing

a. $T = -\frac{\ln(1-X)}{4}$
d. $T = -\frac{\ln X}{4}$
b. $T = 1 - e^{-4X}$
e. $T = e^{-4X}$
c. Both (a) & (d) are correct
f. Both (b) & (e) are correct

___ 19. The CDF of the distribution in (18) above, i.e., the inter-arrival times, is F(t) =

a. $1 - e^{-4X}$
b. $4e^{-4t}$
c. $1 - 4e^{-4t}$
d. $4 - e^{-4t}$
e. e^{-4t}
f. *None of the above*

___ 20. The "Cumulative Distribution Function" (CDF) of a random variable T is

a. $f(t) = P\{T|t\}$
b. $F(t) = P\{T=t\}$
c. $F(t) = P\{T\leq t\}$
d. $f(t) = P\{t\}$
e. $F(t) = P\{T\geq t\}$
f. $f(t) = P\{t \mid T\}$

@@@@@@@@@@ Part 2 @@@@@@@@@@

The time between arrivals of fifty cars are measured. It is expected that these observations have an exponential distribution with mean of 4 minutes, although the actual average value of the observations was 3.68 minutes. We wish to decide whether the discrepancy between the assumed arrival rate (1 every 4 minutes) and the observed arrival rate (1 every 3.68 minutes) is so large as to disqualify our assumption. The number of observations O_i falling within each of several intervals is shown in the table below. We wish to test the "goodness of fit" of the exponential distribution with mean 4 minutes, and have prepared the table below:

i	Interval	O_i	p_i	$E_i = 50p_i$	$\frac{(E_i - O_i)^2}{E_i}$
1	0-1	12	0.221199	11.06	0.0798984
2	1-2	11	0.17227	8.61351	0.661212
3	2-3	6	0.134164	6.70821	0.0747674
4	3-5	6	0.185862	9.29309	1.16693
5	5-9	10	0.181106	9.05528	0.0985611
6	9-∞	5	0.105399	5.26996	0.0138291

total = 2.0952

A portion of a table of the chi-square distribution is given below:

deg.of freedom	Chi-square Dist'n $P\{D \geq \chi^2\}$ 99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475

Indicate whether true or false, using "+" for true, "**o**" for false.

____ 1. The CDF of the inter-arrival time distribution is $F(t) = P\{T \leq t\}$

____ 2. The parameter of the exponential distribution was assumed to be $\lambda = 1/4$ min. = 0.25/minute.

____ 3. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D exceeds 2.0952 is less than 10%.

____ 4. The quantity $\sum_{i=1}^{6} (O_i - E_i)^2 / E_i$ is assumed to have a "chi-square" distribution.

____ 5. The smaller the value of D, the better the fit for the distribution being tested.

____ 6. The quantity E_i is the expected number of observations in interval #i, if the assumption is true.

____ 7. The probability p_i that a car arrives in interval #4, i.e., [3,4], is F(4) - F(3), where F(t) is the CDF of the interarrival times.

____ 8. The random variable D (which has the observed value 2.0952) is assumed to have the chi-square distribution.

____ 9. If the assumption above (that the times above have exponential distribution) is correct, the arrivals of the cars forms a Poisson process.

____ 10. The number of "degrees of freedom" of the chi-square distribution for this test will be 6, the number of cells in the histogram.

____ 11. Based upon these observations, the exponential distribution with mean 4 minutes should <u>not</u> be rejected as a model for the interarrival times of the vehicles.

____ 12. The chi-square distribution for this goodness-of-fit test will have 4 degrees of freedom.

____ 13. The number of observations O_i in interval #i is a random variable with approximately binomial distribution with n=50 and probability of "success" $p=p_i$.

____ 14. The quantity E_i is a random variable with approximately a Poisson distribution.

____ 15. The quantity D is assumed to have approximately a Normal distribution.

____ 16. If T actually does have an exponential distribution with mean 4 minutes, then (based upon the table above) the probability that D is less than 2.0952 is more than 10%.

____ 17. If the gender of the car's driver is recorded also, with $X_n=1$ if the driver of car #n is female (0 otherwise), then the sequence $\{X_1, X_2, X_3, X_4, ...\}$ forms a Bernouilli process.

@@@@@@@@@@ Part 3 @@@@@@@@@@

Suppose that 500 light bulbs are tested by simultaneously lighting them and recording the number of failures every 100 hours. The test is interrupted at the end of 1000 hours, when 291 bulbs have failed. As in your homework assignment, a Weibull probability model is then "fit" to the data.

For each statement, indicate "+" for true, "o" for false:

___ 1. The quantity which is recorded in Cricket Graph as R_t is the fraction of the 500 bulbs which are surviving at time t .

___ 2. The Weibull distribution is usually appropriate for the minimum of a large number of nonnegative random variables.

___ 3. If the failure rate is known to be decreasing, it may be more appropriate to use the exponential distribution than the Weibull.

___ 4. We assume that the number of survivors at time t , $N_s(t)$, has a Weibull distribution.

___ 5. The Weibull CDF, i.e., F(t), gives, for each bulb, the probability that at time t it has already failed.

___ 6. The method used in this homework to estimate the Weibull parameters u & k requires that you compute the mean and standard deviation of the 291 bulbs which have failed.

___ 7. The Cricket Graph program fits a line which minimizes the maximum error, i.e., the vertical distance between each data point and the line.

___ 8. Given a coefficient of variation for the Weibull distribution (the ratio σ/μ), the Weibull shape parameter k but not the scale parameter u can be computed.

___ 9. The sum of the CDF (cumulative distribution function) F(t) and the Reliability function R(t) is always equal to 1 for *every* probability distribution.

___ 10. The exponential distribution is a special case of the Weibull distribution, with constant failure rate.

___ 11. A positive value of ln k indicates an increasing failure rate, and negative ln k indicates a decreasing failure rate.

___ 12. If each bulb's lifetime has an exponential distribution, the time of the 10^{th} failure has Erlang-10 distribution.

___ 13. If 6 bulbs are installed in an office's light fixtures, the number still functioning after 1000 hours has a Weibull distribution.

___ 14. If k=1, then $\Gamma\left(1+\frac{1}{k}\right) = 1$.

___ 15. If 10 bulbs are installed in an office's light fixtures, the number still functioning after 1000 hours has a Poisson distribution.

___ 16. The "gamma" function has the property $\Gamma(x) = x!$ for all nonnegative integer values of x.

Select the letter ("A" through "X") below which indicates each correct answer: When preparing a plot so as to estimate the Weibull parameters, ...

___ 17. The label on the vertical axis should be ...

___ 18. The label on the horizontal axis should be ...
___ 19. The slope of the line fit by Cricket Graph should be approximately ...
___ 20. The vertical intercept of the line fit by Cricker Graph should be approximately ...

A shape parameter k
B. +k ln u
C. -k ln u
D. t
E. ln $^1/_t$
F R_t
G. ln $^1/_{R_t}$
H. mean value μ

I. scale parameter u
J. u ln k
K. -u ln k
L. ln t
M. ln ln t
N. ln R_t
O. ln ln R_t
P. standard deviation σ

Q. coefficient of variation $^\sigma/_\mu$
R. +ln k
S. -ln k
T. -ln t
U. ln ln $^1/_t$
V. - ln R_t
W. ln ln $^1/_{R_t}$
X. *None of the above*

@@@@@@@@@@@@ Part 4 @@@@@@@@@@@@

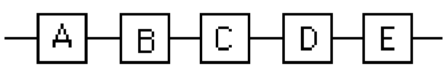
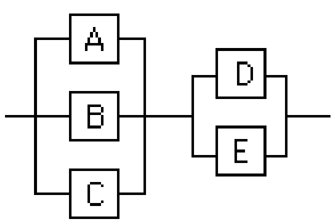
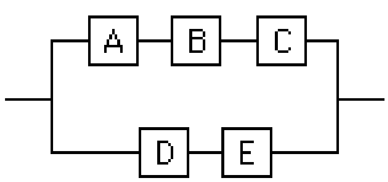
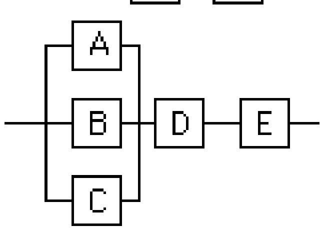
Five components (A,B,C,D, & E) are available for constructing a system. The probability that each component *survives* the first year of operation is 70% for A, B, & C, and 80% for D & E. For each system ((1) through (5) below, indicate:

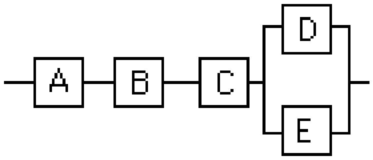
(i) the letter of the ***SLAM network*** model which represents the system

(ii) for each of the three scenarios (a,b,c) below, whether the system will Fail or Survive (circle "F" or "S"):

(a) components A and C fail.
(b) components B and D fail.
(c) components C, D, & E fail.

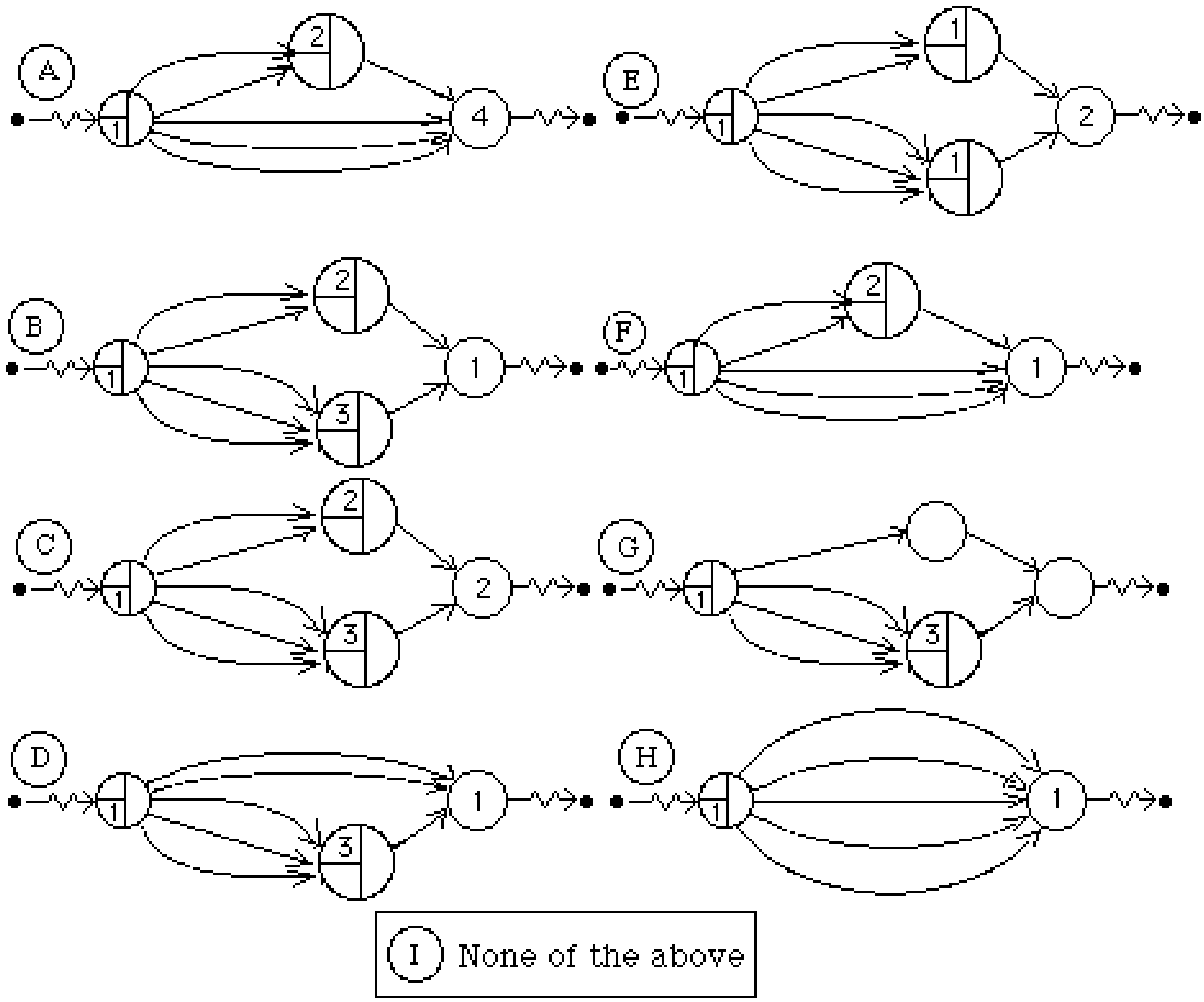
(iii) the letter with the ***computation*** of the 1-year reliability (i.e., survival probability)

		SLAM MODEL	SCENARIO (a)	SCENARIO (b)	SCENARIO (c)	RELIABILITY
1.		____	F / S	F / S	F / S	____
2.		____	F / S	F / S	F / S	____
3.		____	F / S	F / S	F / S	____
4.		____	F / S	F / S	F / S	____

5. 

| ______ | F/S ______ F/S ______ F/S ______ | ______ |

SLAM network models:



Reliabilities:

J. $1 - (0.3)^3(0.2)^2 = 0.99892$

K. $1 - (0.3)^3[1 - (0.8)^2] = 0.99028$

L. $[1 - (0.3)^3] [1 - (0.2)^2] = 0.93408$

M. $(0.7)^3(0.8)^2 = 0.21952$

N. $(0.7)^3[1- (0.2)^2] = 0.32928$

O. $1 - (0.3)^3[1- (0.8)^2] = 0.99028$

P. $[1 - (0.3)^3] (0.2)^2 =0.03892$

Q. None of the above