57:022 Principles of Design II
Midterm Exam #2 Solutions

Part:	I	II	III	IV	V	Total
Possible Pts:	20	15	12	16	12	75

PART ONE

Indicate "+" if True and "O" if False:

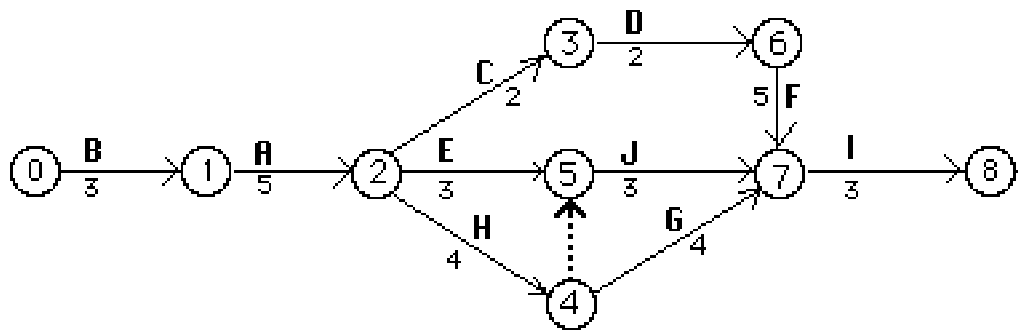
_+_a. If a component's lifetime has exponential distribution, its failure rate ("hazard rate") is constant.
_o_b. "Dummy" activities are unnecessary in the "Activity-on-Arrow" representation of a project.
_o_c. In a Poisson arrival process, the time between arrivals has the Poisson distribution.
_o_d. If two components of a system have a parallel configuration with respect to system reliability, then both are required to function in order for the system to function.
_o_e. If two components of a system have a series configuration with respect to system reliability, then the second component replaces the first when it fails, and the system then fails when the second component fails.
_+_f. Disregarding "Begin" and "End" activities,"Dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
_+_g. PERT assumes that the project duration has a Normal distribution.
_+_h. R(t)+F(t)=1 for all t, where R and F are the reliability function and the cumulative distribution function of the failure time, respectively.
_+_i. If two components of a system have a series configuration with respect to system reliability, the system lifetime is the minimum of the two component lifetimes.
_o_j. PERT assumes that each activity's duration has a Normal distribution.
_+_k. The exponential distribution is a special case of an Erlang distribution.
_o_l. "Birth/death" processes are a special class of discrete-time Markov chains.
_o_m. The gamma function $\Gamma(n)$ is equal to n! for positive integer values of n.
_o_n. If 3 components of a system have a parallel configuration with respect to system reliability, then the system lifetime is the sum of the component lifetimes.
_+_o. In a Poisson arrival process, the number of arrivals during an hour has the Poisson distribution.
_+_p. "Birth/death" processes are a special class of continuous-time Markov chains.
_o_q. The exponential distribution is a special case of a Weibull distribution with k=0.
_o_r. If we...
(i) test 100 batteries, recording the failure time of each,
(ii) prepare a histogram indicating number of failures on each of six consecutive days (where the last failure occurred on the sixth day),
(iii) use the mean and standard deviation of the failure times to estimate the parameter of the Weibull distribution,
then we would assume 3 degrees of freedom when performing the Chi-Square goodness of fit test.
_+_s. If t_i is the i^{th} failure time of the 100 batteries in (r), and the lifetimes do in fact have a Weibull distribution, then a plot of $\ln t_i$ vs $\ln \ln {}^{100}/_{(100-i)}$, i=1,2,...100 should be (approximately) a straight line with slope equal to the "shape" parameter k.
_+_t. The Weibull parameters could still be estimated by drawing the plot as in (s) without including all of the 100 failure times.

PART TWO

Consider the project:

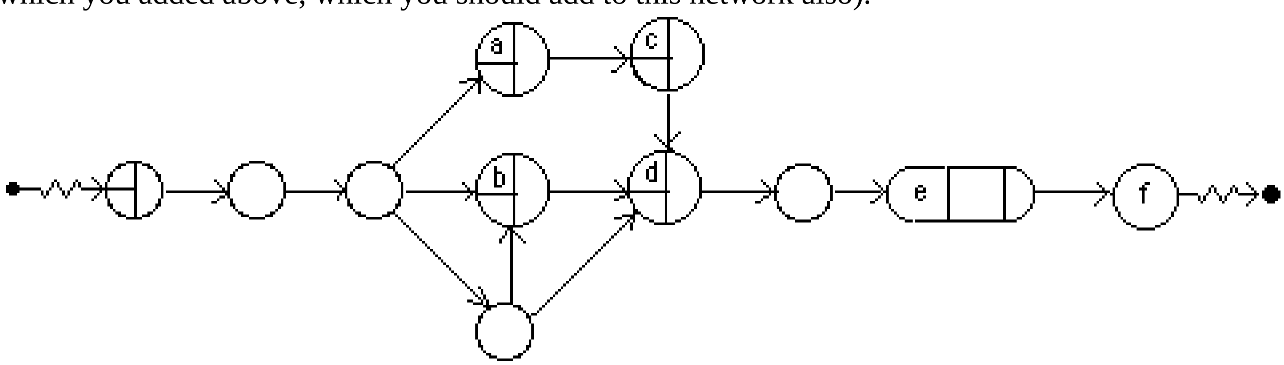
Activity	Description	Predecessor Activities	Duration (days) Mean	Std Dev
A	Walls & ceiling	B	5	2
B	Foundation	none	3	1
C	Roof timbers	A	2	1
D	Roof sheathing	C	2	1
E	Electrical wiring	A	3	1
F	Roof shingles	D	5	2
G	Exterior siding	H	4	1
H	Windows	A	4	1
I	Paint	F,G,J	3	2
J	Inside wall board	E,H	3	1

1. Complete the AOA & the corresponding SLAM networks below by inserting any "dummy" activities which are necessary, and labeling the nodes.



Nodes labeled 3,4, & 5 may be correctly labeled in several ways!

Below is the SLAM network which simulates this project schedule (minus any "dummy" activities which you added above, which you should add to this network also).

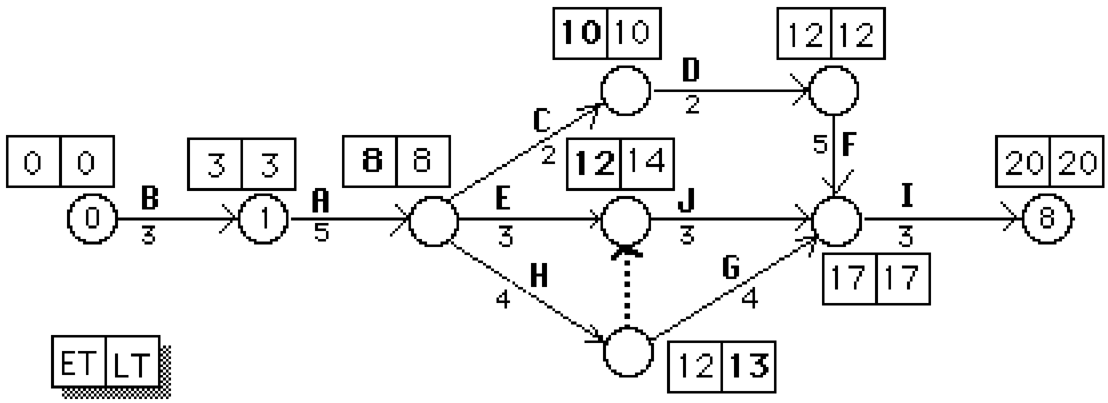


2. What are the values (0,1,2,3 or ∞) which should be used on the four "ACCUMULATE" nodes above?
   a __1_ b __ 2 _ c __1_ d __3_

3. The parameter "e" on the "COLCT" node on the SLAM network above should indicate which type of statistic is to be collected?
   Circle: LAST INT(1) BETWEEN FIRST

4. What should be the value of "f" (0,1,2,3 or ∞) on the "TERMINATE" node? __1__

5. Complete the ETs (earliest times) & LTs (latest times) in the network below. *Don't forget any "dummy" activities which you entered above!*



6. What are the critical activities? (A B C D E F G H I J)

7. What is the expected completion time of the project? __20__

... the standard deviation of the project completion time? $\sqrt{15} \approx 3.87$

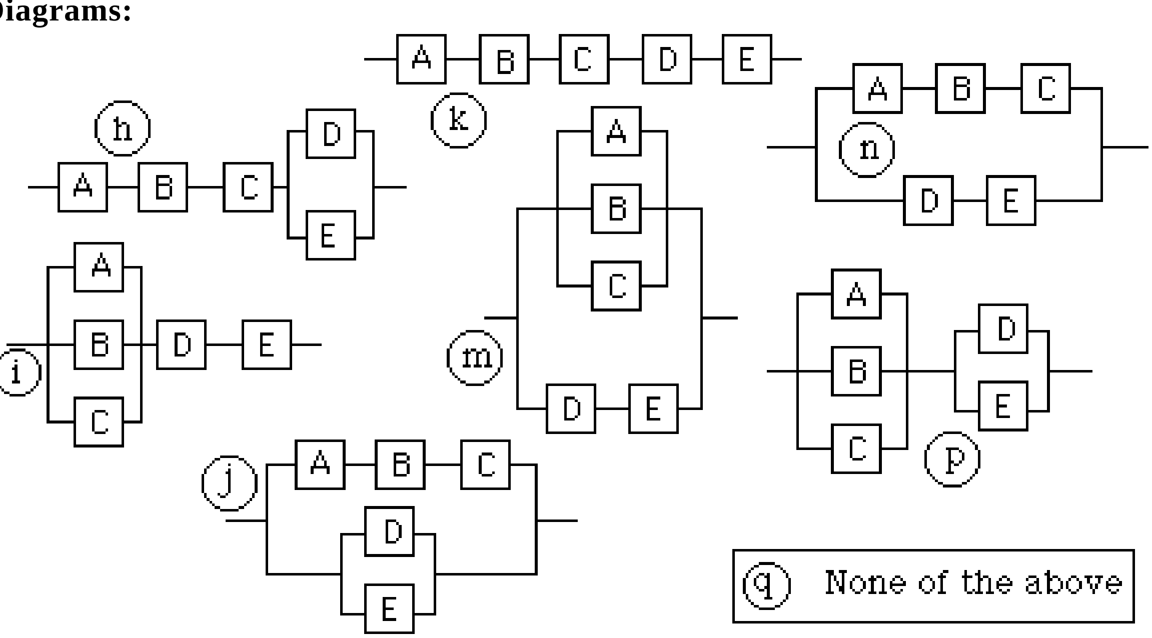
PART THREE

A system consists of five components (A,B,C,D, & E). The probability that each component *survives* the first year of operation is 70% for A, B, & C, and 80% for D & E. For each alternative of (1) through (4), indicate:

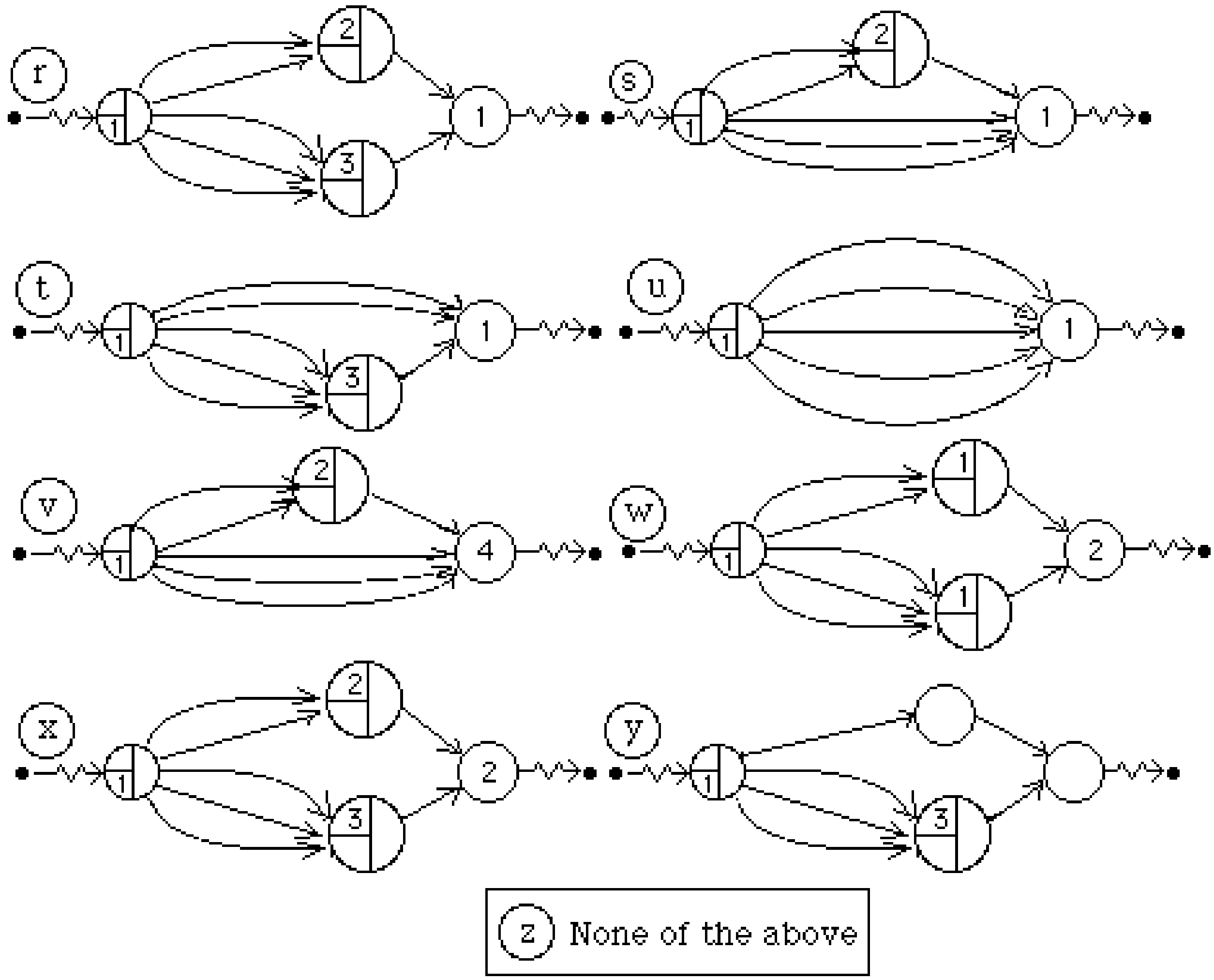
(i) the letter of the *reliability diagram* below which represents the system
(ii) the letter of the *SLAM network* model which represents the system
(iii) the letter with the *computation* of the 1-year reliability (i.e., survival probability)

i.	ii.	iii.	
h	s	e	1. The system requires all of A, B, & C, and at least one of D & E.
p	r	c	2. The system requires at least one of A, B, & C, and at least one of D & E.
k	u	d	3. The system requires all of A, B, C, D, & E.
m	z	b or f	4. The system requires either (at least one of A ,B, & C) or (both of D & E) .

Diagrams:



SLAM networks:



Reliabilities:

a. $1 - (0.3)^3(0.2)^2 = 0.99892$
b. $1 - (0.3)^3[1 - (0.8)^2] = 0.99028$
c. $[1 - (0.3)^3]\,[1 - (0.2)^2] = 0.93408$
d. $(0.7)^3(0.8)^2 = 0.21952$
e. $(0.7)^3[1- (0.2)^2] = 0.32928$
f. $1 - (0.3)^3[1- (0.8)^2] = 0.99028$
g. $1- (0.3)^3(0.2)^2 = 0.97372$
h. None of the above

PART FOUR

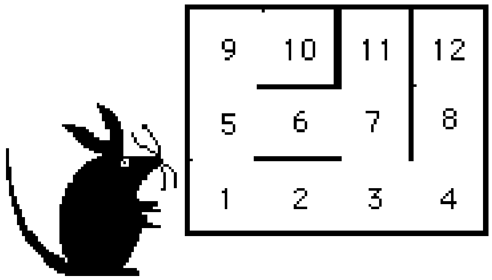
A rat is placed in location #1 of a maze shown below on the left. A Markov chain model has been built where the state of the "system" is the location of the rat after each move. In assigning transition probabilities, it is assumed that the rat is equally likely to leave a location by any of the available paths. (If he arrives at a "dead end", he will retrace his last move with probability 1.)

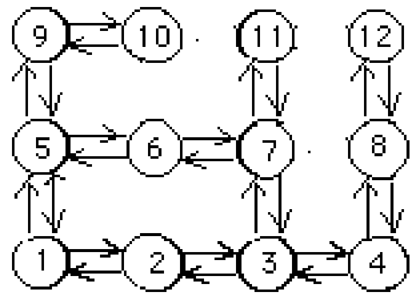
__d_ 1. If we record the rat's location over a period of several days, which location do you expect to be visited most frequently by the rat?

a. all equally often
b. location 7 more often than others
c. locations 3 & 5 equally often
d. locations 3, 5, &7 equally often
e. locations 1, 3, 5, &7 equally often

__c_ 2. If the rat begins in location #10, what is the expected number of moves required to reach location #11?

a. five
b. between 5 and 20
c. between 20 and 50
d. between 50 and 75
e. between 75 and 100
f. over 100





Steady State Distribution

i	π_i
1	0.0833
2	0.0833
3	0.125
4	0.0833
5	0.125
6	0.0833
7	0.125
8	0.0833
9	0.0833
10	0.0417
11	0.0417
12	0.0417

P=

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.5	0	0	0.5	0	0	0	0	0	0	0
2	0.5	0	0.5	0	0	0	0	0	0	0	0	0
3	0	0.333	0	0.333	0	0	0.333	0	0	0	0	0
4	0	0	0.5	0	0	0	0	0.5	0	0	0	0
5	0.333	0	0	0	0	0.333	0	0	0.333	0	0	0
6	0	0	0	0	0.5	0	0.5	0	0	0	0	0
7	0	0	0.333	0	0	0.333	0	0	0	0	0.333	0
8	0	0	0	0.5	0	0	0	0	0	0	0	0.5
9	0	0	0	0	0.5	0	0	0	0	0.5	0	0
10	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	1	0	0	0	0

M=

	1	2	3	4	5	6	7	8	9	10	11	12
1	12	9.33	12.7	31.7	8.67	15.3	16	52.7	29.7	52.7	39	75.7
2	10.7	12	7.33	26.3	15.3	18	14.7	47.3	36.3	59.3	37.7	70.3
3	19.3	12.7	8	19	20	18.7	11.3	40	41	64	34.3	63
4	24.3	17.7	5	12	25	23.7	16.3	21	46	69	39.3	44
5	11.3	16.7	16	35	8	10.7	15.3	56	21	44	38.3	79
6	16.7	18	13.3	32.3	9.33	12	8.67	53.3	30.3	53.3	31.7	76.3
7	20	17.3	8.67	27.7	16.7	11.3	8	48.7	37.7	60.7	23	71.7
8	27.3	20.7	8	3	28	26.7	19.3	12	49	72	42.3	23
9	14.3	19.7	19	38	3	13.7	18.3	59	12	23	41.3	82
10	15.3	20.7	20	39	4	14.7	19.3	60	1	24	42.3	83
11	21	18.3	9.67	28.7	17.7	12.3	1	49.7	38.7	61.7	24	72.7
12	28.3	21.7	9	4	29	27.7	20.3	1	50	73	43.3	24

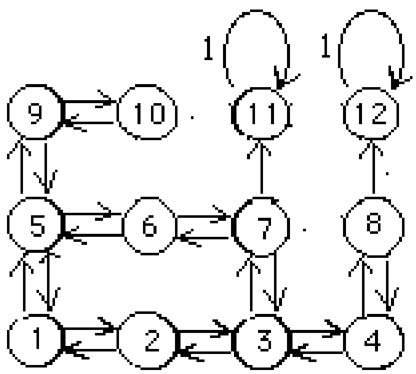
__b_ 3. If the rat begins in location #5, how many moves will we expect the rat to make before returning to his starting point, location #5?

a. five   b. between 5 and 20   c. between 20 and 50
d. between 50 and 75   e. between 75 and 100   f. over 100

__a_ 4. The number of *transient* states in this Markov chain model is

a. 0   b. 6   c. 9
d. 10   e. 12   f. none of the above

Now suppose that food is placed in locations 11 and 12, so that the rat does not leave when he finds it. States 11 & 12 then become absorbing states, and the Markov chain model becomes:



The matrices A and E for this Markov chain are:

A =

	11	12
1	0.69	0.31
2	0.655	0.345
3	0.621	0.379
4	0.414	0.586
5	0.724	0.276
6	0.759	0.241
7	0.793	0.207
8	0.207	0.793
9	0.724	0.276
10	0.724	0.276

E =

	1	2	3	4	5	6	7	8	9	10
1	4.07	2.97	2.79	1.24	4.76	2.28	2.07	0.621	3.17	1.59
2	2.97	3.52	3.1	1.38	3.62	1.86	1.97	0.69	2.41	1.21
3	1.86	2.07	3.41	1.52	2.48	1.45	1.86	0.759	1.66	0.828
4	1.24	1.38	2.28	2.34	1.66	0.96	1.24	1.17	1.1	0.552
5	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	3.93	1.97
6	2.28	1.86	2.17	0.96	4.03	3.1	2.28	0.483	2.69	1.34
7	1.38	1.31	1.86	0.82	2.17	1.52	2.38	0.414	1.45	0.724
8	0.62	0.69	1.14	1.17	0.82	0.48	0.62	1.59	0.552	0.276
9	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	5.93	2.97
10	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.552	5.93	3.97

__a_ 5. If the rat begins at location #10, the probability that the rat finds the food at location #12 first (before the food at #11) is (nearest to)
a. 50% b. 60% c. 70%
d. 80% e. 90% f. 95%

__a_ 6. The expected number of times that the rat returns to his initial location (#10) before finding food is
a. less than 4 b. between 4 and 9 c. between 9 and 25
d. between 25 and 40 e. between 40 and 80 f. more than 80

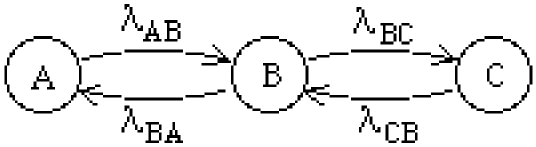
__d_ 7. If the rat, starting at location #10, manages to reach location #8 before finding food, the probability that he first finds the food at location #12 is
a. 50% b. 60% c. 70%
d. 80% e. 90% f. 95%

__d_ 8. The number of *transient* states in this second Markov chain model is
a. 0 b. 2 c. 9
d. 10 e. 12 f. none of the above

PART FIVE

A repairman is responsible for maintaining two machines in working condition. When both are in good condition, they operate simultaneously. However, a machine operates for an average of only 1 hour, when it fails and repair begins. Repair of a machine requires an average of 30 minutes. (Only one machine at a time can be repaired.) Define a continuous-time Markov chain with states:

A. Both machines have failed, with repair in progress on one machine
B. One machine is operable, and the other is being repaired
C. Both machines are in operating condition



__e_ 1. In this model, the probability distribution of the time required to repair a machine is assumed to be:

a. Uniform b. Markov c. Poisson
d. Normal e. exponential f. None of the above

__c_ 2. The transition rate λ_{AB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{BA}$ e. λ_{BA} f. None of the above

__c_ 3. The transition rate λ_{CB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{CB}$ e. λ_{BC} f. None of the above

__b_ 4. The repair time will be less than t with probability

a. e^{-2t} b. $1 - e^{-2t}$ c. $1- e^{2t}$
d. $1- 2e^{-t}$ e. $2e^{t}$ f. None of the above

a,b,e 5. The steady-state probability distribution must satisfy the equation(s) *(choose one or more):*

a. $\pi_A + \pi_B + \pi_C = 1$
b. $\lambda_{AB}\pi_A = \lambda_{BA}\pi_B$
c. $\lambda_{BA}\pi_A = \lambda_{AB}\pi_B$
d. $\pi_A = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$
e. $\lambda_{BC}\pi_B = \lambda_{CB}\pi_C$
f. $\pi_B = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$

__f_ 6. The average utilization of *each* of these machines in steady state (i.e., the fraction of maximum capacity at which they will operate), is:

a. $\pi_B+\pi_C$ b. $(\pi_B+\pi_C)/2$ c. $\pi_B+2\pi_C$
d. $\pi_A+\pi_B+\pi_C$ e. $2(\pi_B+\pi_C)$ f. $(\pi_B+2\pi_C)/2$

7. Is this continuous-time Markov chain a "birth/death" process? (Yes No)