57:022 Principles of Design II
Midterm Exam #1 - Spring 1995

Part:	I	II	III	IV	V	Total
Possible Pts:	10	10	10	8	12	50
Your score:	_____	_____	_____	_____	_____	______

PART I

Consider passengers arriving at a Cambus stop at the average rate of 4/minute in a completely random fashion, starting at time t=0. Ten percent of the persons are engineering students. The capacity of each Cambus is 40, and they depart every fifteen minutes.

Write the alphabetic letter corresponding to the name of the probability distribution which each of the following random variables has. Warning: some distributions may apply in more than one case, while others not at all!

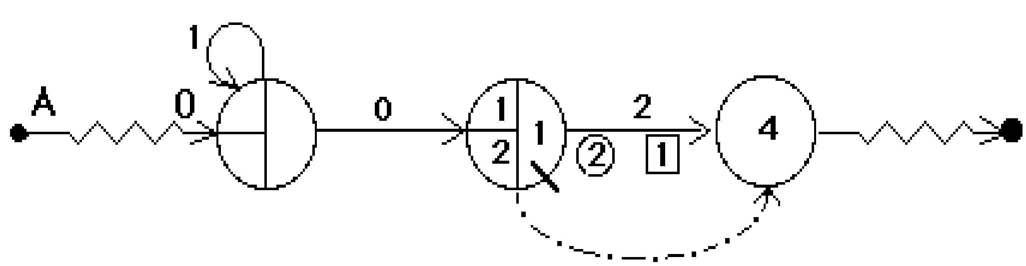
___ 1. the time of arrival of the fourth passenger
___ 2. the time between arrival of first and second passenters
___ 3. the number of student engineers among the first 10 persons to arrive
___ 4. the time of arrival of the first passenger
___ 5. the total weight of the passengers when the bus is full
___ 6. the number of passengers arriving between the departure of two buses
___ 7. the number of the first passenger who is an engineering student.
___ 8. the weight of the heaviest passenger when the bus is full

a. normal
b. uniform
c. Poisson
d. Weibull
e. geometric
f. Erlang
g. Gumbel
h. Bernouilli
i. chi-square
j. exponential
k. binomial
l. beta
m. *none of the above*

9. Circle all of the probability distributions of *continuous* random variables:
a. normal
b. uniform
c. Poisson
d. Weibull
e. geometric
f. Erlang
g. Gumbel
h. Bernouilli
i. chi-square
j. exponential
k. binomial
l. beta

PART II

Note that
- all activity durations in the SLAM network below are *constants* , and none are random!
- first entity is created at time=0



___ 1. In Network **A**, the first entity to *leave the system* (& is terminated) leaves at time =
a. 0 b. 1 c. 2 d. 3 e. 4

f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 2. In Network **A**, the first created entity *begins being served* at time =

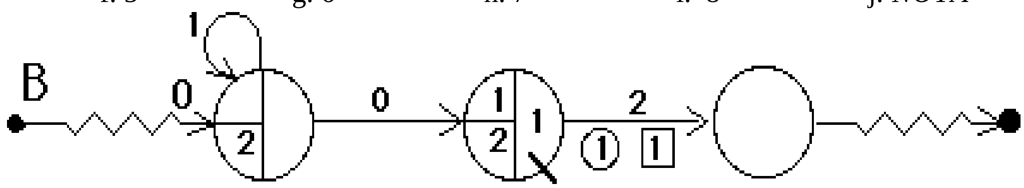
a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 3. In Network **A**, the first created entity *begins being served* at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 4. In Network **A**, the simulation will terminate at time=

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

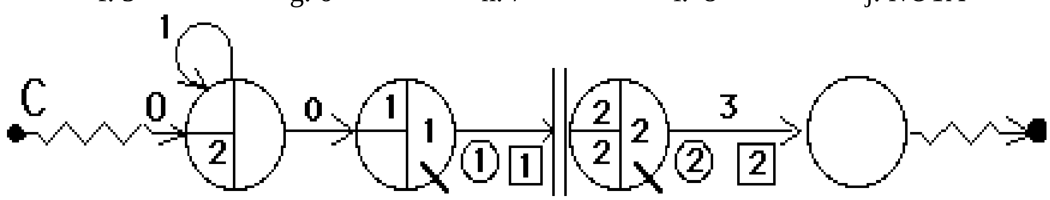


___ 5. In Network **B**, the first created entity *begins being served* at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 6. In Network **B**, the first entity which *cannot enter the queue* will arrive at the queue at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*



___ 7. In Network **C**, *before* the first created entity, there are how many entities already in the network?

a. none b. one c. two d. three
e. four f. five g. can't be determined h. *NOTA*

___ 8. In Network **C**, the first created entity *enters the queue* at time =

a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 9. In Network **C**, the total number of servers is

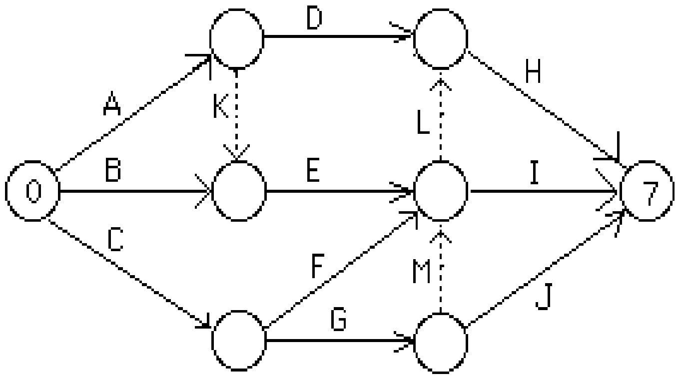
a. 0 b. 1 c. 2 d. 3 e. 4
f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 10. Of the three SLAM networks, the network in which "balking" may occur is

a. A b. B c. C d. both A & C
e. both A & B f. both B & C g. *NOTA*

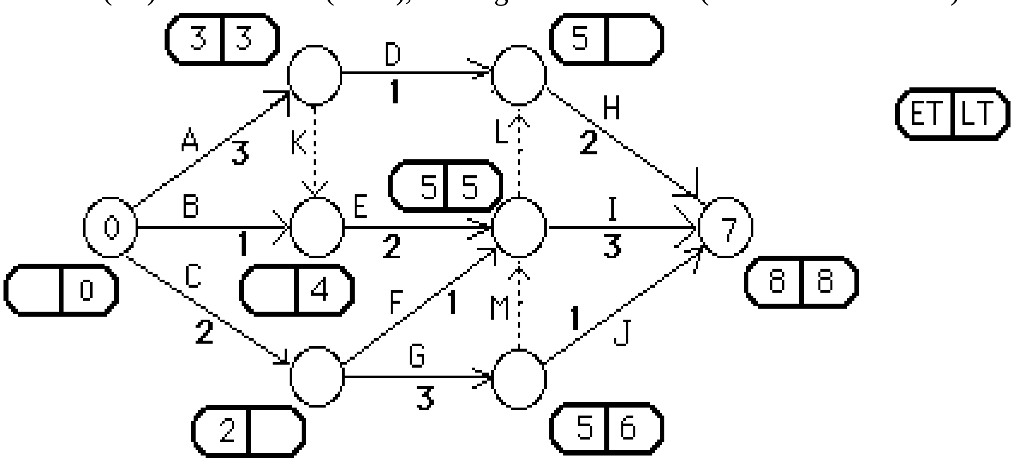
PART III

Consider the project depicted by the A-O-A network below:



a. Complete the labeling of the nodes on the A-O-A project network above.

b. The activity durations are given below on the arrows. Finish computing the Early Times (ET) and Late Times (LT) for each node (event), writing them in the box (with rounded corners) beside each node.



d. Find the slack ("total float") for activity D. _________

e. Which activities are critical? (circle: A B C D E F G H I J K L M)

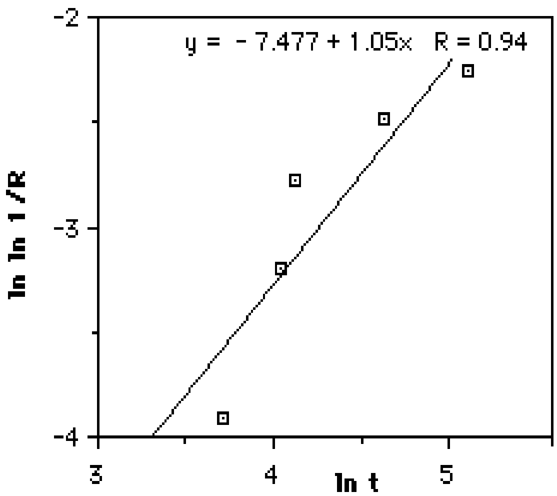
f. What is the earliest completion time for the project? ___________

gggg PART IV gggg

An electronic device is made up of a large number of components. Every component is essential, so that the device will fail when the first component fails. The lifetime of each component is random, but its probability distribution is unknown. The manufacturer has provided a 90-day warranty on this device.

A test of the device is performed, in which *fifty* units of the device are operated simultaneously, and the time of the first six failures is noted, namely 41, 57, 62, 102, 165, and 185 days. (The test was then terminated at 185 days.) Letting R be the fraction of the devices surviving, "Cricket Graph" was used to prepare the following table and plot, with line fit:

t	R	ln t	1/R	ln ln 1/R
41	.98	3.714	1.020	-3.912
57	.96	4.043	1.042	-3.194
62	.94	4.127	1.064	-2.781
102	.92	4.625	1.087	-2.489
165	.90	5.106	1.111	-2.254
185	.88	5.220	1.136	-2.056



We will make the assumption that the unit's lifetime has a Weibull distribution. Let ε_i denote the "error", i.e., the vertical distance between data point #i and the line determined by Cricket Graph. *(Use the table of the Gamma function below, interpolating as necessary).*

___ 1. The Cricket Graph program fits a line through the data points which minimizes

a. $\sum_{i=1}^{6} \varepsilon_i$ b. $\sum_{i=1}^{6} |\varepsilon_i|$ c. $\sum_{i=1}^{6} (\varepsilon_i)^2$

d. $\max_i \{ \varepsilon_i \}$ e. $\max_i \{ |\varepsilon_i| \}$ f. none of the above

2. Based upon the above plot, the value of the "shape" parameter (k) of the probability dist'n is approximately _________ .

3. Based upon the above plot, the value of the "location" parameter (u) of the probability dist'n is approximately _________ .

___ 4. For the distribution with the parameters you specified in (1) & (2), the failure rate is
a. increasing b. decreasing c. constant d. cannot be determined

5. According to this probability model, what is the probability that the device will survive the 90-day warranty period? ______ %

6. According to this probability model, the average failure time is _________ days.

$$\Gamma\left(1 + \frac{1}{k}\right)$$

k	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	3628800	120	9.2605	3.3234	2.0000	1.5046	1.2658	1.1330	1.0522
1	1.0000	0.9649	0.9407	0.9236	0.9114	0.9027	0.8966	0.8922	0.8893	0.8874
2	0.8862	0.8857	0.8856	0.8859	0.8865	0.8873	0.8882	0.8893	0.8905	0.8917
3	0.8930	0.8943	0.8957	0.8970	0.8984	0.8997	0.9011	0.9025	0.9038	0.9051

PART V

The times T_1, … T_{50} (in seconds) *between* arrivals of the first fifty vehicles at an intersection are recorded (the table on the left below):

Observed interarrival times

0.0226392	0.768035	1.65885	3.08626	6.29563
0.0485026	0.790591	1.66189	3.64492	7.04469
0.236294	1.14222	1.68663	3.70833	7.58034
0.412293	1.17618	1.98549	4.06761	7.97349
0.44836	1.20924	2.03548	4.87876	7.98124
0.477881	1.30452	2.07311	4.98918	9.20103
0.480905	1.33905	2.12645	5.07361	10.5373
0.514895	1.3464	2.15331	5.16394	13.7621
0.603458	1.56215	2.62304	5.2581	14.9808
0.716652	1.64656	3.0584	5.70407	16.0848

t	$P\{T \le t\}$
1	0.23758100
2	0.41871726
3	0.55681899
4	0.66211038
5	0.74238653
6	0.80359060
7	0.85025374
8	0.88583060
9	0.91295508
10	0.93363530
11	0.94940229
12	0.96142335
13	0.97058843
14	0.97757606
15	0.98290356

The average of these interarrival times is 3.6865 seconds. We believe that the arrival process is Poisson. Based upon the computed average interarrival time above, the table on the right above is computed. The number of interarrival times are grouped into seven "cells":

i	interval	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
1	0-1	12	11.879	0.00123
2	1-2	12	9.05681	0.95644
3	2-3	5	6.90509	0.52560
4	3-4	4		
5	4-6	7	7.07401	0.00077
6	6-8	5	4.112	0.19176
7	8-∞	5	5.70848	0.08792

The total of the numbers in the last column is D = 2.06751.

Indicate, for each statement, whether *true* ("+") or *false* ("**o**"):

____ 1. The value of E_4 (blanked in the table above) is between 6 and 7.
____ 2. The probability p_i that a car arrives in an interval $[t_1, t_2]$, is $F(t_2) - F(t_1)$
____ 3. The CDF of a random variable T is $F(t) = P\{T \ge t\}$
____ 4. The CDF of the distribution is assumed to be $F(t) = 1 - \lambda e^{-\lambda t}$ where $\lambda = 1/3.6865$ sec.
____ 5. The number of observations, O_i, in interval #i should have the binomial distribution.
____ 6. The quantity D is assumed to have the chi-square distribution.
____ 7. The chi-square distribution for this test will have 5 "degrees of freedom".
____ 8. The quantity $(E_i-O_i)^2/E_i$ is assumed to have the normal N(0,1) distribution.
____ 9. The chi-square distribution for this test will have 7 "degrees of freedom".
____ 10. The number of observations O_i in interval #i is a random variable with approximately Poisson distribution.
____ 11. <u>If</u> T actually has a mean value of 3.6865 seconds, the probability that D exceeds the observed value 2.0675 is less than 10%.
____ 12. The exponential distribution with mean 3.6865 seconds should be accepted as a model for the interarrival times of the vehicles.
____ 13. The quantity E_i is a random variable with approximately Poisson distribution.
____ 14. The chi-square distribution for this test will have 6 "degrees of freedom".
____ 15. The quantity D is assumed to have approximately a Normal distribution.
____ 16. The smaller the value of D, the worse the fit for the distribution being tested.

____ 17. The quantity E_i is the expected number of observations in interval #i

____ 18. The sum of several N(0,1) random variables has chi-square distribution.

____ 19. The number of observations, O_i, in interval #i should have the Poisson distribution.

deg.of			Chi-square Dist'n $P\{D \geq \chi^2\}$			
freedom	99%	95%	90%	10%	5%	1%
2	0.0201	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.341
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475