Solutions

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57:022 Principles of Design II
Final Exam Solutions -- Spring 1999

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Part	I	II	III	IV	V	Total
Possible	7	18	12	15	18	70

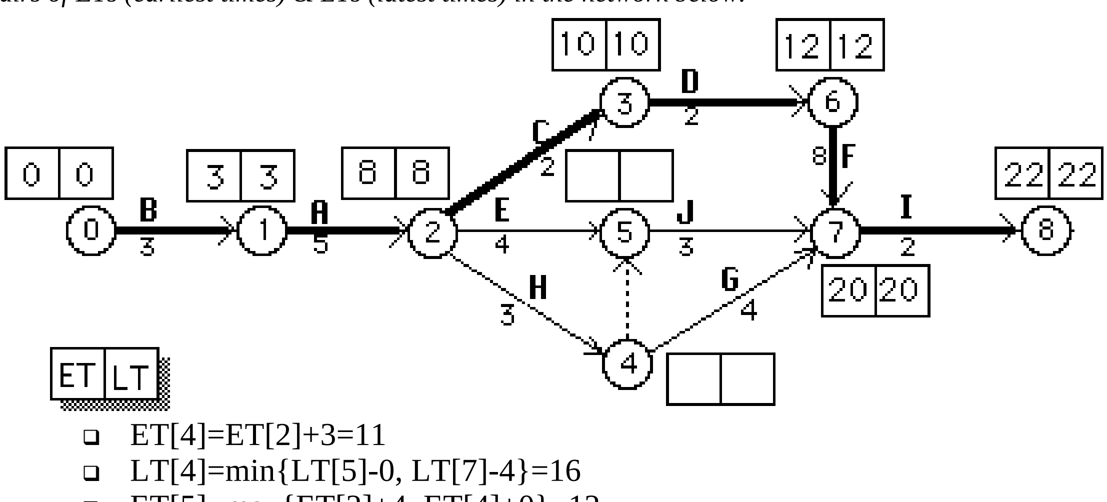
Part I. Probability Distributions *For each probability distribution below, indicate by "C" or "D" whether the corresponding random variable is Continuous or Discrete.*

_D_1. Bernouilli
_D_2. Binomial
_D_3. Poisson
_C_4. Uniform
_C_5. Beta
_D_6. Geometric
_C_7. Exponential
_C_8. Normal
_C_9. Weibull
_C_10. Triangular
_D_11. Pascal (negative binomial)
_C_12. Erlang-k with k>1
_C_13. Gumbel
_C_14. Chi-square

Part II. Project Scheduling: *indicate by "+" or"O" whether "true" or "false", respectively.*

_+_1. An ARENA model of a project is more similar to an **AON** network model than an **AOA** network model of the project.
_O_2. The quantity LT(i) [i.e. latest time] for each node i is determined by a forward pass through the network.
_O_3. If an activity is represented by an arrow from node i to node j, then LS (latest start time) for that activity is LT(i).
_+_4. If an activity is represented by an arrow from node i to node j, then ES (early start time) for that activity is ET(i).
_O_5. If an activity is represented by an arrow from node i to node j, then that activity has zero "float" or "slack" if and only if ET(i)=LT(j).
_+_6. An activity is critical if and only if its total float ("slack") is zero.
_O_7. A "dummy" activity cannot be critical.
_O_8. The mean value of the duration of activity is equal to its most likely value, if the probability distribution is Beta.
_O_9. PERT assumes that each activity's duration has a Normal distribution.
_+_10. PERT assumes that the project duration has a Normal distribution.
_+_11. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
_+_12. The project network below is of the AOA form.

In the project network below, each activity is assumed to require its expected duration. *Complete the two missing pairs of ETs (earliest times) & LTs (latest times) in the network below.*



- ET[4]=ET[2]+3=11
- LT[4]=min{LT[5]-0, LT[7]-4}=16
- ET[5]=max{ET[2]+4, ET[4]+0}=12
- LT[5]=LT[7]-3=17

Solutions

The critical path is shown in bold above. If the durations are random, with expected values as shown and *standard deviations all equal to 1.0,* what is

13. ... the expected completion time of the project, according to PERT? __22__ days

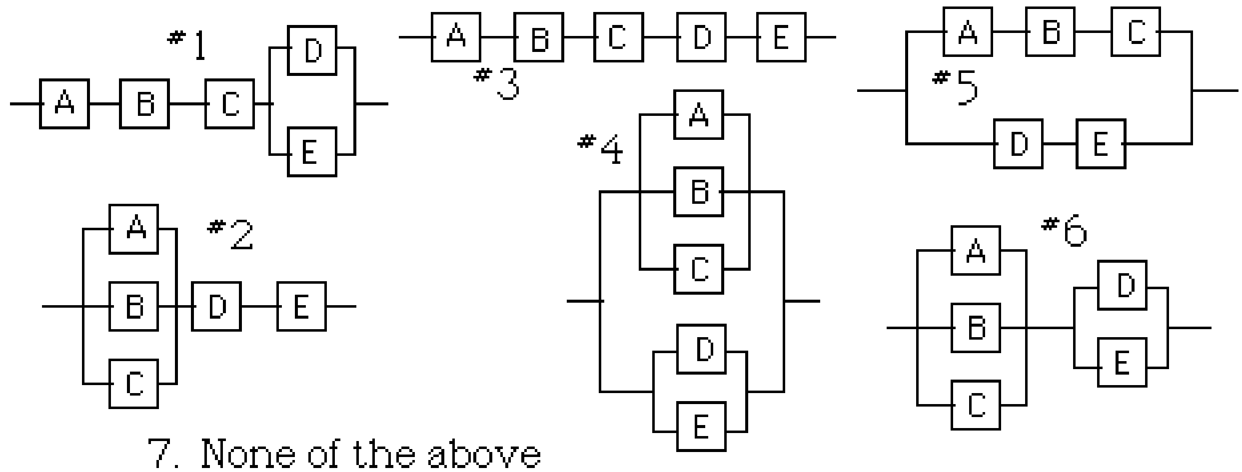
14. ... the standard deviation of the project completion time, according to PERT? $\sqrt{6}$ days

Part III: A system consists of five components (A,B,C,D, &E). The probability that each component *fails during the first year* of operation is 30% for A, B, and C, and 40% for D and E. For each alternative (a) and (b), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)

Diagram	Reliability	
_2_	_7_	1. The system requires that at least one of A, B, & C function, <u>and</u> that both D and E function.
_5_	_2_	2. The system will fail if one of A, B, and C fails <u>and</u> if either D or E fails.

Diagrams:



Reliabilities:

1. $(0.7)^3(0.6)^2$ = 12.3%
2. 1- $[1-(0.7)^3]$ $[1-(0.6)^2]$ = 57.9%
2. $(.7)^3(1-[.4]^2)$ = 28.8%
4. 1 - $(0.3)^3(0.4)^2$ = 94.5%
5. $[1-(0.3)^3]$ $[1-(0.4)^2]$ = 81.7%
6. 1 - $[1-(0.3)^3]$ $[1- (0.4)^2]$ = 18.3%
7. $[1-(0.3)^3]$ $(0.6)^2$ = 35.0%
8. *None of the above*

Part IV. Weibull Model of Reliability. An electronic device is made up of a large number of components. Every component is essential, so that the device will fail when the first component fails. The lifetime of each individual component is random, but its probability distribution is unknown. The manufacturer, who has provided a 90-day warranty on this device, has decided to use the Weibull reliability model.

For each statement, indicate "+" for true, "o" for false:

_+_ 1. A positive value of ln k indicates an increasing failure rate, and negative ln k indicates a decreasing failure rate.

_O_ 2. We assume that the number of survivors at time t , $N_S(t)$, has a Weibull distribution.

_+_ 3. The Weibull CDF, i.e., F(t), gives, for each bulb, the probability that at time t it has already failed.

_O_ 4. The exponential distribution is a special case of the Weibull distribution, with failure rate zero. *Note: failure rate is <u>constant</u> if exponential distribution!*

_+_ 5. The sum of the CDF (cumulative distribution function) F(t) and the Reliability function R(t), i.e. F(t) + R(t), is always equal to 1 if the Weibull probability model is assumed.

_+_ 6. If 4 of the devices are installed in a manufacturing system, the number still functioning after 100 days has a binomial distribution.

It has been determined that *average* lifetime of the device is 400 days and the *standard deviation* is 500 days.

_b_ 7. Based upon the above information, the value of the "shape" parameter (k) of the probability dist'n is approximately *(choose nearest value).*

a. 0.1 b. 1.0 c. 10.0 d. 100.0 e. 1000.0

Note: coefficient of variation (σ/μ) *= 500/400 = 1.25, and from Table 2, we obtain 0.8<k<0.9.*

_d_ 8. The value of the "location" parameter (u) of the probability dist'n is approximately *(choose nearest value).*

a. 0.1 b. 1.0 c. 10.0 d. 100.0 e. 1000.0

Note: $\mu = u\Gamma(1+1/k) \Rightarrow u = \mu/\Gamma(1+1/k) \approx 400(1.13) \approx 450$.

_b_ 9. The failure rate is

a. increasing b. decreasing c. constant d. cannot be determined

_h_ 10. The percent of the units which are expected to fail during the 90-day warranty period is *(choose nearest value):*

a. 1% b. 2% c. 3% d. 4%
e. 5% f. 6% g. 7% h. 8%

Note: $F(90) = 1 - e^{-(90/450)^{0.81}} = 0.24$

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Table 1: $\Gamma\left(1 + \frac{1}{k}\right)$

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	362880.	120.000	9.26053	3.32335	2.00000	1.50458	1.26582	1.13300	1.05218
1	1.00000	0.96491	0.94066	0.92358	0.91142	0.90275	0.89657	0.89224	0.88929	0.88736
2	0.88623	0.88569	0.88562	0.88591	0.88648	0.88726	0.88821	0.88928	0.89045	0.89169
3	0.89298	0.89431	0.89565	0.89702	0.89838	0.89975	0.90111	0.90245	0.90379	0.90510
4	0.90640	0.90768	0.90894	0.91017	0.91138	0.91257	0.91374	0.91488	0.91600	0.91710
5	0.91817	0.91922	0.92025	0.92125	0.92224	0.92320	0.92414	0.92507	0.92597	0.92685

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Table 2: Coefficient of variation $\frac{\sigma}{\mu}$ of the Weibull distribution, as a function of k alone:

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	---	---	15.84298	5.40769	3.14086	2.23607	1.75807	1.46242	1.26051	1.11303
1	1.00000	0.91022	0.83690	0.77572	0.72375	0.67897	0.63991	0.60548	0.57487	0.54745
2	0.52272	0.50029	0.47983	0.46108	0.44384	0.42791	0.41314	0.39942	0.38662	0.37466
3	0.36345	0.35292	0.34300	0.33365	0.32482	0.31646	0.30853	0.30101	0.29385	0.28704
4	0.28054	0.27435	0.26842	0.26276	0.25733	0.25213	0.24714	0.24235	0.23775	0.23332
5	0.22905	0.22495	0.22099	0.21717	0.21348	0.20991	0.20647	0.20314	0.19992	0.19680

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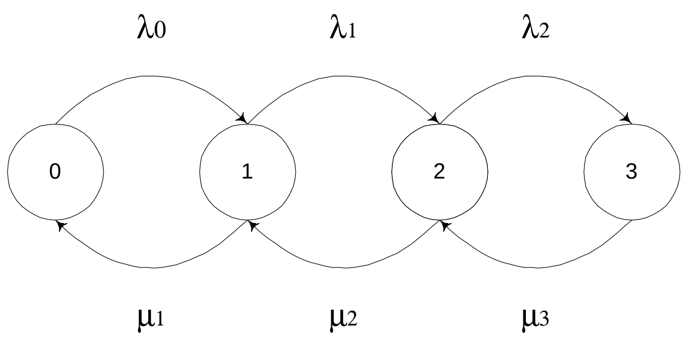
Part V. Stochastic Processes. A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading & reloading) has exponential distribution with mean 15 minutes. The machine will then run unattended for an average of 1 hour (but with actual time having exponential distribution) before it requires the operator's attention again.



_b,c,g_ 1. The Markov chain model diagrammed above is *(select one or more):*

a. a discrete-time Markov chain b. a continuous-time Markov chain
c. a Birth-Death process d. an M/M/1 queue

e. an M/M/3 queue f. an M/M/1/3 queue
g. an M/M/1/3/3 queue h. a Poisson process

Note: in the answers below, the state of the system is defined to be the number of machines which require the operator's attention.

_a_ 2. The value of λ_2 is

a. 1/hr. b. 2/hr.
c. 3/hr. d. 4/hr.
e. 0.5/hr. f. *none of the above*

_d_ 3. The value of μ_2 is

a. 1/hr. b. 2/hr.
c. 3/hr. d. 4/hr.
e. 0.5/hr. f. *none of the above*

_c_ 4. The value of λ_0 is

a. 1/hr. b. 2/hr.
c. 3/hr. d. 4/hr.
e. 0.5/hr. f. *none of the above*

_b_ 5. The steady-state probability π_0 is computed by solving

a. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 \approx \frac{1}{0.366}$
b. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2}\times\frac{3}{4} + \frac{1}{4}\times\frac{1}{2}\times\frac{3}{4} \approx \frac{1}{0.451}$
c. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \approx \frac{1}{0.4}$
d. $\frac{1}{\pi_0} = 1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.753}$
e. $\frac{1}{\pi_0} = 1 + \frac{3}{4} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^3 \approx \frac{1}{0.496}$
f. *none of the above*

_f_ 6. The operator will be busy what fraction of the time? *(choose nearest value)*

a. 30% b. 35% c. 40%
d. 45% e. 50% f. 55%
g. 70% h. 65% i. 70%

_b_ 7. What fraction of the time will the operator be busy but with no machine waiting to be serviced? *(choose nearest value*)

a. 30% b. 35% c. 40%
d. 45% e. 50% f. 55%
g. 70% h. 65% i. 70%

Note: $\pi_1 = \pi_0(3/4)$ = 34%, etc.
i.e., π_0=0.4507, π_1= 0.338, π_2= 0.169, π_3= 0.04225

_f_ 8. Approximately 2.2 machines per hour require the operator's attention. What is the average length of time that a machine waits before the operator begins to ready the machine for the next job? *(select nearest value)*

a. 0.1 hr. (i.e.,6 min.) b. 0.15 hr. (i.e., 9 min.)
c. 0.2 hr. (i.e., 12 min.) d. 0.25 hr. (i.e., 15 min.)
e. 0.3 hr. (i.e., 18 min.) f. greater than 0.33 hr. (i.e., >20 min.)

Note: $L = \sum_{n=0}^{3} n\pi_0 = 0.8$, $W = L / \lambda = 0.8 / 2.2 = 0.365$

_i_ 9. What will be the utilization of this group of 3 machines? *(choose nearest value)*

a. 30% b. 35% c. 40%
d. 45% e. 50% f. 55%
g. 60% h. 65% i. 70%

Note: The average number of machines in operation is 3-L = 2.197. Hence, each machine is in use about 2.197/3 = 73% of the time.