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57:022 Principles of Design II
Final Exam Solutions – Spring 2002

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Match the name of the distribution to the random variable:

Production of parts by a machine is a Poisson process, at the average rate of 2 parts per hour. Inspection will find that 20% of the processed parts are defective.

_J_ 1. the number of parts which are produced during the first hour?
_F_ 2. the time between production of defective parts?
_J_ 3. the number of defective parts which are produced during the first eight hours?
_O_ 4. the time that the second defective part is produced?
_D_ 5. the number of defective parts among the first eight which are produced?

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_N_ 6. The strength of a concrete pillar.
_B_ 7. The total weight of the university football team.
_N_ 8. The failure time of a television.
_M_ 9. The maximum daily rainfall each year in Iowa City.

Some common probability distributions:

[A]. Bernouilli	I. Uniform
B. Normal	[J]. Poisson
C. Lambda	[K] Pascal
[D] Binomial	L. Random
E. Chi-square	M. Gumbel
F. Exponential	N. Weibull
G. Beta	O. Erlang
[H]. Geometric	P. *None of the above*

10. Circle the distributions of discrete random variables in the list above.

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Indicate "+" for true, "O" for false.

_+_ 11. The inverse transformation method can always be used to generate a random number with distribution function F, provided you can calculate its inverse $F^{-1}(\bullet)$.
_+_ 12. The inverse transformation method (if it can be used) will always require fewer uniformly-generated random numbers than the rejection method.
_o_ 13. In a Poisson process, the time between arrivals has a Poisson distribution.
_o_ 14. The rejection method to generate a random number can be used to simulate interarrival times for a Poisson process.
_+_ 15. In a Poisson process with arrival rate λ/minute, the number of arrivals in t minutes is random, with a Poisson distribution having mean λt.
_+_ 16. The exponential distribution is a special case of the Erlang distribution.
_o_ 17. The Weibull distribution is a special case of the exponential distribution.
_+_ 18. If F(t) is the CDF of the interarrival time for a Poisson process, the probability that the next arrival occurs in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$

_o_ 19. If F is the CDF of a random variable X, then F(0) = 0.
_o_ 20. If F is the CDF of a random variable X with mean value μ, then F(μ)= 0.5.
_+_ 21. In linear regression, the "error" of a curve fitted to data points (x_i, y_i) is the vertical distance between the curve and the point (x_i, y_i).
_o_ 22. Linear regression requires solving a linear programming problem.

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One hundred identical devices are tested simultaneously, and the test is terminated after 50 days, at which time 27 of them have failed. The values of $\ln \ln 1/R_i$ vs $\ln t_i$ are plotted, where t_i is the i^{th} failure time, and R_i is (100–i)/100, i.e., the fraction failed. *Assume a Weibull distribution for estimating reliability.*

_o_ 23. If 10 units of this device were to be installed in a facility, the number still functioning after 50 days has a binomial distribution.

_o_ 24. To estimate the time at which 50% of the devices will have failed, evaluate 1 – F(0.50).

_+_ 25. To estimate the Weibull parameters u & k given the data above, we cannot use the "Method of Moments".

_o_ 26. The number of failures at time t , $N_f(t)$, is assumed to have a Weibull distribution.

_+_ 27. The Weibull CDF, i.e., F(t), gives, for each device, the probability that it has failed at time t.

_o_ 28. The time between the failures in the group of 100 units was assumed to have the Weibull distribution.

_+_ 29. The *secant method* is a method which is used to solve a nonlinear equation.

_o_ 30. The exponential distribution is a special case of the Weibull distribution, with λ=u.

_+_ 31. The exponential distribution is a special case of the Weibull distribution, with k=1.

_o_ 32. A value of k>0 indicates an increasing failure rate, while k<0 indicates a decreasing failure rate.

_+_ 33. The slope of the straight line fit by linear regression to the data points ([ln ln 1/R], ln t) will be an estimate of the "shape" parameter k.

_o_ 34. In general, given only a coefficient of variation (i.e., the ratio σ/μ) for the Weibull distribution, the parameters k and u can be determined.

_+_ 35. The probability of a motor failing in the time interval $[t_{i-1},t_i]$ is $F(t_i) - F(t_{i-1})$ where F(t) is the CDF of the failure time distribution.

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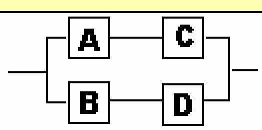
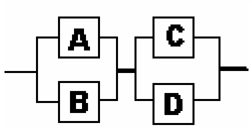
(36-47) Four components (A,B,C, & D) are available for constructing a system. The probability that each component *survives* the first year of operation is

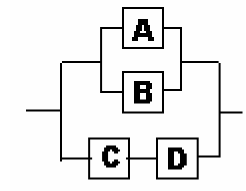
- 80% for A & B
- 90% for C & D.

For each system (1) through (3) below:

For each of these three scenarios (a,b,c), indicate whether the system will **F**ail or **S**urvive ***(write "F" or "S" in the table):***:

(i) only components A and B fail.
(ii) only components B and D fail.
(iii) only components A and D fail.

System #	Diagram	Scenario (i)	Scenario (ii)	Scenario (iii)	Reliability
1		F	S	F	d
2		F	S	S	c

3		S	S	S	b

For each system (#1-3) above, write the letter below indicating the ***computation*** of the 1-year reliability (i.e., survival probability):

a. $[1 - (0.9)(0.8)]^2 = 0.0784$
b. $1 - (0.2)^2(1 - [0.9]^2) = 0.9924$
c. $[1 - (0.2)^2][1 - (0.1)^2] = 0.9504$
d. $1 - [1 - (0.8)(0.9)]^2 = 0.9216$
e. $(0.9)^2(0.8)^2 = 0.5184$
j. *None of the above*

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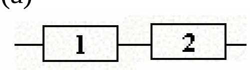
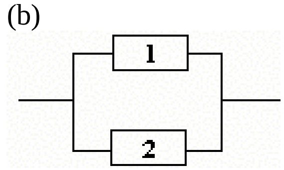
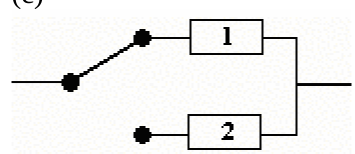
Consider components 1 & 2 with random time-to-failure of T_1 & T_2, respectively, having exponential distributions each with failure rate λ. Assume that any switches are 100% reliable.

(a)	(b)	(c)
		

Match the expression for system lifetime with the diagram (a, b, or c) above:

__c__ 48. $T_1 + T_2$ __b__ 49. $\text{Max}\{T_1, T_2\}$ __a__ 50. $\text{Min}\{T_1, T_2\}$

_+_ 51. A system with "cold" standby is at least as reliable as one with "hot" standby.
_o_ 52. Block diagram [c] above represents "hot" standby of the redundant unit.
_+_ 53. The failure time of system [a] has an exponential distribution with rate 2λ.
_o_ 54. In the case of "cold" standby, there is always some probability that the standby unit cannot be started.
_+_ 55. In the block diagram [c], unit #2 does not begin its lifetime until unit #1 has failed.
_+_ 56. The reliability of system [c] is at least as large as that of system [b].
_o_ 57. The failure time of system [b] has Erlang-2 distribution.
_o_ 58. When lifetimes have exponential distribution, there is no difference in reliability between a system with "hot" and "cold" standby.

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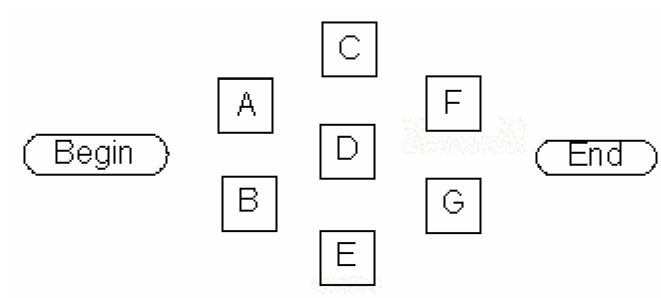
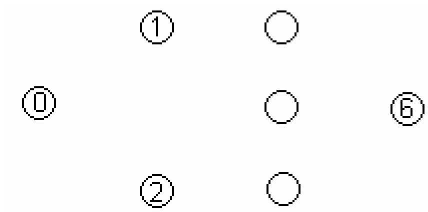
Project Scheduling. The activity descriptions and estimated durations for a project are:

Activity	Predecessor(s)	Duration (days)
A	none	3
B	none	2
C	A	4
D	A	1
E	B	2
F	C & D	3
G	C, D, & E	1

59. Draw the arrows to complete the *AON* (activity-on-node) network representing this project:



60. Draw the arrows to complete the *AOA* (activity-on-arrow) network representing this project, including any "dummy" activities:



61. Complete the labeling of the nodes of the AOA network so that i<j if there is an arrow from i to j.
62. Determine (by inspection if you can) the critical path and circle the critical activities:

$\boxed{A}$ B $\boxed{C}$ D E $\boxed{F}$ G

63. Suppose that the activity durations are actually random variables, with the expected values given as in the table and standard deviation equal to 1 for every activity. Then the expected completion time of the project is _10_ and its standard deviation is $\sqrt{3}$.

_+_ 64. A "dummy" activity always has zero duration.
_+_ 65. The quantity ET(i) [i.e. earliest time] for each node i is determined by a *forward* pass through the network.
_+_ 66. If an activity is represented by an arrow from node i to node j, then ES (earliest start time) for that activity is ET(i).
_o_67. If an activity is represented by an arrow from node i to node j, then LS (late start time) for that activity is LT(j).
_o_68. If an activity is represented by an arrow from node i to node j, then that activity has zero "float" or "slack" if and only if ET(i)=LT(j).
_+_ 69. An activity is critical if and only if its total float ("slack") is zero.
_o_70. A "dummy" activity *cannot* be critical.

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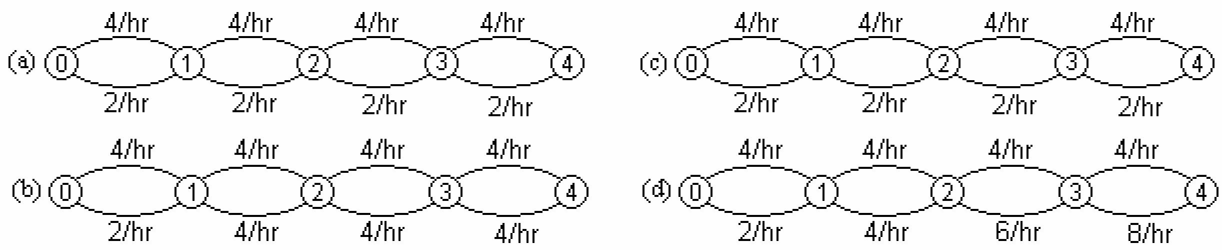
Birth-death model of queue. A small parking lot consists of two spaces. Cars making use of these spaces arrive according to a Poisson process on an average of once every fifteen minutes. Time that a car remains parked is exponentially distributed with mean of *30 minutes*. Cars who cannot find an empty space immediately on arrival will temporarily wait inside the lot until a parked car leaves, but this temporary space can hold only two cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Model this system as a birth-death process, with states 0, 1, … 4.

_b_ 71. Which are the correct transition rates?



_b_ 72. The classification of the above queueing system is

a. M/M/1/2/4 c. M/M/1/4 e. M/M/2/4/4 g. M/M/4
b. M/M/2/4 d. M/M/4/4 f. M/M/4/4/4 h. *None of the above*

Suppose that the steady-state probability distribution of the number of cars in the system is:

n	0	1	2	3	4
π_n	1/9	2/9	2/9	2/9	2/9

_c_ 73. What is the fraction of the time that there is at least one empty space? *(Choose nearest value!)*

a. 10% c. 30% *(33%)* e. 50% g. 70% i. 90%
b. 20% d. 40% f. 60% h. 80%

_d_ 74. What is the average number of cars in the lot (both parked & waiting)? *(Choose nearest value!)*

a. 0.5 c. 1.5 e. 2.5 g. 3.5
b. 1.0 d. 2.0 *(2.22)* f. 3.0 h. 4.0

_a_ 75. What is the average number of cars waiting in the lot? *(Choose nearest value!)*

a. 0.5 *(0.67)* c. 1.5 e. 2.5 g. 3.5
b. 1.0 d. 2.0 f. 3.0 h. 4.0

_e_ 76. What is the *average* arrival rate (keeping in mind that the arrival rate is zero when n=4)? *(Choose nearest value!)*

a. 1/hr c. 2/hr e. 3/hr *(3.11/hr)* g. 4/hr
b. 1.5/hr d. 2.5/hr f. 3.5/hr

_d_ 77. What is the average time that a car waits for a parking space? *(Choose nearest value!)*

a. 0.05 hr c. 0.15 hr e. 0.25 hr g. 0.35 hr i. 0.45 hr
b. 0.1hr d. 0.2 hr *(0.21 hr)* f. 0.3 hr h. 0.4 hr. j. 0.5 hr

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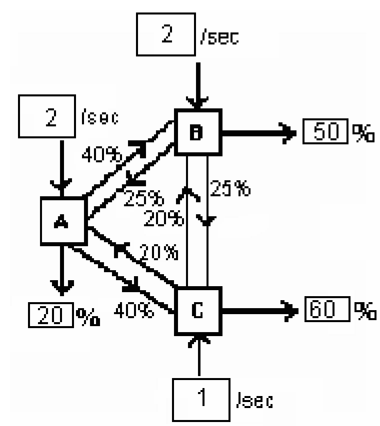
A network of three computer centers (A, B, & C) each receive messages from outside, as shown. The messages may then be routed to another computer in the network for further processing, also as shown, or a reply sent to the sender of the message. Each center has two computers, each processing messages at the rate of 4/sec. Consult the RAQS *(Rapid Analysis of Queueing Systems)* model output below to answer the questions.



_b_ 78. Which is the busiest computer center?

a. A b. B c. C

_J_ 79. What is the average time to respond to a message, (i.e., time from message arrival to reply)? *(Choose nearest value)*

a. ≤ 0.3 sec. b. 0.35 sec. c. 0.4 sec
d. 0.45 sec. e. 0.5 sec. f. 0.55 sec. g. 0.6 sec.
h. 0.65 sec. i. 0.7 sec. j. 0.75 sec. k. 0.8 sec. l. 0.85 sec. l. ≥ 0.9 sec.

80. Complete the three arrival rates and the two fractions of messages routed back to sender in the diagram above. *(Six boxes to fill.)*

Input information
This model has been developed in the Basic Mode
Type of network—Open Network
Number of nodes = 3

Node #	Number of servers	Arrival Rate	Arrival SCV	Mean Serv Time	Service time SCV
1	2	2.000	1.000	0.250	1.00
2	2	2.000	1.000	0.250	1.00
3	2	1.000	1.000	0.250	1.00

Routing (Pij) Matrix

0.000	0.400	0.400
0.250	0.000	0.250
0.200	0.200	0.000

Output Report
This Model has been developed in the Basic Mode
Type of Network – Open Network

Network Measures
Average Number in the Network = 3.782
Average time spent in the Network = 0.756

Node Measures

Node	Util	AvTIQ	VarTIQ	AvNIQ	AvTAN	VarTAN	AvNAN	VarNAN
1	0.471	0.071	0.016	0.268	0.321	0.079	1.210	1.195
2	0.527	0.096	0.026	0.406	0.346	0.088	1.461	1.673
3	0.445	0.062	0.013	0.220	0.312	0.076	1.111	1.033

Util – the Utilization at a Node
AvTIQ, VarTIQ – Mean and Variance of the waiting time in queue at a node
AvNIQ – Mean queue length at a node
AvTAN, VarTAN – Mean and Variance of time spent at a node
AvNAN, VarNAN – Mean and Variance of the number of customers at a node

_o 81. "Reneging" in a queueing system occurs when a potential customer is discouraged from joining the queue to be served.
_o 82. Little's Law states that the time spent in a queueing system has Erlang distribution.
_+ 83. In a birth/death model of a queueing system, the population size includes not only the waiting customers, but also any customers currently being served.
_+ 84. "Balking" in a queueing system occurs when a potential customer refuses to enter the queue.
_+ 85. In a birth/death model of a queueing system, a "birth" refers to a customer's joining the queue.
_+ 86. The "utilization" of the server in an M/M/1 system is equal to $1-\pi_0$.
_+ 87. Little's Law applies to <u>any</u> queueing system in steady state, whether or not it is a birth/death process.
_+ 88. An M/M/1 queueing system is a birth/death process.
_+ 89. The notation W_q generally refers to the average time that a customer spends waiting in the queueing system, exclusive of time being served.
_+ 90. In an M/M/1 queueing system, the number of customers arriving per unit time has Poisson distribution.