Match the name of the distribution to the random variable:

Production of parts by a machine is a Poisson process, at the average rate of 2 parts per hour. Inspection will find that 20% of the processed parts are defective.

1. the number of parts which are produced during the first hour?
   - J
2. the time between production of defective parts?
   - F
3. the number of defective parts which are produced during the first eight hours?
   - J
4. the time that the second defective part is produced?
   - O
5. the number of defective parts among the first eight which are produced?
   - D
6. The strength of a concrete pillar.
   - N
7. The total weight of the university football team.
   - B
8. The failure time of a television.
   - N
9. The maximum daily rainfall each year in Iowa City.
   - M

10. Circle the distributions of discrete random variables in the list above.

    ■ ■ ■ ■ ■ ■ ■ ■ ■

Indicate “+” for true, “O” for false.

11. The inverse transformation method can always be used to generate a random number with distribution function F, provided you can calculate its inverse $F^{-1}(\cdot)$.
+  
12. The inverse transformation method (if it can be used) will always require fewer uniformly-generated random numbers than the rejection method.
+  
13. In a Poisson process, the time between arrivals has a Poisson distribution.
×  
14. The rejection method to generate a random number can be used to simulate interarrival times for a Poisson process.
+  
15. In a Poisson process with arrival rate $\lambda$/minute, the number of arrivals in t minutes is random, with a Poisson distribution having mean $\lambda t$.
+  
16. The exponential distribution is a special case of the Erlang distribution.
+  
17. The Weibull distribution is a special case of the exponential distribution.
×  
18. If F(t) is the CDF of the interarrival time for a Poisson process, the probability that the next arrival occurs in the time interval $[t_i-1, t_i]$ is $F(t_i) - F(t_i-1)$
×  
19. If F is the CDF of a random variable X, then F(0) = 0.
×  
20. If F is the CDF of a random variable X with mean value $\mu$, then $F(\mu) = 0.5$.
×  
21. In linear regression, the “error” of a curve fitted to data points $(x_i, y_i)$ is the vertical distance between the curve and the point $(x_i, y_i)$.
×  
22. Linear regression requires solving a linear programming problem.
×
One hundred identical devices are tested simultaneously, and the test is terminated after 50 days, at which time 27 of them have failed. The values of ln ln 1/Ri vs ln ti are plotted, where ti is the ith failure time, and R, is (100–i)/100, i.e., the fraction failed. Assume a Weibull distribution for estimating reliability.

23. If 10 units of this device were to be installed in a facility, the number still functioning after 50 days has a binomial distribution.

24. To estimate the time at which 50% of the devices will have failed, evaluate 1 − F(0.50).

25. To estimate the Weibull parameters u & k given the data above, we cannot use the “Method of Moments”.

26. The number of failures at time t, Nf(t), is assumed to have a Weibull distribution.

27. The Weibull CDF, i.e., F(t), gives, for each device, the probability that it has failed at time t.

28. The time between the failures in the group of 100 units was assumed to have the Weibull distribution.

29. The secant method is a method which is used to solve a nonlinear equation.

30. The exponential distribution is a special case of the Weibull distribution, with \( \lambda = u \).

31. The exponential distribution is a special case of the Weibull distribution, with \( k = 1 \).

32. A value of \( k > 0 \) indicates an increasing failure rate, while \( k < 0 \) indicates a decreasing failure rate.

33. The slope of the straight line fit by linear regression to the data points ( [ln ln 1/R], ln t ) will be an estimate of the "shape" parameter k.

34. In general, given only a coefficient of variation (i.e., the ratio \( \sigma /\mu \)) for the Weibull distribution, the parameters k and u can be determined.

35. The probability of a motor failing in the time interval \([t_{i-1}, t_i]\) is \( F(t_i) − F(t_{i-1}) \) where F(t) is the CDF of the failure time distribution.

(36-47) Four components (A, B, C, & D) are available for constructing a system. The probability that each component survives the first year of operation is

- 80% for A & B
- 90% for C & D.

For each system (1) through (3) below:
For each of these three scenarios (a, b, c), indicate whether the system will Fail or Survive (write "F" or "S" in the table):

(i) only components A and B fail.
(ii) only components B and D fail.
(iii) only components A and D fail.

<table>
<thead>
<tr>
<th>System #</th>
<th>Diagram</th>
<th>Scenario (i)</th>
<th>Scenario (ii)</th>
<th>Scenario (iii)</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image" alt="Diagram 1" /></td>
<td>F</td>
<td>S</td>
<td>F</td>
<td>d</td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="Diagram 2" /></td>
<td>F</td>
<td>S</td>
<td>S</td>
<td>c</td>
</tr>
</tbody>
</table>
For each system (#1-3) above, write the letter below indicating the computation of the 1-year reliability (i.e., survival probability):

a. \[1 - (0.9)(0.8)^2 = 0.0784\]  
b. \[1 - (0.2)^2(1 - [0.9]^2) = 0.9924\]  
c. \[(1 - (0.2)^2)(1 - (0.1)^2) = 0.9504\]  
d. \[1 - [1 - (0.8)(0.9)]^2 = 0.9216\]  
e. \[(0.9)^2(0.8)^2 = 0.5184\]  
j. None of the above

Consider components 1 & 2 with random time-to-failure of T1 & T2, respectively, having exponential distributions each with failure rate \(\lambda\). Assume that any switches are 100% reliable.

(a) \(\max\{T_1, T_2\}\)  
(b) \(\min\{T_1, T_2\}\)  
(c) \(T_1 + T_2\)

51. A system with “cold” standby is at least as reliable as one with “hot” standby.
52. Block diagram [c] above represents “hot” standby of the redundant unit.
53. The failure time of system [a] has an exponential distribution with rate \(2\lambda\).
54. In the case of “cold” standby, there is always some probability that the standby unit cannot be started.
55. In the block diagram [c], unit #2 does not begin its lifetime until unit #1 has failed.
56. The reliability of system [c] is at least as large as that of system [b].
57. The failure time of system [b] has Erlang-2 distribution.
58. When lifetimes have exponential distribution, there is no difference in reliability between a system with “hot” and “cold” standby.

**Project Scheduling.** The activity descriptions and estimated durations for a project are:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Predecessor(s)</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>none</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>none</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>F</td>
<td>C &amp; D</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>C, D, &amp; E</td>
<td>1</td>
</tr>
</tbody>
</table>

59. Draw the arrows to complete the AON (activity-on-node) network representing this project:
60. Draw the arrows to complete the AOA (activity-on-arrow) network representing this project, including any “dummy” activities:

61. Complete the labeling of the nodes of the AOA network so that $i < j$ if there is an arrow from $i$ to $j$.

62. Determine (by inspection if you can) the critical path and circle the critical activities:

63. Suppose that the activity durations are actually random variables, with the expected values given as in the table and standard deviation equal to 1 for every activity. Then the expected completion time of the project is $10$ and its standard deviation is $\sqrt{3}$.

64. A “dummy” activity always has zero duration.

65. The quantity ET(i) [i.e. earliest time] for each node $i$ is determined by a forward pass through the network.

66. If an activity is represented by an arrow from node $i$ to node $j$, then ES (earliest start time) for that activity is ET(i).

67. If an activity is represented by an arrow from node $i$ to node $j$, then LS (late start time) for that activity is LT(j).

68. If an activity is represented by an arrow from node $i$ to node $j$, then that activity has zero “float” or “slack” if and only if ET(i)=LT(j).

69. An activity is critical if and only if its total float (“slack”) is zero.

70. A “dummy” activity cannot be critical.

**Birth-death model of queue.** A small parking lot consists of two spaces. Cars making use of these spaces arrive according to a Poisson process on an average of once every fifteen minutes. Time that a car remains parked is exponentially distributed with mean of 30 minutes. Cars who cannot find an empty space immediately on arrival will temporarily wait inside the lot until a parked car leaves, but this temporary space can hold only two cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Model this system as a birth-death process, with states 0, 1, ... 4.
71. Which are the correct transition rates?

- (a) 4/hr 4/hr 4/hr 4/hr
  - 2/hr 2/hr 2/hr 2/hr
  - 4/hr 4/hr 4/hr 4/hr
- (b) 2/hr 2/hr 2/hr 2/hr
  - 4/hr 4/hr 4/hr 4/hr
  - 2/hr 2/hr 2/hr 2/hr
- (c) 4/hr 4/hr 4/hr 4/hr
  - 2/hr 2/hr 2/hr 2/hr
  - 4/hr 4/hr 4/hr 4/hr
- (d) 4/hr 4/hr 4/hr 4/hr
  - 2/hr 2/hr 2/hr 2/hr
  - 6/hr 6/hr 6/hr 6/hr

72. The classification of the above queueing system is

- a. M/M/1/2/4  
- b. M/M/2/4  
- c. M/M/1/4  
- d. M/M/4/4  
- e. M/M/2/4/4  
- f. M/M/4/4/4  
- g. M/M/4  
- h. None of the above

Suppose that the steady-state probability distribution of the number of cars in the system is:

<table>
<thead>
<tr>
<th>n</th>
<th>πn</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/9</td>
</tr>
<tr>
<td>1</td>
<td>2/9</td>
</tr>
<tr>
<td>2</td>
<td>2/9</td>
</tr>
<tr>
<td>3</td>
<td>2/9</td>
</tr>
<tr>
<td>4</td>
<td>2/9</td>
</tr>
</tbody>
</table>

73. What is the fraction of the time that there is at least one empty space? (Choose nearest value!)

- a. 10%  
- b. 20%  
- c. 30% (33%)  
- d. 40%  
- e. 50%  
- f. 60%  
- g. 70%  
- h. 80%  
- i. 90%

74. What is the average number of cars in the lot (both parked & waiting)? (Choose nearest value!)

- a. 0.5  
- b. 1.0  
- c. 1.5  
- d. 2.0 (2.22)  
- e. 2.5  
- f. 3.0  
- g. 3.5  
- h. 4.0

75. What is the average number of cars waiting in the lot? (Choose nearest value!)

- a. 0.5 (0.67)  
- b. 1.0  
- c. 1.5  
- d. 2.0  
- e. 2.5  
- f. 3.0  
- g. 3.5  
- h. 4.0

76. What is the average arrival rate (keeping in mind that the arrival rate is zero when n=4)? (Choose nearest value!)

- a. 1/hr  
- b. 1.5/hr  
- c. 2/hr  
- d. 2.5/hr  
- e. 3/hr (3.11/hr)  
- f. 3.0/hr  
- g. 3.5/hr

77. What is the average time that a car waits for a parking space? (Choose nearest value!)

- a. 0.05 hr  
- b. 0.1 hr  
- c. 0.15 hr  
- d. 0.2 hr (0.21 hr)  
- e. 0.25 hr  
- f. 0.3 hr  
- g. 0.35 hr  
- h. 0.4 hr  
- i. 0.45 hr  
- j. 0.5 hr

78. Which is the busiest computer center?

- a. A  
- b. B  
- c. C

79. What is the average time to respond to a message, (i.e., time from message arrival to reply)? (Choose nearest value)

- a. ≤ 0.3 sec.  
- b. 0.35 sec.  
- c. 0.4 sec  
- d. 0.45 sec.  
- e. 0.5 sec.  
- f. 0.55 sec.  
- g. 0.6 sec.  
- h. 0.65 sec.  
- i. 0.7 sec.  
- j. 0.75 sec.  
- k. 0.8 sec.  
- l. 0.85 sec.  
- m. ≥ 0.9 sec.

80. Complete the three arrival rates and the two fractions of messages routed back to sender in the diagram above. (Six boxes to fill.)
Input information
This model has been developed in the Basic Mode
Type of network—Open Network
Number of nodes = 3

<table>
<thead>
<tr>
<th>Node</th>
<th>Number</th>
<th>Arrival # of servers</th>
<th>Arrival Rate</th>
<th>Arrival Mean</th>
<th>Serv SCV</th>
<th>Time SCV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2.000</td>
<td>1.000</td>
<td>0.250</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2.000</td>
<td>1.000</td>
<td>0.250</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1.000</td>
<td>1.000</td>
<td>0.250</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Routing (P_{ij}) Matrix

\[
\begin{pmatrix}
0.000 & 0.400 & 0.400 \\
0.250 & 0.000 & 0.250 \\
0.200 & 0.200 & 0.000 \\
\end{pmatrix}
\]

Output Report
This Model has been developed in the Basic Mode
Type of Network - Open Network

Network Measures
Average Number in the Network = 3.782
Average time spent in the Network = 0.756

Node Measures

<table>
<thead>
<tr>
<th>Node</th>
<th>Util</th>
<th>AvTIQ</th>
<th>VarTIQ</th>
<th>AvNIQ</th>
<th>AvTAN</th>
<th>VarTAN</th>
<th>AvNAN</th>
<th>VarNAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.471</td>
<td>0.071</td>
<td>0.016</td>
<td>0.268</td>
<td>0.321</td>
<td>0.079</td>
<td>1.210</td>
<td>1.195</td>
</tr>
<tr>
<td>2</td>
<td>0.527</td>
<td>0.096</td>
<td>0.026</td>
<td>0.406</td>
<td>0.346</td>
<td>0.088</td>
<td>1.461</td>
<td>1.673</td>
</tr>
<tr>
<td>3</td>
<td>0.445</td>
<td>0.062</td>
<td>0.013</td>
<td>0.220</td>
<td>0.312</td>
<td>0.076</td>
<td>1.111</td>
<td>1.033</td>
</tr>
</tbody>
</table>

Util – the Utilization at a Node
AvTIQ, VarTIQ – Mean and Variance of the waiting time in queue at a node
AvNIQ – Mean queue length at a node
AvTAN, VarTAN – Mean and Variance of time spent at a node
AvNAN, VarNAN – Mean and Variance of the number of customers at a node

81. ＿“Reneging” in a queueing system occurs when a potential customer is discouraged from joining the queue to be served.
82. ＿Little’s Law states that the time spent in a queueing system has Erlang distribution.
83. ＿In a birth/death model of a queueing system, the population size includes not only the waiting customers, but also any customers currently being served.
84. ＿“Balking” in a queueing system occurs when a potential customer refuses to enter the queue.
85. ＿In a birth/death model of a queueing system, a “birth” refers to a customer’s joining the queue.
86. ＿The “utilization” of the server in an M/M/1 system is equal to 1−π_0.
87. ＿Little’s Law applies to any queueing system in steady state, whether or not it is a birth/death process.
88. ＿An M/M/1 queueing system is a birth/death process.
89. ＿The notation W_q generally refers to the average time that a customer spends waiting in the queueing system, exclusive of time being served.
90. ＿In an M/M/1 queueing system, the number of customers arriving per unit time has Poisson distribution.