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57:022 Principles of Design II
Final Exam -- May 13, 1999
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| Part | I | II | III | IV | V | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Your score: |  |  |  |  |  |  |
| Possible | 7 | 18 | 12 | 15 | 18 | 70 |
|  |  |  |  |  |  |  |

Part I. Probability Distributions For each probability distribution below, indicate by " $C$ " or " $D$ " whether the corresponding random variable is Continuous or Discrete.
__1. Bernouilli
_2. Binomial
-3. Poisson
4. Uniform
5. Beta
_6. Geometric
7. Exponential
8. Normal
9. Weibull
10. Triangular
__11. Pascal (negative binomial)
_12. Erlang-k with $\mathrm{k}>1$
—13. Gumbel
—_14. Chi-square

Part II. Project Scheduling
__1. An ARENA model of a project is more similar to an AON network model than an AOA network model of the project.
___ 2. The quantity LT(i) [i.e. latest time] for each node i is determined by a forward pass through the network.
_3. If an activity is represented by an arrow from node $i$ to node $j$, then LS (latest start time) for that activity is LT(i).
__4. If an activity is represented by an arrow from node i to node $j$, then ES (early start time) for that activity is ET(i).
__5. If an activity is represented by an arrow from node i to node j , then that activity has zero "float" or "slack" if and only if $E T(i)=L T(j)$.
__6. An activity is critical if and only if its total float ("slack") is zero.
__7. A "dummy" activity cannot be critical.
__8. The mean value of the duration of activity is equal to its most likely value, if the probability distribution is Beta.
__ 9. PERT assumes that each activity's duration has a Normal distribution.
10. PERT assumes that the project duration has a Normal distribution.
—_11. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-onNode" representation of a project.
__12. The project network below is of the AOA form.
In the project network below, each activity is assumed to require its expected duration. Complete the two missing pairs of ETs (earliest times) \& LTs (latest times) in the network below.

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The critical path is shown in bold above. If the durations are random, with expected values as shown and standard deviations all equal to 1.0 , what is
13. ... the expected completion time of the project, according to PERT? $\qquad$ days
14. ... the standard deviation of the project completion time, according to PERT? $\qquad$ days

Part III: A system consists of five components (A,B,C,D, \&E). The probability that each component fails during the first year of operation is $30 \%$ for A, B, and C, and $40 \%$ for D and E. For each alternative (a) and (b), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1 -year reliability (i.e., survival probability)
Diagram $\quad$ Reliability

1. The system requires that at least one of $A, B, \& C$ function, and that both $D$ and
E function.

## Diagrams:



Reliabilities:

1. $(0.7)^{3}(0.6)^{2}=12.3 \%$
2. $1-\left[1-(0.7)^{3}\right]\left[1-(0.6)^{2}\right]=57.9 \%$
3. $(.7)^{3}\left(1-[.4]^{2}\right)=28.8 \%$
4. $1-(0.3)^{3}(0.4)^{2}=94.5 \%$
5. $\left[1-(0.3)^{3}\right]\left[1-(0.4)^{2}\right]=81.7 \%$
6. $1-\left[1-(0.3)^{3}\right]\left[1-(0.4)^{2}\right]=18.3 \%$
7. $\left[1-(0.3)^{3}\right](0.6)^{2}=35.0 \%$
8. None of the above

Part IV. Weibull Model of Reliability. An electronic device is made up of a large number of components. Every component is essential, so that the device will fail when the first component fails. The lifetime of each individual component is random, but its probability distribution is unknown. The manufacturer, who has provided a 90-day warranty on this device, has decided to use the Weibull reliability model.

For each statement, indicate " + " for true, " 0 " for false:

1. A positive value of $\ln \mathrm{k}$ indicates an increasing failure rate, and negative $\ln \mathrm{k}$ indicates a decreasing failure rate.
2. We assume that the number of survivors at time $t, N_{S}(t)$, has a Weibull distribution.
3. The Weibull CDF, i.e., $F(t)$, gives, for each bulb, the probability that at time $t$ it has already failed.
4. The exponential distribution is a special case of the Weibull distribution, with failure rate zero.
5. The sum of the CDF (cumulative distribution function) $\mathrm{F}(\mathrm{t})$ and the Reliability function $\mathrm{R}(\mathrm{t})$, i.e. $\mathrm{F}(\mathrm{t})+$ $R(t)$, is always equal to 1 if the Weibull probability model is assumed.
6. If 4 of the devices are installed in a manufacturing system, the number still functioning after 100 days has a binomial distribution.

## Name

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It has been determined that average lifetime of the device is 400 days and the standard deviation is 500 days. 7. Based upon the above information, the value of the "shape" parameter $(\mathrm{k})$ of the probability dist'n is approximately (choose nearest value).
a. 0.1
b. 1.0
c. 10.0
d. 100.0
e. 1000.0
$\qquad$
8. The value of the "location" parameter (u) of the probability dist'n is approximately (choose nearest value).
a. 0.1
b. 1.0
c. 10.0
d. 100.0
e. 1000.0
9. The failure rate is
a. increasing
b. decreasing
c. constant
d. cannot be determined
10. The percent of the units which are expected to fail during the 90-day warranty period is (choose nearest value):
a. $1 \%$
b. $2 \%$
c. $3 \%$
d. $4 \%$
e. $5 \%$
f. $6 \%$
g. $7 \%$
h. $8 \%$

Table 1: $\Gamma\left(1+\frac{1}{k}\right)$

|  | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | m | 362880 | 120.000 | 9.26053 | 3.32335 | 2.00000 | 1.50458 | 1.26582 | 1.13300 |
| 1 | 1.00000 | 0.96491 | 0.94066 | 0.92358 | 0.91142 | 0.90275 | 0.89657 | 0.89224 | 0.88929 | 0.88736 |
| 2 | 0.88623 | 0.88569 | 0.88562 | 0.88591 | 0.88648 | 0.88726 | 0.88821 | 0.88928 | 0.89045 | 0.89169 |
| 3 | 0.89298 | 0.89431 | 0.89565 | 0.89702 | 0.89838 | 0.89975 | 0.90111 | 0.90245 | 0.90379 | 0.90510 |
| 4 | 0.90640 | 0.907688 | 0.90894 | 0.91017 | 0.91138 | 0.91257 | 0.91374 | 0.91488 | 0.91600 | 0.91710 |
| 5 | 0.91817 | 0.91922 | 0.92025 | 0.92125 | 0.92224 | 0.92320 | 0.92414 | 0.92507 | 0.92597 | 0.92685 |

Table 2: Coefficient of variation $\frac{\sigma}{\mu}$ of the Weibull distribution, as a function of $k$ alone:

|  | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | --- | --- | 15.84298 | 5.40769 | 3.14086 | 2.23607 | 1.75807 | 1.46242 | 1.26051 | 1.11303 |
| 1 | 1.00000 | 0.91022 | 0.83690 | 0.77572 | 0.72375 | 0.67897 | 0.63991 | 0.60548 | 0.57487 | 0.54745 |
| 2 | 0.52272 | 0.50029 | 0.47983 | 0.46108 | 0.44384 | 0.42791 | 0.41314 | 0.39942 | 0.38662 | 0.37466 |
| 3 | 0.36345 | 0.35292 | 0.34300 | 0.33365 | 0.32482 | 0.31646 | 0.30853 | 0.30101 | 0.29385 | 0.28704 |
| 4 | 0.28054 | 0.27435 | 0.26842 | 0.26276 | 0.25733 | 0.25213 | 0.24714 | 0.24235 | 0.23775 | 0.23332 |
| 5 | 0.22905 | 0.22495 | 0.22099 | 0.21717 | 0.21348 | 0.20991 | 0.20647 | 0.20314 | 0.19992 | 0.19680 |

Part V. Stochastic Processes. A machine operator has sole responsibility for keeping three semi-automatic machines busy. The time required to ready a machine (unloading \& reloading) has exponential distribution with mean 15 minutes. The machine will then run unattended for an average of 1 hour (but with actual time having exponential distribution) before it requires the operator's attention again.


1. The Markov chain model diagrammed above is (select one or more):
a. a discrete-time Markov chain
b. a continuous-time Markov chain
c. a Birth-Death process
d. an $M / M / 1$ queue
e. an $M / M / 3$ queue
f. an $M / M / 1 / 3$ queue
g. an $M / M / 1 / 3 / 3$ queue
h. a Poisson process
2. The value of $\lambda_{2}$ is
a. $1 / \mathrm{hr}$.
b. $2 / \mathrm{hr}$.
c. $3 / \mathrm{hr}$.
d. $4 / \mathrm{hr}$.
e. $0.5 / \mathrm{hr}$.
f. none of the above
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3. The value of $\mu_{2}$ is
a. $1 / \mathrm{hr}$.
b. $2 / \mathrm{hr}$.
c. $3 / \mathrm{hr}$.
d. $4 / \mathrm{hr}$.
e. $0.5 / \mathrm{hr}$.
f. none of the above
4. The value of $\lambda_{0}$ is
a. $1 / \mathrm{hr}$.
b. $2 / \mathrm{hr}$.
c. $3 / \mathrm{hr}$.
d. $4 / \mathrm{hr}$.
e. $0.5 / \mathrm{hr}$.
f. none of the above
5. The steady-state probability $\pi_{0}$ is computed by solving
a. $\frac{1}{\pi_{0}}=1+\frac{3}{4}+\left(\frac{3}{4}\right)^{2}+\left(\frac{3}{4}\right)^{3} \approx \frac{1}{0.366}$
b. $\frac{1}{\pi_{0}}=1+\frac{3}{4}+\frac{1}{2} \times \frac{3}{4}+\frac{1}{4} \times \frac{1}{2} \times \frac{3}{4} \approx \frac{1}{0.451}$
c. $\frac{1}{\pi_{0}}=1+\frac{3}{4}+\frac{1}{2}+\frac{1}{4} \approx \frac{1}{0.4}$
d. $\frac{1}{\pi_{0}}=1+\frac{1}{4}+\left(\frac{1}{4}\right)^{2}+\left(\frac{1}{4}\right)^{3} \approx \frac{1}{0.753}$
e. $\frac{1}{\pi_{0}}=1+\frac{3}{4}+\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{4}\right)^{3} \approx \frac{1}{0.496}$
f. none of the above
6. The operator will be busy what fraction of the time? (choose nearest value)
a. $30 \%$
b. $35 \%$
c. $40 \%$
d. $45 \%$
e. $50 \%$
f. $55 \%$
g. $70 \%$
h. $65 \%$
i. $70 \%$
7. What fraction of the time will the operator be busy but with no machine waiting to be serviced? (choose nearest value)
a. $30 \%$
b. $35 \%$
c. $40 \%$
d. $45 \%$
e. $50 \%$
f. $55 \%$
g. $70 \%$
h. $65 \%$
i. $70 \%$
8. Approximately 2.2 machines per hour require the operator's attention. What is the average length of time that a machine waits before the operator begins to ready the machine for the next job? (select nearest value)
a. 0.1 hr . (i.e., 6 min .)
b. 0.15 hr . (i.e., 9 min .)
c. 0.2 hr . (i.e., 12 min .)
d. 0.25 hr . (i.e., 15 min .)
e. 0.3 hr . (i.e., 18 min .)
f. greater than 0.33 hr . (i.e., >20 min.)
9. What will be the utilization of this group of 3 machines? (choose nearest value)
a. $30 \%$
b. $35 \%$
c. $40 \%$
d. $45 \%$
e. $50 \%$
f. $55 \%$
g. $70 \%$
h. $65 \%$
i. $70 \%$
