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57:022 Principles of Design II
Final Exam - Spring 1997

Part:	I	II	III	IV	V	VI	Total
Possible Pts:	52	10	12	16	13	12	115
Your score:	___	___	___	___	___	___	______

PART ONE

Indicate "+" if True and "O" if False:

___a. If a component's lifetime has an exponential distribution, its failure rate ("hazard rate") is constant.

___b. The steadystate distribution π for a continuous-time Markov chain must satisfy $\pi\Lambda=0$ where Λ is the transition rate matrix.

___c. If two components of a system have a parallel configuration with respect to system reliability, then both are required to function in order for the system to function.

___d. If two components of a system have a series configuration with respect to system reliability, then the second component replaces the first when it fails, and the system then fails when the second component fails.

___e. In a discrete-time Markov chain, the sum of each column of the transition probability matrix P must be 1.

___f. In an M/M/1 queueing system, π_0 represents the probability that the server is busy.

___g. If the server of an M/M/1 system is modified so that the service time is constant, but equal to the average service time of the M/M/1 system, the average time spent in the queue is reduced.

___h. In a continuous-time Markov chain, it is assumed that the time between transitions has an exponential distribution.

___i. In a discrete-time Markov chain, a state that is not transient is called "recurrent".

___j. In a discrete-time Markov chain with absorbing states, the diagonal elements of the matrix E must be greater than or equal to 1.

___k. PERT assumes that the project duration has a Normal distribution.

___l. R(t)+F(t)=1 for all t, where R and F are the reliability function and the cumulative distribution function of the failure time, respectively.

___m. If two components of a system have a series configuration with respect to system reliability, the system lifetime is the maximum of the two component lifetimes.

___n. PERT assumes that each activity's duration has a Normal distribution.

___o. The exponential distribution is a special case of an Erlang distribution.

___p. "Birth/death" processes are a special class of discrete-time Markov chains.

___q. In a discrete-time Markov chain, an absorbing state may also be a recurrent state.

___r. The argument x of the gamma function $\Gamma(x)$ may be any real nonnegative number, and is equal to (x-1)! for positive integer values of x.

___s. If 3 components of a system have a parallel configuration with respect to system reliability, then the system lifetime is the sum of the component lifetimes.

___t. In a Poisson arrival process, the number of arrivals during an hour has the Poisson distribution.

___u. In an M/M/2 queueing system, it is assumed that the each of two servers has its own queue, and customers cannot change queues once they have entered the system.

___v. In an M/M/2 queueing system, it is assumed that the arrival process is Poisson.

___w. In a continuous-time Markov chain, the transition rate λ_{ii} from a state to itself must be defined to be zero.

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___x. In an M/M/2 queueing system, it is assumed that the servers work at the same rate.
___y. Disregarding "Begin" and "End" activities,"Dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
___z. "Birth/death" processes are a special class of continuous-time Markov chains.
___A. In a discrete-time Markov chain, the steadystate probability distribution π must satisfy the equation $P\pi=\pi$.
___B. The exponential distribution is a special case of a Weibull distribution with k=0.
___C. An $M/E_k/1$ queueing system can be modelled as a birth-death process.
___D. In a discrete-time Markov chain having a steadystate distribution, the mean recurrence time (m_{ii}) times the probability π_i is a constant.
___E. If t_i is the i^{th} failure time of 100 batteries (i=1,2,...25), and the lifetimes have a Weibull distribution, then a plot of $\ln t_i$ vs $\ln \ln {}^{100}/_{(100-i)}$, i=1,2,...25 should be (approximately) a straight line with slope equal to the "shape" parameter k.
___F. An ARENA model of a project employs the "Activity-on-Node" rather than "Activity-on-Arrow" representation of the project.
___G. In a discrete-time Markov chain, the steadystate probability distribution π must satisfy the equation $P^t\pi=\pi$.
___H. Both the triangular and beta distributions for a project activity's duration are uniquely specified when you give the minimum & maximum durations and a most likely duration.
___I. If a component's lifetime has Weibull distribution with k>0, its failure rate ("hazard rate") is increasing.
___J. In a Poisson arrival process, the time between arrivals has the geometric distribution.
___K. The "Rejection" technique may be used to generate random numbers representing activity durations in a project, if those durations have either triangular or beta distribution.
___L. In a continuous-time Markov chain, the sum of each row of the transition rate matrix Λ must be 1.
___M. In an M/M/1 queueing system, with arrival rate 1/minute and service time averaging 15 seconds, we would expect the server to be busy lesss than 50% of the time.
___N. In a discrete-time Markov chain, a state that is a member of a "minimal closed set" is called "recurrent".
___O. In an M/M/1/N queueing system, N represents the capacity of the waiting line.
___P. In an M/M/2 queueing system, at most 2 customers may be in the system at any time.
___Q. In an M/M/1 queueing system, the greek letter ρ denotes the probability that the server is idle.
___R. For an infinite-capacity single-server queue (M/M/1), a steady state exists only if the arrival rate is less than the service rate.
___S. In a discrete-time Markov chain with absorbing states, the sum of each row of the matrix E is 1.
___T. In a continuous-time Markov chain, it is assumed that the time until the next transition of the system has a Poisson distribution.
___U. If two components of a system have a parallel configuration with respect to system reliability, the system lifetime is the maximum of the two component lifetimes.
___V. In an M/M/1 queueing system, with an average of 30 seconds between arrivals and an average of 3 customers in the system, we would expect the average time spent by a customer in the system to be at least 2 minutes.
___W. Each replication in a simulation experiment must begin with a different random number seed, or else the results of the replications will be identical.
___X. In an M/G/1 queueing system, the service time distribution is assumed to be Gamma.
___Y. An M/M/1/N queueing system has a steadystate distribution if and only if the arrival rate λ is strictly less than the service rate μ.
___Z. A Poisson process is a special case of a birth-death process, in which death is caused by poissoning.

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PART TWO

Multiple Choice

___ a. The "Rejection" method for generating random numbers can be used to simulate
1. time between arrivals   2. number of arrivals   3. probability of an arrival
4. time to serve a “customer”   5. *More than one of above*   6. *None of the above*

for an **M/M/1** queueing system.

___ b. If F(t) is the CDF of the failure time for a component and N components are tested simultaneously, the expected number of components which fail during the time interval $[t_{i-1}, t_i]$ is

1. $N \times F(t_i - t_{i-1})$ 2. $N \times [F(t_i) - F(t_{i-1})]$ 3. $F(t_i - t_{i-1}) / N$
4. $f(t_i) \times (t_i - t_{i-1}) \times N$ 5. $N \times [f(t_i) - f(t_{i-1})]$ 6. *None of the above*

___ c. The number of states in a Markov chain model of an $M/E_2/1/3$ queueing system is
1. three   2. four   3. five
4. six   5. seven   6. *None of the above*

___d. If you use the Cricket Graph program to fit a line to n data points (x_i, y_i), i=1,2,...n, it will find the coefficients a & b of the straight line y = ax+b which

1. minimizes $\sum_{i=1}^{n} |ax_i + b - y_i|$ 2. maximizes the # of points such that $y_i = ax_i + b$
3. minimizes $\sum_{i=1}^{n} (y_i - ax_i - b)^2$ 4. minimizes $\sum_{i=1}^{n} |y_i - ax_i - b|$
5. minimizes $\max_i \{ax_i + b - y_i\}$ 6. *None of the above*

___ e. The "Cumulative Distribution Function" (CDF) of any random variable X is defined as
1. $f(x) = P\{x \mid X\}$ 2. $F(x) = P\{X \geq x\}$ 3. $F(x) = P\{X \leq x\}$
4. $f(x) = P\{X \mid x\}$ 5. $F(x) = P\{X = x\}$ 6. $f(x) = P\{x\}$

PART THREE

Write the ***number*** *corresponding to the correct probability distribution in each blank below. Note that some distributions may apply in more than one case, while others not at all!*

_____ a. the number of cars passing through an intersection during a 1-minute green light.
_____ b. the number of trucks among the first 10 vehicles to arrive at an intersection during a red light
_____ c. The sum of ten N(0,1) random variables
_____ d. the time until the arrival of the <u>second</u> car at an intersection after a traffic light has turned red.
_____ e. the completion time of a large project with random task durations
_____ f. the time between the arrivals of the first and second vehicle during a red light.
_____ g. the interarrival time for the M/M/1 queueing system
_____ h. the magnitude of the highest rate of flow into the Coralville Reservoir next year
_____ i. the number of heads obtained by tossing a single coin once.
_____ j. number of defective items found when testing a batch of size 10
_____ k. The sum of the squares of ten N(0,1) random variables
_____ l. the number of items produced in order to obtain 4 acceptable items, if each is tested before producing the next

<u>Probability distributions:</u>

1. Bernouilli   6. Geometric   11. Pascal (negative binomial)
2. Binomial   7. Exponential   12. Erlang-k with k>1
3. Poisson   8. Normal   13. Gumbel
4. Uniform   9. Weibull   14. Chi-square
5. Beta   10. Triangular

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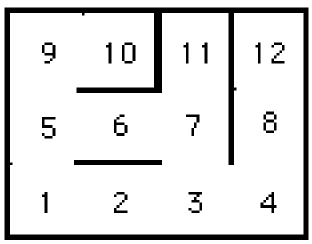
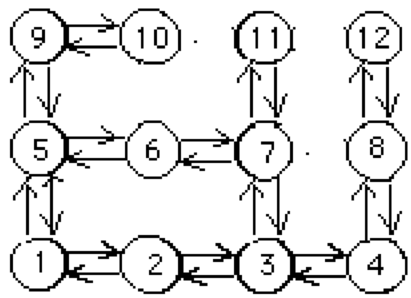
PART FOUR

A rat is placed in room #1 of a maze shown below on the left. A Markov chain model has been built where the state of the "system" is the location of the rat after each move. In assigning transition probabilities, it is assumed that the rat is equally likely to leave a location by any of the available doors, including the door by which he entered. (If he arrives at a "dead end", he will retrace his last move with probability 1.) Consult the given computer output to answer the following questions:





Steady State Distribution

i	π_i
1	0.0833
2	0.0833
3	0.125
4	0.0833
5	0.125
6	0.0833
7	0.125
8	0.0833
9	0.0833
10	0.0417
11	0.0417
12	0.0417

P=

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.5	0	0	0.5	0	0	0	0	0	0	0
2	0.5	0	0.5	0	0	0	0	0	0	0	0	0
3	0	0.333	0	0.333	0	0	0.333	0	0	0	0	0
4	0	0	0.5	0	0	0	0	0.5	0	0	0	0
5	0.333	0	0	0	0	0.333	0	0	0.333	0	0	0
6	0	0	0	0	0.5	0	0.5	0	0	0	0	0
7	0	0	0.333	0	0	0.333	0	0	0	0	0.333	0
8	0	0	0	0.5	0	0	0	0	0	0	0	0.5
9	0	0	0	0	0.5	0	0	0	0	0.5	0	0
10	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	1	0	0	0	0

M=

	1	2	3	4	5	6	7	8	9	10	11	12
1	12	9.33	12.7	31.7	8.67	15.3	16	52.7	29.7	52.7	39	75.7
2	10.7	12	7.33	26.3	15.3	18	14.7	47.3	36.3	59.3	37.7	70.3
3	19.3	12.7	8	19	20	18.7	11.3	40	41	64	34.3	63
4	24.3	17.7	5	12	25	23.7	16.3	21	46	69	39.3	44
5	11.3	16.7	16	35	8	10.7	15.3	56	21	44	38.3	79
6	16.7	18	13.3	32.3	9.33	12	8.67	53.3	30.3	53.3	31.7	76.3
7	20	17.3	8.67	27.7	16.7	11.3	8	48.7	37.7	60.7	23	71.7
8	27.3	20.7	8	3	28	26.7	19.3	12	49	72	42.3	23
9	14.3	19.7	19	38	3	13.7	18.3	59	12	23	41.3	82
10	15.3	20.7	20	39	4	14.7	19.3	60	1	24	42.3	83
11	21	18.3	9.67	28.7	17.7	12.3	1	49.7	38.7	61.7	24	72.7
12	28.3	21.7	9	4	29	27.7	20.3	1	50	73	43.3	24

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____ 1. If we record the rat's location over a period of several days, which location do you expect to be visited most frequently by the rat?

a. all equally often
b. location 7 more often than others
c. locations 3 & 5 equally often
d. locations 3, 5, &7 equally often
e. locations 1, 3, 5, &7 equally often

____ 2. If the rat begins in location #10, what is the expected number of moves required to reach location #11?

a. five
b. between 5 and 20
c. between 20 and 50
d. between 50 and 75
e. between 75 and 100
f. over 100

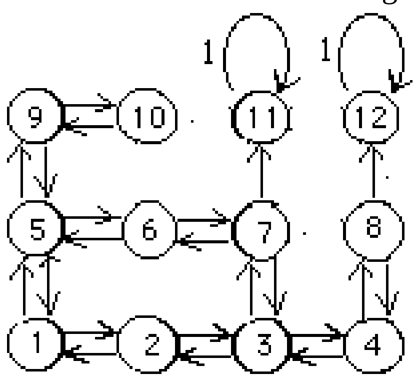
____ 3. If the rat begins in location #5, how many moves will we expect the rat to make before returning to his starting point, location #5?

a. five
b. between 5 and 10
c. between 10 and 25
d. between 25 and 50
e. between 50 and 100
f. over 100

____ 4. The number of *transient* states in this Markov chain model is

a. 0
b. 6
c. 9
d. 10
e. 12
f. none of the above

Now suppose that food is placed in locations 11 and 12, so that the rat does not leave when he finds it. States 11 & 12 then become absorbing states, and the Markov chain model becomes:



The *absorption* and *expected number of visit* matrices are:

A =

	11	12
1	0.69	0.31
2	0.655	0.345
3	0.621	0.379
4	0.414	0.586
5	0.724	0.276
6	0.759	0.241
7	0.793	0.207
8	0.207	0.793
9	0.724	0.276
10	0.724	0.276

E =

	1	2	3	4	5	6	7	8	9	10
1	4.07	2.97	2.79	1.24	4.76	2.28	2.07	0.62	3.17	1.59
2	2.97	3.52	3.1	1.38	3.62	1.86	1.97	0.69	2.41	1.21
3	1.86	2.07	3.41	1.52	2.48	1.45	1.86	0.75	1.66	0.82
4	1.24	1.38	2.28	2.34	1.66	0.96	1.24	1.17	1.1	0.55
5	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.55	3.93	1.97
6	2.28	1.86	2.17	0.96	4.03	3.1	2.28	0.48	2.69	1.34
7	1.38	1.31	1.86	0.82	2.17	1.52	2.38	0.41	1.45	0.72
8	0.62	0.69	1.14	1.17	0.82	0.48	0.62	1.59	0.55	0.27
9	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.55	5.93	2.97
10	3.17	2.41	2.48	1.1	5.9	2.69	2.17	0.55	5.93	3.97

____ 5. If the rat begins at location #4, the probability that the rat finds the food at location #12 first (before the food at #11) is (nearest to)

a. 50%
b. 60%
c. 70%
d. 80%
e. 90%
f. 95%

____ 6. The expected number of times that the rat returns to his initial location (#4) before finding food is

a. less than 4
b. between 4 and 8
c. between 8 and 15
d. between 15 and 25
e. between 25 and 50
f. more than 50

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____ 7. If the rat, starting at location #4, manages to reach location #8 before finding food, the probability that he first finds the food at location #12 instead of #11 is (nearest to)

a. 50% b. 60% c. 70%
d. 80% e. 90% f. 95%

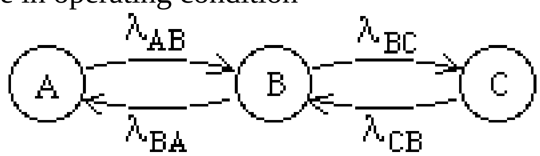
____ 8. The number of *transient* states in this second Markov chain model is

a. 0 b. 2 c. 9
d. 10 e. 12 f. none of the above

PART FIVE

A single repairman is responsible for maintaining two machines in working condition. When both machines are in good condition, they both operate simultaneously. However, a machine operates for an average of only 1 hour, when it fails and repair begins. Repair of a machine requires an average of 30 minutes. (Only one machine at a time can be repaired.) Define a continuous-time Markov chain with states:

A. Both machines have failed, with repair in progress on one machine
B. One machine is operable, and the other is being repaired
C. Both machines are in operating condition



____ 1. In this model, the probability distribution of the time required to repair a machine is assumed to be:

a. Uniform b. Markov c. Poisson
d. Normal e. exponential f. None of the above

____ 2. The transition rate λ_{BA} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{BA}$ e. λ_{BA} f. None of the above

____ 3. The transition rate λ_{CB} is

a. 0.5/hour b. 1/hour c. 2/hour
d. $-\lambda_{BC}$ e. λ_{BC} f. None of the above

____ 4. The repair time will be less than t with probability

a. e^{-2t} b. $1 - e^{-2t}$ c. $1- e^{2t}$
d. $1- 2e^{-t}$ e. $2e^{t}$ f. None of the above

____ 5. The steady-state probability distribution must satisfy the equation(s) *(choose one or more):*

a. $\pi_A + \pi_B + \pi_C = 1$
b. $\lambda_{AB}\pi_A = \lambda_{BA}\pi_B$
c. $\lambda_{BA}\pi_A = \lambda_{AB}\pi_B$
d. $\pi_A = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$
e. $\lambda_{BC}\pi_B = \lambda_{CB}\pi_C$
f. $\pi_B = \lambda_{AB}\pi_A + (\lambda_{BA} + \lambda_{BC})\pi_B + \lambda_{CB}\pi_C$

____ 6. The average utilization of *each* of these machines in steady state (i.e., the fraction of maximum capacity at which they will operate), is:

a. $\pi_B+\pi_C$ b. $(\pi_B+\pi_C)/2$ c. $\pi_B+2\pi_C$
d. $\pi_A+\pi_B+\pi_C$ e. $2(\pi_B+\pi_C)$ f. $(\pi_B+2\pi_C)/2$

7. Is this continuous-time Markov chain a "birth/death" process? (*Circle:* Yes No)

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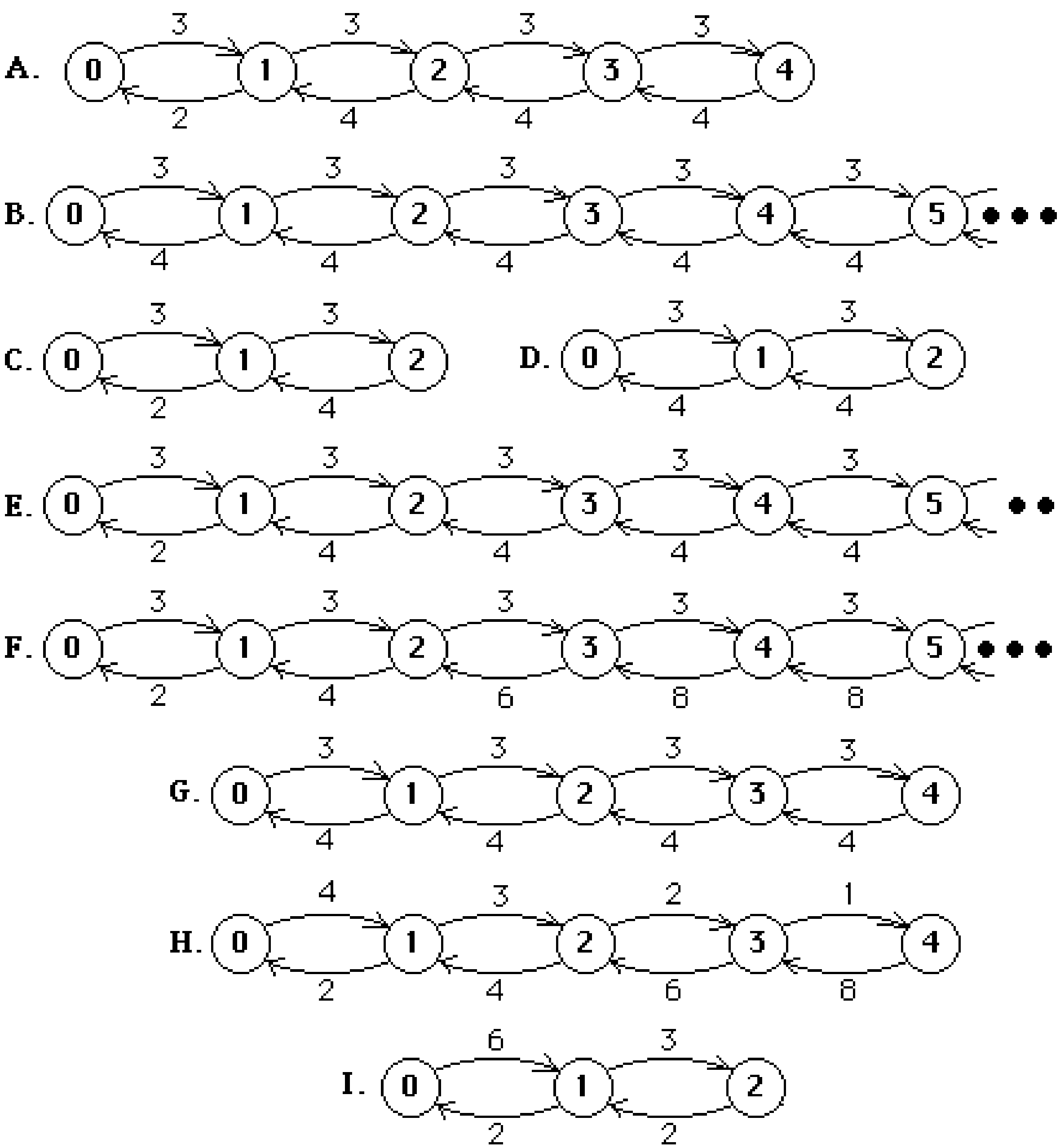
PART SIX

For each class of queueing system below, find a matching birth/death model which might belong to that class. If none of the b/d models fit the classification, write *"none"*

Queue	B/D Model	Queue	B/D Model
M/M/2	____	M/M/2/4/4	____
M/M/1/2	____	M/M/2/2	____
M/M/4	____	M/M/1	____

Birth/Death Models



AT LAST, THE END!