57:022 Principles of Design II
Final Examination - Spring 1995

Part:	I	II	III	IV	V	VI	VII	Total
Possible Pts:								
Your score:								

PART ONE

In the game of "craps", we roll a pair of six-sided dice.

- On the *first* throw, if we roll a 7 or an 11, we win right away.
- If, on the *first* throw, we roll a 2, 3, or 12, we lose right away.
- If, however, we first roll a 4, 5, 6, 8, 9, or 10, then we keep rolling the dice until either
  - we get a 7 (in which case we lose), or
  - we get again the total which was rolled on the first throw (in which case we win)

Model a crap game as a Markov chain:

state 1 : initial state, before rolling the first time,
state 2 : a 4 or 10 was rolled on the first throw
state 3 : a 5 or 9 was rolled on the first throw
state 4: a 6 or 8 was rolled on the first throw
state 5 : win,
state 6 : loss.

The transition probability matrix is :

$$P = \begin{bmatrix} 0 & 6/36 & 8/36 & 10/36 & 8/36 & 4/36 \\ 0 & 27/36 & 0 & 0 & 3/36 & 6/36 \\ 0 & 0 & 26/36 & 0 & 4/36 & 6/36 \\ 0 & 0 & 0 & 25/36 & 5/36 & 6/36 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

A = Absorbing Probabilities

from	5	6
1	0.49292929	0.50707071
2	0.33333333	0.66666667
3	0.4	0.6
4	0.45454545	0.54545455

E = Expected NO. Visits to Transient States

from	1	2	3	4
1	1	0.6666667	0.8	0.90909091
2	0	4	0	0
3	0	0	3.6	0
4	0	0	0	3.2727273

Assume that you are the person who is to roll the dice.

___ 1. What is your probability of winning? *(choose nearest value)*

a. 0 b. 33% c. 49%
d. 50% e. 51% f. 66%

___ 2. If your first roll is a "5", what is your probability of winning? *(choose nearest value)*

a. 0 b. 20% c. 40%
d. 50% e. 60% f. 80%

___ 3. If the first roll of the dice is a "5", how many *additional* rolls are expected in the game? *(choose nearest value)*

a. one	b. two	c. three
d. four	e. five	f. six

___ 4. Which value of the first roll has the highest probability of winning?

a. four	b. five	c. six
d. eight	e. nine	f. ten

___ 5. What is the expected number of rolls of the dice in a crap game? *(choose nearest value)*

a. one	b. two	c. three
d. four	e. five	f. six

___ 6. Which states are transient?

a. 1	b. 2,3,4	c. 1,2,3,4
d. 5,6	e. 1,2,3,4,5,6	f. *NOTA*

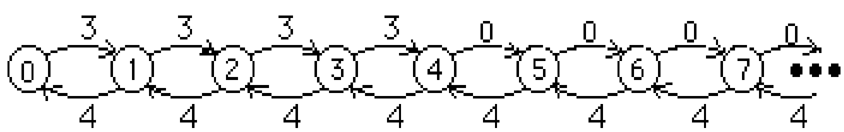
___ 7. Which states are recurrent?

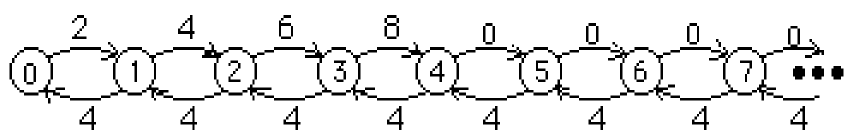
a. 1	b. 2,3,4	c. 1,2,3,4
d. 5,6	e. 1,2,3,4,5,6	f. *NOTA*

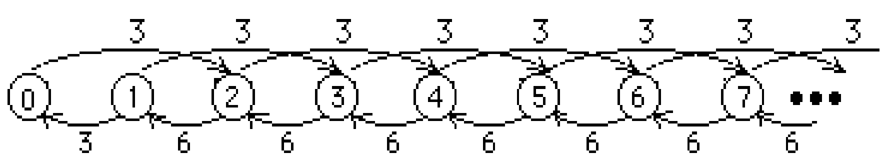
PART TWO

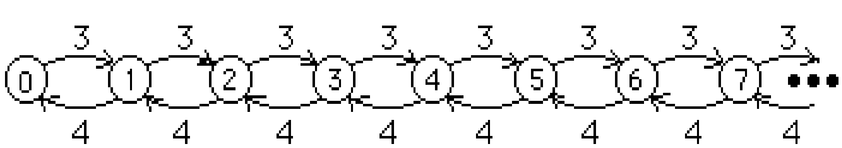
Birth-Death Processes For each birth-death model of a queue in diagrams (1) through (8) below, indicate the correct Kendall's classification from among the following choices. (Note that some classifications might not be matched with any birth-death diagram, while others might be matched to more than one!)

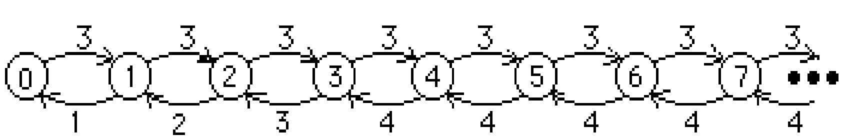
a. M/M/1	d. M/M/2	g. M/M/1/4	j. M/M/2/3
b. M/M/4	e. M/M/2/4	h. M/M/2/4/4	k. M/M/2/2/4
c. M/M/1/4/4	f. M/M/4/2	i. M/M/4/4	l. *None of Above*

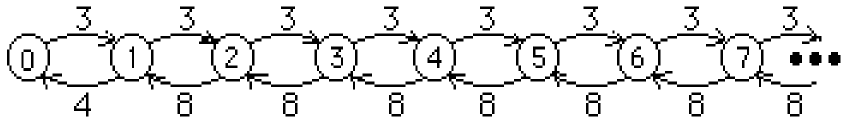
____1 

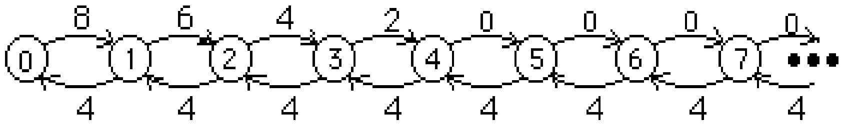
____2 

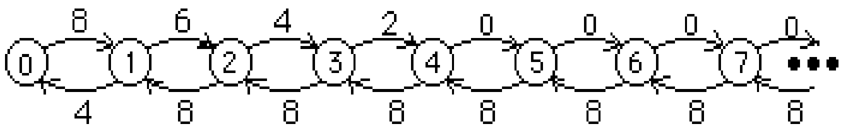
____3 

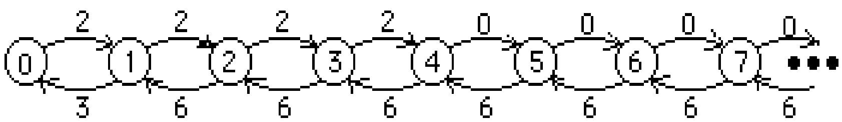
____4 

____5 

____6 

____7 

____8 

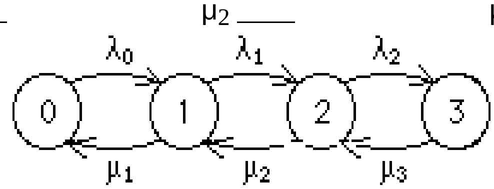
____9 

PART THREE

A machine operator has the task of keeping three machines running. Each machine runs for an average of one hour before it becomes jammed or otherwise needs the operator's attention. He then spends an average of ten minutes restoring the machine to running condition. Define a continuous-time Markov chain, with the state of the system being *the number of machines which are **not** running.*

1. True or False (circle): This Markov chain is a birth/death process.
2. Specify the letter for each of the transition rates:

λ_0 ____	λ_1 ____	λ_2 ____
μ_1 ____	μ_2 ____	μ_3 ____



a. 1/hr	b. 2/hr	c. 3/hr
d. 4/hr	e. 6/hr	f. 8/hr
g. 12/hr	h. 18/hr	i. None of the above

____3. Which equation is used to compute the steady-state probability π_0?

(a.) $\pi_0 = \left(\frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2} + \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3}\right)^{-1}$

(b.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0\lambda_1}{\mu_1\mu_2} + \frac{\lambda_0\lambda_1\lambda_2}{\mu_1\mu_2\mu_3}$

(c.) $\frac{1}{\pi_0} = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu_3}$

(d.) $\pi_0 = \dfrac{1 + \frac{\lambda_0}{\lambda_1} + \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_3}}{1 + \frac{\mu_0}{\mu_1} + \frac{\mu_1}{\mu_2} + \frac{\mu_2}{\mu_3}}$

(e.) $\pi_0 = 1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu_3}$

(f.) $\pi_0 = 1 + \left(\frac{\lambda_0}{\mu_1}\right)^1 + \left(\frac{\lambda_1}{\mu_2}\right)^2 + \left(\frac{\lambda_2}{\mu_3}\right)^3$

(g.) $\frac{1}{\pi_0} = 1 + \left(\frac{\lambda_0}{\mu_1}\right)^1 + \left(\frac{\lambda_1}{\mu_2}\right)^2 + \left(\frac{\lambda_2}{\mu_3}\right)^3$

(h.) None of the above

____4. What is the relationship between π_0 and π_1 *for this particular system?*

a. $\pi_1 = \pi_0$ b. $\pi_1 = 2\pi_0$ c. $\pi_1 = 6\pi_0$

d. $\pi_1 = \frac{1}{2}\pi_0$ e. $\pi_1 = \frac{1}{6}\pi_0$ f. None of the above

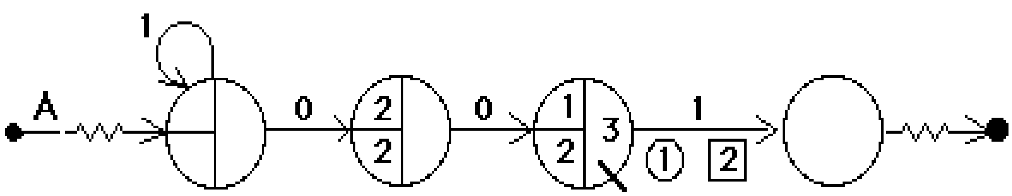
____5. If the average number of machines *not* running (i.e., the number in the queueing system of the operator) is approximately 0.5 and the average number of machine jams per hour is 2.5 (i.e., approximately one every 0.4 hr., what is the average turnaround time (waiting time plus operator's service time) to restore a machine to running condition? *(Choose nearest answer.)*

a. 0.1 hour b. 0.2 hour c. 0.3 hour

d. 0.4 hour e. 0.5 hour f. 0.6 hour

PART FOUR

Note that
- all activity durations in the SLAM networks below are *constants* , and none are random!
- first entity is created at time=0



___ 1. In Network **A**, *before* the first created entity, there are how many entities are already in the network?

a. none b. one c. two d. three

e. four f. five g. can't be determined h. *NOTA*

___ 2. In Network **A**, the first entity to *leave the system* (& is terminated) leaves at time =

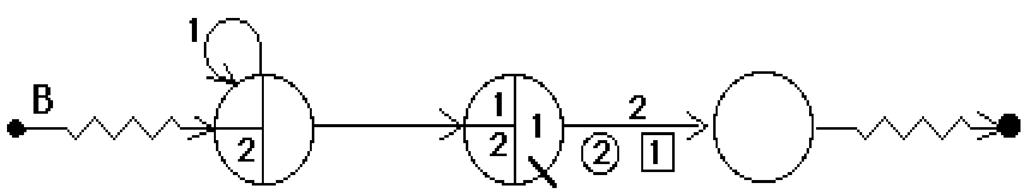
a. 0 b. 1 c. 2 d. 3 e. 4

f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 3. In Network **A**, the first created entity *enters the queue* at time =

a. 0 b. 1 c. 2 d. 3 e. 4

f. 5 g. 6 h. 7 i. 8 j. *NOTA*



___ 4. In Network **B**, the first created entity *begins being served* at time =

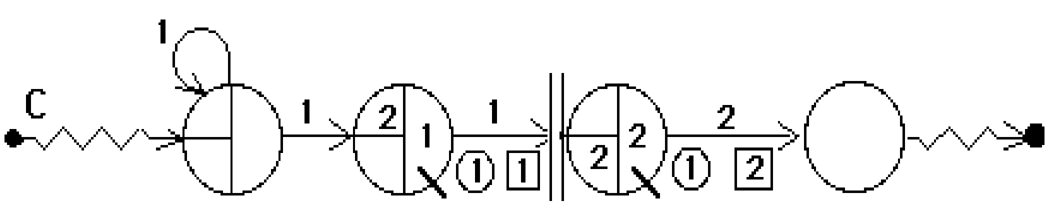
a. 0 b. 1 c. 2 d. 3 e. 4

f. 5 g. 6 h. 7 i. 8 j. *NOTA*

___ 5. In Network **B**, the total number of entities which will *leave* the system is

a. 0 b. 1 c. 2 d. 3 e. 4

f. 5 g. 6 h. 7 i. 8 j. *NOTA*

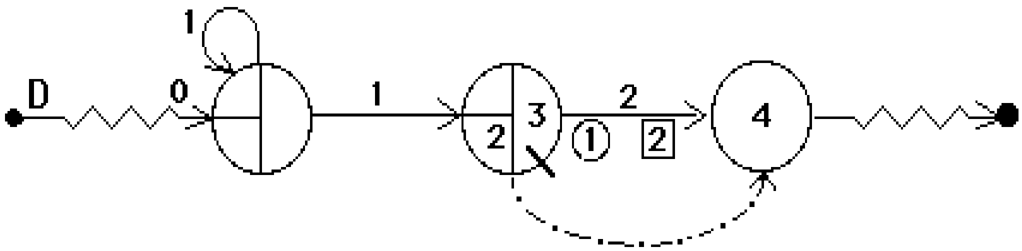


___ 6. In Network **C**, the total number of servers is

a. 0   b. 1   c. 2   d. 3   e. 4
f. 5   g. 6   h. 7   i. 8   j. *NOTA*

___ 7. In Network **C**, the total number of queues is

a. 0   b. 1   c. 2   d. 3   e. 4
f. 5   g. 6   h. 7   i. 8   j. *NOTA*



___ 8. In Network **D**, the first entity which *cannot enter the queue* will arrive at the queue at time =

a. 0   b. 1   c. 2   d. 3   e. 4
f. 5   g. 6   h. 7   i. 8   j. *NOTA*

___ 9. In Network **D**, the simulation will terminate at time=

a. 0   b. 1   c. 2   d. 3   e. 4
f. 5   g. 6   h. 7   i. 8   j. *NOTA*

___ 10. In Network **D**, the first entity to leave the system will leave at time =

a. 0   b. 1   c. 2   d. 3   e. 4
f. 5   g. 6   h. 7   i. 8   j. *NOTA*

___ 11. Of the four SLAM networks, the network in which "blocking" may occur is

a. A   b. B   c. both C & D   d. both A & B
e. C   f. D   g. both B & D   h. *NOTA*

___ 12. Of the four SLAM networks, the network in which "balking" may occur is

a. A   b. B   c. both C & D   d. both A & B
e. C   f. D   g. both B & D   h. *NOTA*

PART FIVE

___ 1. The "Cumulative Distribution Function" (CDF) of a random variable X is defined to be

a. f(x) = P{x | X}
b. f(x) = P{x}
c. F(x) = P{X=x}
d. F(x) = P{X≥x}
e. f(x) = P{X|x}
f. F(x) = P{X≤x}

___ 2. The "Reliability" function of system with random lifetime X is defined to be

a. R(x) = P{x | X}
b. R(x) = P{x}
c. R(x) = P{X=x}
d. R(x) = P{X≥x}
e. R(x) = P{X|x}
f. R(x) = P{X≤x}

___ 3. Suppose that a steel chain is made up of many links. The strength of the chain is, of course, the strength of its weakest link, since the chain fails whenever any link fails. A reasonable assumption for the probability distribution for the strength of the chain is

a. Normal distribution
b. Exponential distribution
c. Uniform distribution
d. Weibull distribution
e. Gumbel distribution
f. *None of the above*

___ 4. The CDF, i.e., F(x), of the Gumbel distribution with parameters a and u is

a. $e^{\left(-e^{-\alpha(x-u)}\right)}$
b. $1 - e^{-\alpha(x-u)}$
d. $u - \dfrac{\ln(-\ln x)}{\alpha}$
e. $1 - \dfrac{u \ln(-\ln x)}{\alpha}$

c. $1 - e^{-(x/u)^{\alpha}}$

f. *None of the above*

___ 5. The CDF, i.e., F(x), of the Weibull distribution with parameters k and u is

a. $e^{\left(- e^{- k(x-u)}\right)}$

b. $1 - e^{- k (x-u)}$

c. $1 - e^{- (x/u)^{k}}$

d. $u - \frac{\ln(-\ln x)}{k}$

e. $1 - \frac{u \ln(-\ln x)}{k}$

f. *None of the above*

___ 6. The "coefficient of variation" of a probability distribution with mean μ and variance σ^2, is

a. $\sqrt{\mu^2 + \sigma^2}$

b. μ^2/σ^2

c. σ^2/μ^2

d. σ/μ

e. μ/σ

f. none of the above

___ 7. The "Gamma" function Γ is related to the factorial function for integers by

a. $\Gamma(1\text{-}k) = k!$

b. $\Gamma(1+k) = k!$

c. $\Gamma(k) = k!$

d. $\Gamma(k) = (k+1)!$

e. $\Gamma\left(1+1/k\right) = k!$

f. *None of the above*

___ 8. Given a set of data points (x_i,y_i), i=1,2,...n, "linear regression" is a method for determining a relationship y = f(x) which

a. minimizes the maximum error max $\{y_i - f(x_i)\}$

b. minimizes the sum of the absolution values of the errors: $\sum_i |y_i - f(x_i)|$

c. minimizes the sum of the errors $\sum_i y_i - f(x_i)$

d. minimizes the sum of the squares of the errors: $\sum_i (y_i - f(x_i))^2$

e. *None of the above*

PART SIX

Match SLAM diagram (**A** through **I**) & Queue Classification. If "none", indicate "N"

____ M/M/1

____ M/M/1/2

____ M/G/1

____ M/M/4

____ M/M/2/4

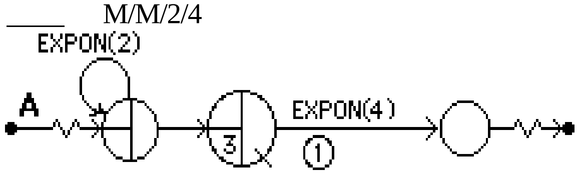
____ M/M/2

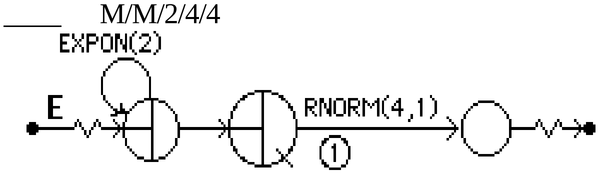
____ M/M/1/2/2

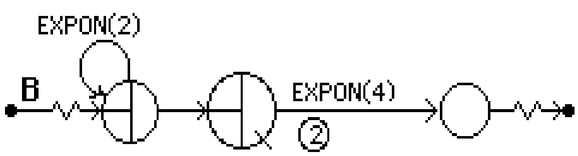
____ M/M/1/4

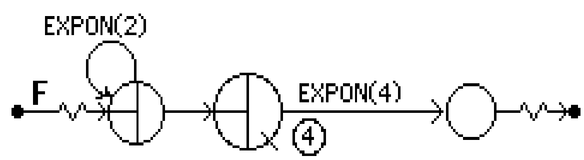
____ M/M/2/2

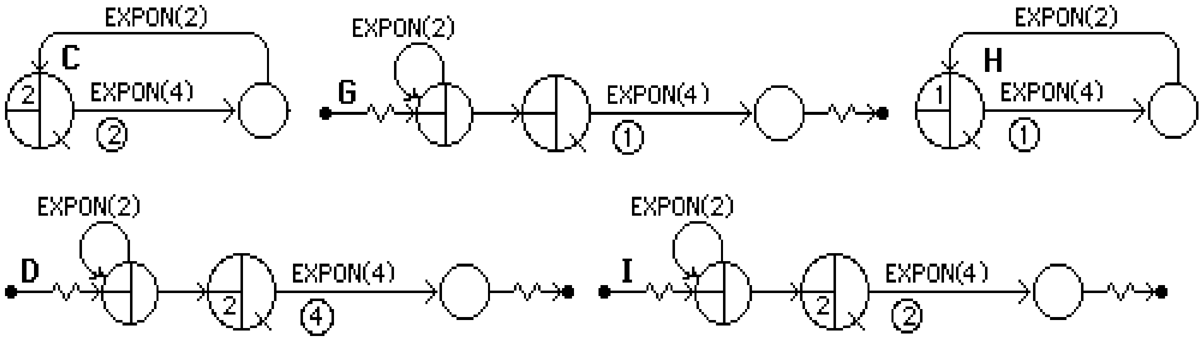
____ M/M/2/4/4







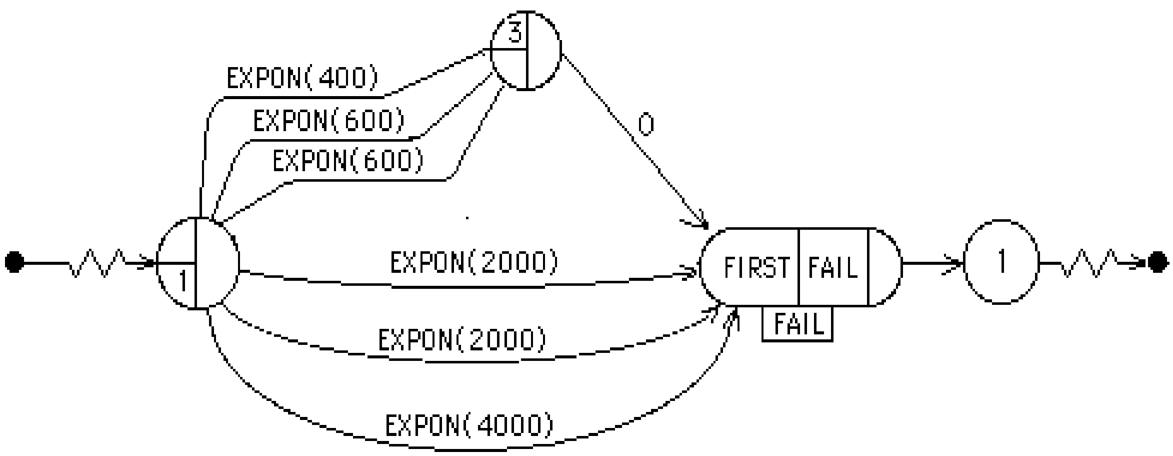




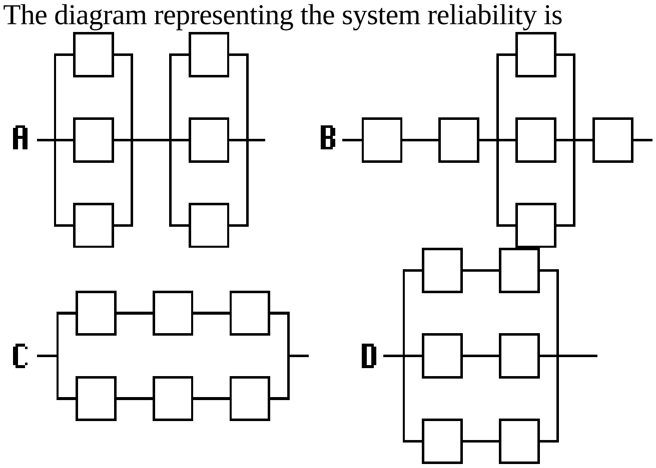
PART SEVEN

The following SLAM network was used to simulate a system consisting of six components (where the time units are days). Refer to the output for five hundred runs to find (or estimate) the quantities below:

____ 1. The average lifetime of the system.
____ 2. The probability that the system survives 800 days.
____ 3. The time which the first failure occurred.
____ 4. The reliability of the system if its designed lifetime is specified to be 60 days.



___ 5. The diagram representing the system reliability is



STATISTICS FOR VARIABLES BASED ON OBSERVATION

```
                  MEAN    STANDARD  COEFF. OF  MINIMUM   MAXIMUM  NO.OF
                  VALUE   DEVIATION VARIATION   VALUE      VALUE    OBS

FAIL TIME       0.489E+03 0.381E+03 0.780E+00 0.209E+00 0.217E+04   500

                          HISTOGRAM NUMBER 1**
                             FAIL TIME

OBS  RELA   UPPER
FREQ FREQ  CELL LIM 0        20        40        60        80       100
                    +    +    +    +    +    +    +    +    +    +    +
 32 0.064 0.500E+02 +***                                              +
 34 0.068 0.125E+03 +***   C                                          +
 65 0.130 0.200E+03 +*******     C                                    +
 44 0.088 0.275E+03 +****             C                               +
 41 0.082 0.350E+03 +****                 C                           +
 44 0.088 0.425E+03 +****                     C                       +
 49 0.098 0.500E+03 +*****                         C                  +
 30 0.060 0.575E+03 +***                              C               +
 30 0.060 0.650E+03 +***                                 C            +
 20 0.040 0.725E+03 +**                                    C          +
 19 0.038 0.800E+03 +**                                      C        +
 12 0.024 0.875E+03 +*                                        C       +
 15 0.030 0.950E+03 +**                                        C      +
 14 0.028 0.103E+04 +*                                           C    +
  7 0.014 0.110E+04 +*                                            C   +
  8 0.016 0.118E+04 +*                                            C   +
  7 0.014 0.125E+04 +*                                             C  +
  9 0.018 0.133E+04 +*                                              C +
  5 0.010 0.140E+04 +*                                              C +
  4 0.008 0.148E+04 +                                                C+
  3 0.006 0.155E+04 +                                                C+
  8 0.016    INF    +*                                                C
---                 +    +    +    +    +    +    +    +    +    +    +
500                 0        20        40        60        80       100
```

Using the mean and standard deviation from the simulation output, the Weibull parameters U=528.961 and k=1.29394 are determined.

____6. According to this result, the failure rate is

a. zero   b. increasing   c. decreasing

d. constant   e. cannot be determined from the information given

Based upon the cumulative probability function F(t) for this Weibull distribution, the following probabilities and expected values for each cell were calculated, where t_i is the upper limit of the cell. The cells at the upper end were grouped, as indicated by the horizontal lines, so as to obtain a more even distribution of observations. Next we calculate for each cell (or group of cells) the square of the deviation of O from E, and divide by E, and then sum to obtain the chi-square statistic in the table on the right:

i	t_i	p_i	E_i	O_i
1	50	0.04615	23.07647	32
2	125	0.09713	48.56294	34
3	200	0.10402	52.00927	65
4	275	0.10151	50.75408	44
5	350	0.09467	47.33290	41
6	425	0.08577	42.88484	44
7	500	0.07610	38.05003	49
8	575	0.06643	33.21305	30
9	650	0.05721	28.60495	30
10	725	0.04871	24.35746	20
11	800	0.04107	20.53625	19
12	875	0.03433	17.16303	12
13	950	0.02846	14.23084	15
14	1025	0.02343	11.71482	14
15	1100	0.01916	9.57989	7
16	1175	0.01557	7.78599	8
17	1250	0.01258	6.29178	7
18	1325	0.01011	5.05699	9
19	1400	0.00809	4.04392	5
20	1475	0.00644	3.21825	4
21	1550	0.00510	2.54946	3

t	E	O	D
50	23.07647	32	3.45068
125	48.56294	34	4.36710
200	52.00927	65	3.24479
275	50.75408	44	0.89880
350	47.33290	41	0.84731
425	42.88484	44	0.02900
500	38.05003	49	3.15116
575	33.21305	30	0.31083
650	28.60495	30	0.06804
800	44.89371	39	0.77373
1025	43.10869	41	0.10315
∞	47.50906	51	0.25651
SUM	500	500	17.5011

____ 7. The number of degrees of freedom for the chi-square goodness-of-fit test is

a. 10 b. 11 c. 12 d. 13

e. 14 f. 15 g. 16 h. *NOTA*

The chi-square probability table indicates that with this # of degrees of freedom , if the system lifetime does have the Weibull distribution with the parameters above, P{D>16.919} is α=5%.

8. Based upon this value, should we accept for the system lifetime the Weibull distribution model with the parameters U=528.961 and k=1.29394? (*circle*: Yes / No / Maybe)