Name ______________________________

57:022 Principles of Design II
Final Exam -- May 8, 2000

Part	I	II	III	IV	V	VI	VII	Total
Your score:								
Possible	7	16	12	18	7	12	8	80

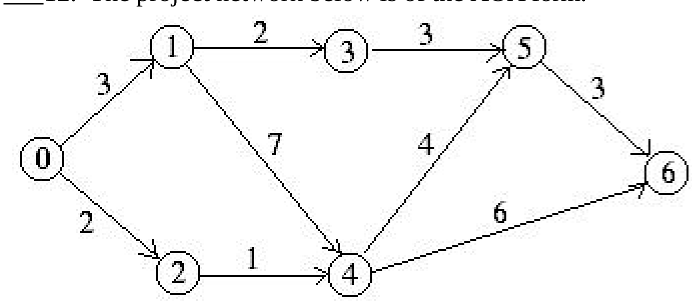
Topics:
Part I: Probability distributions
Part II. Project scheduling
Part III. Weibull model of reliability
Part IV. System reliability
Part V. Reliability estimates from life-test data
Part VI. Birth-death queues
Part VII. Networks of queues

Part I. Probability Distributions *For each probability distribution below, indicate by "C" or "D" whether the corresponding random variable is Continuous or Discrete.*

___1. Uniform
___2. Beta
___3. Geometric
___4. Binomial
___5. Poisson
___6. Weibull
___7. Triangular
___8. Pascal (negative binomial)
___9. Exponential
___10. Normal
___11. Chi-square
___12. Bernouilli
___13. Erlang-k with k>1
___14. Gumbel

Part II. Project Scheduling

___1. An ARENA model of a project is more similar to an **AOA** network model than an **AON** network model of the project.
___2. The quantity ET(i) [i.e. earliest time] for each node i is determined by a forward pass through the network.
___3. If an activity is represented by an arrow from node i to node j, then LS (latest start time) for that activity is LT(i).
___4. If an activity is represented by an arrow from node i to node j, then EF (early finish time) for that activity is ET(j).
___5. If an activity is represented by an arrow from node i to node j, then that activity has zero "float" or "slack" if and only if ET(i)=LT(i).
___6. An activity is critical if and only if its total float ("slack") is zero.
___7. A "dummy" activity cannot be critical.
___8. The mean value of the duration of activity is equal to its most likely value, if the probability distribution is triangular.
___9. PERT assumes that each activity's duration has a Normal distribution.
___10. PERT assumes that the project duration has a Normal distribution.
___11. Except perhaps for "begin" and "end" activities, "dummy" activities are unnecessary in the "Activity-on-Node" representation of a project.
___12. The project network below is of the AOA form.



13. An ARENA model to simulate the above project will require the following numbers of nodes of each type:
_____ ARRIVE nodes
_____ DUPLICATE nodes
_____ DELAY nodes
_____ BATCH nodes
_____ DEPART nodes

14. The critical path in the above project consists of ____ activities and is length _____ .

Name ______________________________

Part III. Weibull Model of Reliability. An electronic device is made up of a large number of components. Every component is essential, so that the device will fail when the first component fails. The lifetime of each individual component is random, but its probability distribution is unknown. The manufacturer, who has provided a 90-day warranty on this device, has decided to use the Weibull reliability model.

For each statement, indicate "+" for true, "o" for false:

___ 1. A k>1 indicates an increasing failure rate, and k<1 indicates a decreasing failure rate.

___ 2. We assume that the lifetime of a component has a Weibull distribution.

___ 3. The Weibull density function, i.e., f(t), gives, for each component, the probability that at time t it has already failed.

___ 4. The exponential distribution is a special case of the Weibull distribution, with failure rate zero.

___ 5. The sum of the CDF (cumulative distribution function) F(t) and the Reliability function R(t), i.e. F(t) + R(t), is always equal to 1 if the Weibull probability model is assumed.

___ 6. If 10 of the devices are installed in a manufacturing system, the number still functioning after 100 days has a Weibull distribution.

It has been determined that *average* lifetime of the device is 300 days and the *standard deviation* is 200 days.

___ 7. Based upon the above information and the table(s) below, the value of the "shape" parameter (k) of the probability dist'n is approximately *(choose nearest value).*

a. 0.1 b. 0.5 c. 1.0 d. 1.5 e. 2.0 f. 2.5

___ 8. The value of the "location" parameter (u) of the probability dist'n is approximately *(choose nearest value).*

a. 100 b. 200 c. 300 d. 400 e. 500 f. ≥600

___ 9. The failure rate is

a. increasing b. decreasing c. constant d. cannot be determined

___ 10. The percent of the units which are expected to fail during the 90-day warranty period is *(choose nearest value):*

a. 1% b. 3% c. 5% d. 7% e. 9%
f. 11% g. 13% h. 15% i. 17% j. 19%

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Table 1: $\Gamma\left(1+\frac{1}{k}\right)$ *(For example, if k=0.5 then G(1 + 1/k) = 2.)*

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	362880.	120.000	9.26053	3.32335	2.00000	1.50458	1.26582	1.13300	1.05218
1	1.00000	0.96491	0.94066	0.92358	0.91142	0.90275	0.89657	0.89224	0.88929	0.88736
2	0.88623	0.88569	0.88562	0.88591	0.88648	0.88726	0.88821	0.88928	0.89045	0.89169
3	0.89298	0.89431	0.89565	0.89702	0.89838	0.89975	0.90111	0.90245	0.90379	0.90510
4	0.90640	0.90768	0.90894	0.91017	0.91138	0.91257	0.91374	0.91488	0.91600	0.91710
5	0.91817	0.91922	0.92025	0.92125	0.92224	0.92320	0.92414	0.92507	0.92597	0.92685

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Table 2: Coefficient of variation $\frac{\sigma}{\mu}$ of the Weibull distribution, as a function of k alone *(For example, s/m = 0.1968 implies k=5.9.)*

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	---	---	15.84298	5.40769	3.14086	2.23607	1.75807	1.46242	1.26051	1.11303
1	1.00000	0.91022	0.83690	0.77572	0.72375	0.67897	0.63991	0.60548	0.57487	0.54745
2	0.52272	0.50029	0.47983	0.46108	0.44384	0.42791	0.41314	0.39942	0.38662	0.37466
3	0.36345	0.35292	0.34300	0.33365	0.32482	0.31646	0.30853	0.30101	0.29385	0.28704
4	0.28054	0.27435	0.26842	0.26276	0.25733	0.25213	0.24714	0.24235	0.23775	0.23332
5	0.22905	0.22495	0.22099	0.21717	0.21348	0.20991	0.20647	0.20314	0.19992	0.19680

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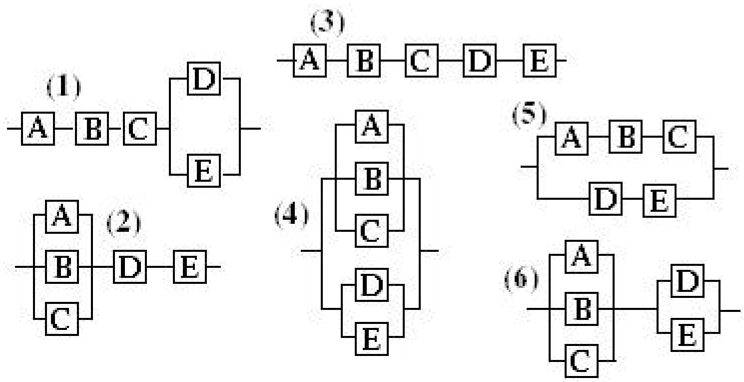
Name ______________________________

Part IV. System Reliability A system consists of five components (A,B,C,D, &E). The probability that each component *fails during the first year* of operation is 30% for A, B, and C, and 40% for D and E. For each alternative (a) and (b), indicate:

- the number of the reliability diagram below which represents the system.
- the computation of the 1-year reliability (i.e., survival probability)
- the ARENA model which would simulate the system's lifetime

Diagram	Reliability	ARENA	
___	___	___	1. The system requires that all of A, B, & C function, and that either D or E function.
___	___	___	2. The system will fail if all of A, B, and C fails or if both D and E fail.

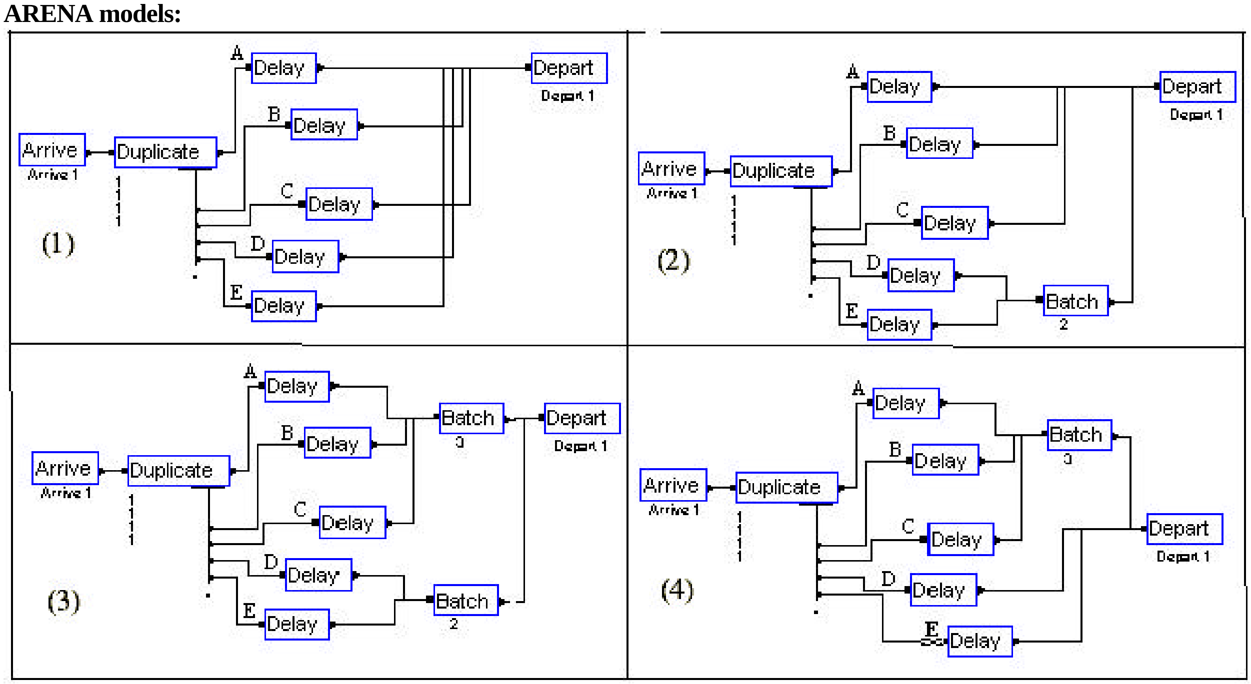
Diagrams:



Reliabilities:

1. $(0.7)^3(0.6)^2 = 12.3\%$
2. $1- [1-(0.7)^3] [1-(0.6)^2] = 57.9\%$
3. $(.7)^3(1-[.4]^2) = 28.8\%$
4. $1 - (0.3)^3(0.4)^2 = 94.5\%$
5. $[1-(0.3)^3] [1-(0.4)^2] = 81.7\%$
6. $1 - [1-(0.3)^3] [1- (0.4)^2] = 18.3\%$
7. $[1-(0.3)^3] (0.6)^2 = 35.0\%$
8. *None of the above*

ARENA models:



(5) *None of the above*

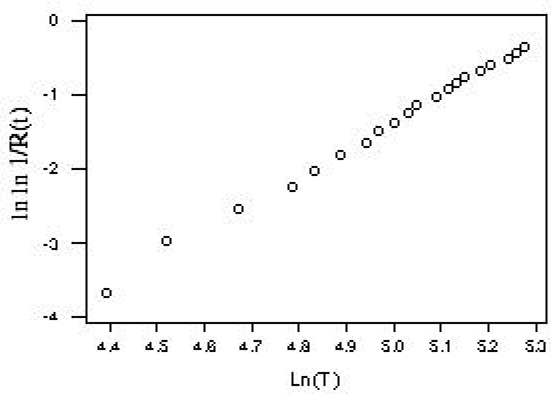
Part V. Reliability from Life-Test data

Name ______________________________

Suppose that your company wishes to estimate the reliability of an electric motor. Two hundred units are tested simultaneously, and the time(in days) of failures is recorded until 200 days have passed. The table below gives the failure times of the 5^{th}, 10^{th}, 15^{th}, etc. motor. After the 95^{th} motor had failed, it was necessary to interrupt the test for lack of time. It is expected that a Weibull reliability model will provide good results.

NF	t	R(t)	ln(t)	ln ln 1/R(t)	NF	t	R(t)	ln(t)	ln ln 1/R(t)
5	81	.975	4.39	-3.67	50	153	0.75	5.03	-1.25
10	92	.95	4.52	-2.97	55	156	0.725	5.05	-1.13
15	107	.925	4.67	-2.55	60	163	0.75	5.09	-1.03
20	120	.9	4.78	-2.25	65	166	0.674	5.12	-0.93
25	125	.875	4.83	-2.01	70	170	0.65	5.13	-0.84
30	132	.85	4.88	-1.82	75	172	0.625	5.15	-0.76
35	140	.825	4.94	-1.65	80	178	0.6	5.18	-0.67
40	144	.8	4.97	-1.5	85	182	0.575	5.21	-0.59
45	148	.775	5.00	-1.37	90	190	0.55	5.25	-0.51
50	156	.75	5.03	-1.25	95	193	0.525	5.26	-0.44

Then Ln Ln 1/R(t) was plotted vs Ln(t):



The line which most nearly fits this data has slope 3.7 and vertical-intercept –20.
The parameters of the Weibull distribution are next estimated:

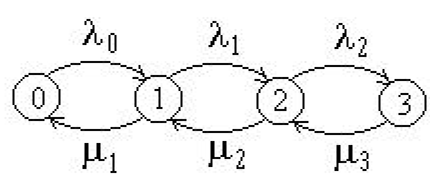
___ 1. the shape parameter (k) is *(choose nearest value):*
a. 1 b. 2 c. 3 d. 4 e. 5 f. 6 g. 7 h. 8 i. 9 j. 10

___ 2. the scale parameter (u) is *(choose nearest value):*
a. 100 b. 200 c. 300 d. 400 e. 500 f. 600 g. 700 h. 800 i. 900 j. 1000

___ 3. If the test had been continued until all of the motors had failed, the mean failure time (in days) would have been (choose nearest value):
a. 100 b. 200 c. 300 d. 400 e. 500 f. 600 g. 700 h. 800 i. 900 j. 1000

Part VI. Birth-death Processes

Customers arrive at the rate of 1/hour at a queue with a single server and a capacity of 2 customers (plus the one being served.) The average time to serve a customer is 30 minutes, with exponential distribution.



____1. The *utilization*, i.e., steady-state probability that the server is busy, is $1 - \pi_0$ = *(choose nearest value):*
a. ≤15% b. 30% c. 45% d. 50%
e. 55% 5. 60% f. 70% g. ≥80%

____2. If the queue had infinite capacity, the utilization of the server would be *(choose nearest value):*
a. ≤15% b. 30% c. 45% d. 50%
e. 55% 5. 60% f. 70% g. ≥80%

Suppose that the average arrival rate is 0.93/hour and the average number of customers in the system (including the customer being served) is 0.73.

____3. From this we can deduce that the average time that each customer *waits* before its service begins is (choose nearest value):

a. 10 min. b. 20 min. c. 30 min. d. 40 min.
e. 50 min. 5. 60 min. f. 70 min. g. 80 min.

Suppose that customers may get impatient of waiting to be served, and that the average time that they are willing to wait is 15 minutes.

____4. The "death" rate μ_2 in state 2 is now *(choose nearest value):*

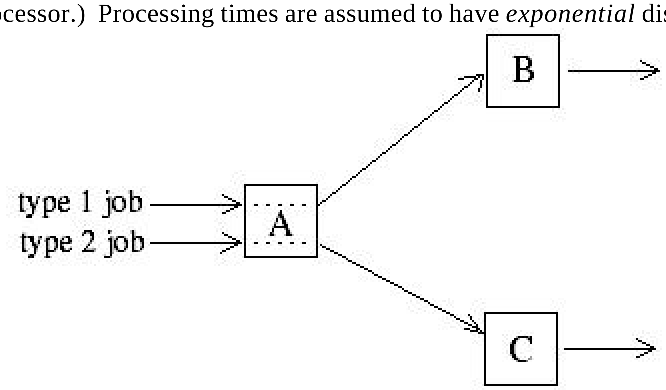
a. 1/hr b. 2/hr c. 3/hr d. 4/hr e. 5/hr
f. 6/hr g. 7/hr h. 8/hr i. 9/hr j. 10/hr

____5. The "death" rate μ_3 in state 3 is now *(choose nearest value):*

a. 1/hr b. 2/hr c. 3/hr d. 4/hr e. 5/hr
f. 6/hr g. 7/hr h. 8/hr i. 9/hr j. 10/hr

Part VII. Queueing networks

Consider a system which processes two types of jobs. Type 1 jobs arrive on average twice per hour, and type 2 jobs arrive on average once per hour. Both jobs arrive first at Station A, where there are three processors. Type 1 jobs require an average of one hour of processing and are then are routed to Station B, where there are two processors. An average of 15 minutes of processing time is required at Station B. Type 2 jobs require an average of 30 minutes first at Station A and then 30 minutes at Station C (which has a single processor.) Processing times are assumed to have *exponential* distributions.



The software system RAQS (Rapid Analysis of Queueing Systems) yields the output below. Use it to answer the following questions:

___ 1. Which type of job spends more time in the system?

a. Type 1 b. Type 2 c. No difference d. Cannot be determined

___ 2. What fraction of the day is a processor at Station A busy? (Choose nearest value)

a. ≤40% b. 50% c. 60% d. 70%
e. 80% f. 90% g. 100% h. Cannot be determined

Suppose that the processing time for job type 1 at station A has *Erlang-2* distribution (instead of *Exponential* distribution), but has the same mean value as before.

___ 3. Then for this distribution the value of SCV in the RAQS dialog box should be revised to: (Choose nearest value)

a. 0.25 b. 0.5 c. 0.707 d. 1.00
e. 1.414 f. 2 g. No change

___ 4. We should expect that the average time spent at Station A will

a. increase b. decrease c. no change

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RAQS Input information

This Model has been developed in the Intermediate Mode
Type of Network - Open Network
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Number of nodes = 3
Number of classes = 2

Node	No.of servers
1	3
2	2
3	1

Class Information

Class	Arrival Rate	Arrival SCV	# of Visits
1	2.00	1.00	2
2	1.00	1.00	2

Route Information

Class 1 information

Visit	Node	Service Mean	SCV
1	1	1.00	1.000
2	2	0.25	1.000

Class 2 information

Visit	Node	Service Mean	SCV
1	1	0.50	1.000
2	3	0.50	1.000

RAQS Output Report

Network Measures
Average Number in the Network = 7.804
Average Response Time = 2.601

Node Measures

Node	Rho	AvTAN	VarTAN	AvNAN	VarNAN	AvTIQ	VarTIQ	AvNIQ
1	0.833	2.088	3.057	6.265	25.668	1.255	2.252	3.765
2	0.250	0.267	0.065	0.535	0.356	0.017	0.002	0.035
3	0.500	1.004	1.003	1.004	2.008	0.504	0.753	0.504

Class Specific Output

Class	AvRT	VarRT
1	2.522	3.316
2	2.759	3.505

Thruput - Output rate per server at a node
AvTAN - Average time spent at a node
AvNAN - Mean number of customers at a node
AvTIQ - Average waiting time in queue at a node
AvNIQ - Mean queue length at a node
AvRT - Average time spent in the network by a customer in a class