Solutions

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57:022 Principles of Design II

Homework #10 Solutions -- Spring 2002

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1. Bectol, Inc. is building a dam. A total of 10,000,000 cu ft of dirt is needed to construct the dam. The dirt is moved via dumpers to the dam site. Only one loader is available, and it rents for $100 per hour. Bectol can rent, at $40 per hour, as many dumpers as desired. Each dumper can hold 1000 cu ft of dirt. It takes an average of ten minutes to load a dumper with dirt, and it takes each dumper an average of ten minutes to deliver the dirt to the dam and return to the loader. Making appropriate assumptions about exponentiality so as to obtain a birth/death model, we wish to determine the optimal number of dumpers and the minimum total expected cost (rental of loader plus dumpers) of moving the dirt needed to build the dam.

   a. How many loads are required to deliver all the dirt?

   Solution: We have to use $\dfrac{10,000,000\ cu\ ft}{1,000\ cu\ ft\ /\ load}$ = 10,000 loads to deliver all the dirt.

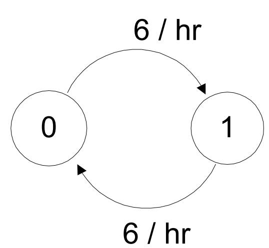
   b. If the loader were to have *100% utilization*, i.e., if there were always a dumper available at the dump site, how many hours would be required to complete the job?

   Solution: If the loader were to have 100% utilization, it would require $\dfrac{10,000\ loads}{6\ loads/hr}$ = 1666.667 hours to complete the job.

As the number of dumpers is increased, the utilization of the loader will *increase* (and the time required to complete the job will *decrease*), so that the cost will *decrease*. Making appropriate assumptions about exponentiality so as to obtain a birth/death model, we wish to determine the optimal number of dumpers and the minimum total expected cost (rental of loader plus dumpers) of moving the dirt needed to build the dam.
Evaluate the cases of one, two, and three dumpers.

<u>Case 1 : One Dumper</u> (M/M/1/1/1)

Define state 0 : no dumper in the system
state 1 : one dumper in the system



Steady-state Distribution

i	π_i	CDF
0	0.5000	0.5000
1	0.5000	1.0000

Utilization of loader is $1-\pi_0$ = 50 % so that the total time to complete the job will be

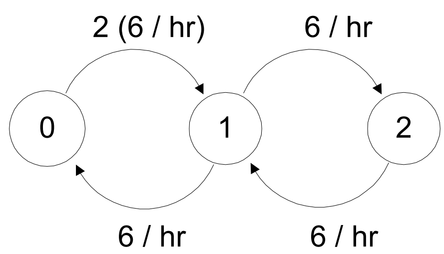
$\dfrac{10,000\ loads}{0.5\times 6\ loads/hr}$ = 3,333.33 hours.

The hourly cost of renting the bulldozer and 1 dumper is $100/hr + $40/hr = $140/hr, and so the total cost of completing the job will be

3,333.33 hours×$140/hr = $466,666.67

<u>Case 2 : Two Dumpers</u> (M/M/1/2/2)

Define state 0 : no dumper in the system
state 1 : one dumper in the system
state 2 : two dumpers in the system, one is being served and another is waiting.



Steady-state Distribution

i	π_i	CDF
0	0.2000	0.2000
1	0.4000	0.6000
2	0.4000	1.0000

Utilization of loader is $1-\pi_0$ = 80 % so that the total time to complete the job will be

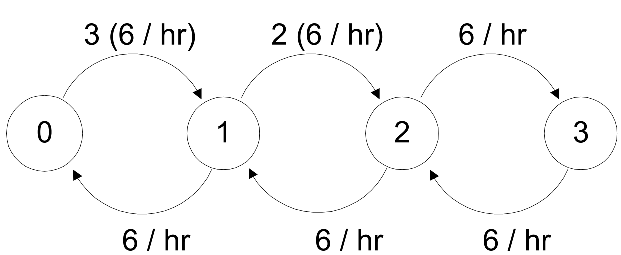
$$\frac{10,000\ loads}{0.8\times 6\ loads/hr} = 2{,}083.33 \text{ hours.}$$

The hourly cost of renting the bulldozer and 2 dumpers is $100/hr + 2($40/hr) = $180/hr, and so the total cost of completing the job will be

2,083.33 hours×$180/hr = $375,000.00

<u>Case 3 : Three Dumpers</u> (M/M/1/3/3)

Define state 0 : no dumper in the system
state 1 : one dumper in the system
state 2 : two dumpers in the system, one is being served and another is waiting.
state 3 : three dumpers in the system, one is being served and the other two are waiting.



Steady-state Distribution

i	π_i	CDF
0	0.0625	0.0625
1	0.1875	0.2500
2	0.3750	0.6250
3	0.3750	1.0000

Utilization of loader is $1-\pi_0$ = 93.75 % so that the total time to complete the job will be

$$\frac{10,000\ loads}{0.9375\times 6\ loads/hr} = 1{,}777.78 \text{ hours.}$$

The hourly cost of renting the bulldozer and 3 dumpers is $100/hr + 3($40/hr) = $220/hr, and so the total cost of completing the job will be

1,777.78 hours×$220/hr = $391,111.11

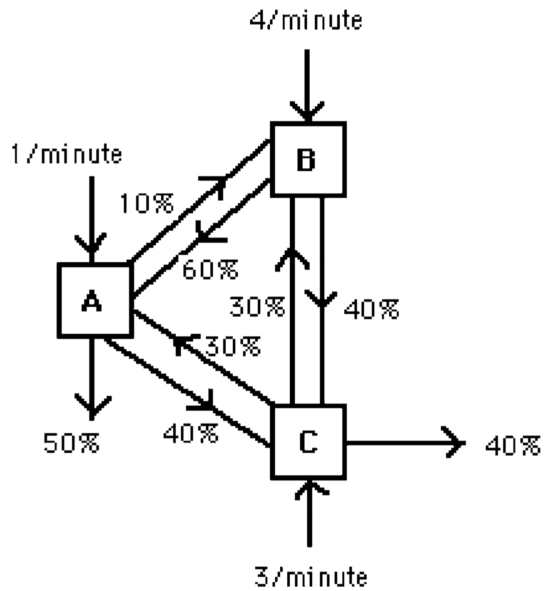
Summary:

Case	Utilization of loader	Time to complete job	Cost of renting loader	Cost of renting dumpers	Total cost
One dumper	50%	333.33 hr	$33,333	$13,333	$46,667
Two dumpers	80%	208.33 hr	$20,833	$16,667	$37,500
Three dumpers	93.75%	177.78 hr	$17,778	$21,333	$39,111

Of the three cases you have evaluated, which is the lowest cost alternative? ***Solution***: _N=2 dumpers

2. Consider a system with three service facilities, and six available servers. Each facility must be assigned one or more servers. Assume unlimited queue capacity at each facility.

Each server has a service rate of 8/minute. The exogenous arrival rates and routing probabilities of jobs shown below:



Jobs arrive at a facility from outside at the rates shown, and after being served at that facility, may be routed to another facility for additional service, according to the probabilities shown.

We want to assign the six servers to the facilities so as to achieve the *smallest average time in the system.*

a. Use the RAQS *(Rapid Analysis of Queueing Systems)* software to evaluate *at least* three cases and record the results below.
Solution:

	# servers assigned to				
Case	Facility A	Facility B	Facility C	Average # in system	Average time in system
1	2	2	2	4.523	0.565
2	3	1	2	26.814	3.352
3	2	1	3	26.62	3.328

Case #1 results in the smallest value both in average # in system and average time in system. Thus, it is the best solution.

b. Using your best solution, verify Little's Law for the system considered as a whole (for which the total arrival rate is 8/minute).

Solution:

Case	A	B	C	Average # in system	Average time in system	Average arrival rate calculated	Average arrival rate given
1	2	2	2	4.523	0.565	8.0053	8
2	3	1	2	26.814	3.352	7.9994	8
3	2	1	3	26.62	3.328	7.9988	8

Little's Law, i.e., $L = \underline{\lambda} W$ is satisfied with all these three cases