1. (Exercise 6, page 1083-1084 of text by Winston) Bectol, Inc. is building a dam. A total of 10,000,000 cu ft of dirt is needed to construct the dam. A bulldozer is used to collect dirt for the dam. Then the dirt is moved via dumpers to the dam site. Only one bulldozer is available, and it rents for $100 per hour. Bectol can rent, at $40 per hour, as many dumpers as desired. Each dumper can hold 1000 cu ft of dirt. It takes an average of 12 minutes for the bulldozer to load a dumper with dirt, and it takes each dumper an average of five minutes to deliver the dirt to the dam and return to the bulldozer. Making appropriate assumptions about exponentiality so as to obtain a birth/death model, determine the optimal number of dumpers and the minimum total expected cost of moving the dirt needed to build the dam.

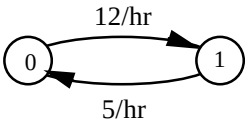
Solutions.

We have to use 10,000,000/1000=10,000 times of dumper to deliver all the dirt.

Case 1 : One dumper :

Define state 0 : no dumper in the system,

state 1 : one dumper in the system.



Steady-state Distribution

i	Pi	CDF
0	0.294118	0.294118
1	0.705882	1.000000

The average departure rate of dumper is $(1-\pi 0)5=0.705882(5)=3.52941$(times/hr)

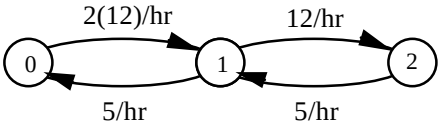
The total cost = (10,000/3.52941)($100+$40)=396667.

Case 2 : Two dumpers.

Define state 0 : no dumper in the system,

state 1 : one dumper in the system,

state 2 : two dumpers in the system, one is being served and another is waiting.



Steady-state Distribution

i	Pi	CDF
0	0.057737	0.057737
1	0.277136	0.334873
2	0.665127	1.000000

The total cost={10,000/[$(1-\pi 0)(5)$]}{$100+2($40)}=382059.

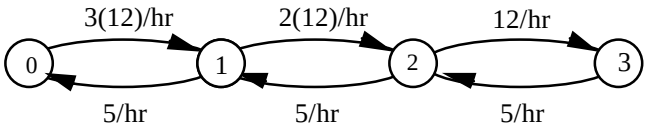
Case 3 : Three dumpers :

Define state 0 : no dumper in the system,

state 1 : one dumper in the system,

state 2 : two dumpers in the system, one is being served and another is waiting.

state 3 : three dumpers in the system, one is being served, and the other two are waiting.



Steady-state Distribution

i	Pi	CDF
0	0.007955	0.007955
1	0.057277	0.065233
2	0.659836	0.340164
3	0.659836	1.000000

Computing the steady-state distribution:

$$\frac{1}{p_0} = 1 + \frac{36}{5} + \frac{36}{5} \times \frac{24}{5} + \frac{36}{5} \times \frac{24}{5} \times \frac{12}{5} = 125.704$$

$$\Rightarrow p_0 = \frac{1}{125.704} = 0.007955$$

$1-\pi_0 = 99.2\%$ is the utilization of the bulldozer loading the trucks.

(The other steady-state probabilities are:

$$p_1 = \frac{36}{5} p_0 = 0.057277$$

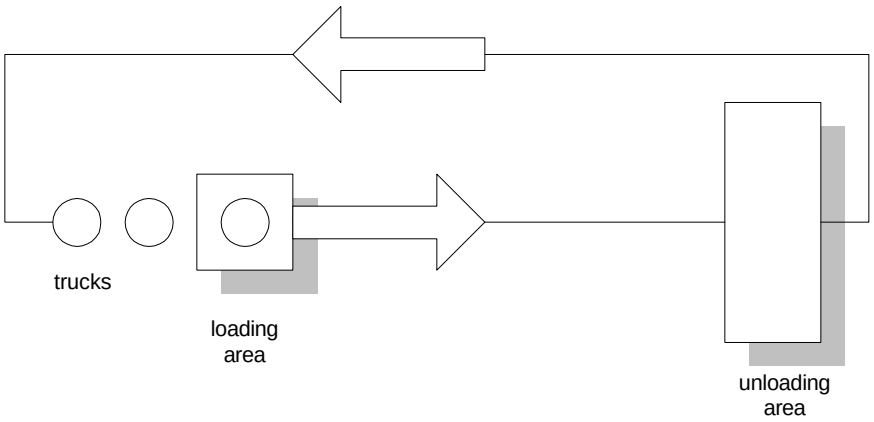
$$p_2 = \frac{36}{5} \times \frac{24}{5} p_0 = 0.2749$$

$$p_3 = \frac{36}{5} \times \frac{24}{5} \times \frac{12}{5} p_0 = 0.6598)$$

The total cost = {10.000/[(1-π0)(5)]}{$100+3($40)}=443528

Thus, the optimal number of dumpers is 2.

RAQS (*Rapid Analysis of Queueing Systems*) software can do the same computation. This will be a *closed* queueing system, i.e., the three trucks will not enter or leave the system, but move between the two nodes (loading area and unloading area). The loading area has one server, and the unloading area (assuming that more than one truck can unload simultaneously) has up to three servers (at least one per truck).



The service time distributions are both exponential, with rates 5/hour (for loading) and 12/hour (for unloading).

In the case of **one** dump truck:

```
Input information
This Model has been developed in the Basic Mode
Type of Network - Closed Network

Number of nodes = 2
Number of customers in the Network = 1

Node  Number       Mean Service  Service time
#     of Servers   Time          SCV
1     1            0.200         1.000
2     3            0.083         1.000
```

The result is:

```
Output Report
This Model has been developed in the Basic Mode
Type of Network - Closed Network

Network Measures
Average Time in the Network =    0.283

Node Measures
Node Util    ThruPut   AvTIQ    AvNIQ    AvTAN    AvNAN
1    0.706   3.52941   0.000    0.000    0.200    0.706
2    0.098   3.52941   0.000    0.000    0.083    0.294
```

In the case of **two** dump trucks:

Input information

This Model has been developed in the Basic Mode
Type of Network - Closed Network

Number of nodes = 2
Number of customers in the Network = 2

Node #	Number of Servers	Mean Service Time	Service time SCV
1	1	0.200	1.000
2	3	0.083	1.000

The result is:

Output Report

This Model has been developed in the Basic Mode
Type of Network - Closed Network

Network Measures

Average Time in the Network = 0.446

Node Measures

Node	Util	ThruPut	AvTIQ	AvNIQ	AvTAN	AvNAN
1	0.897	4.48470	0.163	0.729	0.363	1.626
2	0.125	4.48470	0.000	0.000	0.083	0.374

In the case of **three** dump trucks:

Input information

This Model has been developed in the Basic Mode
Type of Network - Closed Network

Number of nodes = 2
Number of customers in the Network = 3

Node #	Number of Servers	Mean Service Time	Service time SCV
1	1	0.200	1.000
2	1	0.083	1.000

Output Report

This Model has been developed in the Basic Mode
Type of Network - Closed Network

Network Measures

Average Time in the Network = 0.642

Node Measures

Node	Util	ThruPut	AvTIQ	AvNIQ	AvTAN	AvNAN
1	0.934	4.66951	0.330	1.541	0.530	2.475
2	0.389	4.66951	0.029	0.136	0.113	0.525

The utilization of the bulldozer loading the trucks is 93.4%.