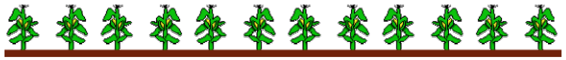
Walnut Orchard



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Walnut Orchard has two farms that grow **wheat** & **corn**.

Because of differing soil conditions, there are differences in the yields and costs of growing crops on the two farms:

	Farm #1	Farm #2
Corn yield/acre	100 bushels	120 bushels
Cost/acre of corn	\$90	\$115
Wheat yield/acre	40 bushels	35 bushels
Cost/acre of wheat	\$90	\$80

Farm #1 has 100 acres available for cultivation, while Farm #2 has 150 acres.
The farm has contracted to grow 11,000 bushels of corn and 6000 bushels of wheat.

Determine a planting plan that will minimize the cost of meeting these contracts.

Note: We are assuming that the costs and yields are known with certainty, which is <u>not</u> the case in the "real world"!

Decision variables:

C1 = # of acres of Farm 1 planted in corn
W1 = # of acres of Farm 1 planted in wheat
C2 = # of acres of Farm 2 planted in corn
W2 = # of acres of Farm 2 planted in wheat

Constraints:

- Restrictions of the number of acres of each farm which are planted in crops.
  $C1 + W1 \le 100$
  $C2 + W2 \le 150$
- Restrictions of the minimum quantity of each crop.
  $100C1 + 120C2 \le 11000$
  $40W1 + 35W2 \le 6000$
- Nonnegativity constraint on each of the four variables.
  $C1 \ge 0,\ C2 \ge 0,\ W1 \ge 0,\ W2 \ge 0$

Objective:

Minimize 90 C1 + 115 C2 + 90 W1 + 80 W2

Walnut Orchard

Complete LP formulation with solution :

```
MIN      90 C1 + 115 C2 + 90 W1 + 80 W2
SUBJECT TO
        2)    C1 + W1 <=    100
        3)    C2 + W2 <=    150
        4)    100 C1 + 120 C2 >=    11000
        5)    40 W1 + 35 W2 >=    6000
END
      OBJECTIVE FUNCTION VALUE
      1)        24096.15

VARIABLE         VALUE          REDUCED COST
      C1          3.846154          0.000000
      C2         88.461540          0.000000
      W1         96.153847          0.000000
      W2         61.538460          0.000000

     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.000000         17.692308
      3)         0.000000         14.230769
      4)         0.000000         -1.076923
      5)         0.000000         -2.692308
```

LINDO

That is, the optimal plan is to plant

- ♦ 3.85 acres of corn on farm #1 ,
- ♦ 88.46 acres of corn on farm #2 ,
- ♦ 96.15 acres of wheat on farm #1 and
- ♦ 61.54 acres of wheat on farm #2.

The total cost will be $24,096.15.

The **LINGO** model (*without using sets, etc.*) is nearly the same:

```
MIN =    90 * C1 + 115 * C2 + 90 * W1 + 80 * W2;

   C1 + W1 <=   100;
   C2 + W2 <=   150;
   100 * C1 + 120 * C2 >=   11000;
   40 * W1 + 35 * W2 >=   6000;
END
```

LINGO

Note that LINGO requires the "" to indicate multiplication, and the semicolon to indicate end of statement.*

Using sets allows us to generalize the model, separating the data from the model.


```
SETS:
   FARM/1..2/:ACRES;
   CROP/CORN, WHEAT/:RQMT;
   PLANT(FARM,CROP):COST,YIELD,X;
ENDSETS

DATA:
   ACRES = 100 150;
   RQMT = 11000 6000;
   YIELD=  100  40
           120  35;
   COST=    90  90
           110  80;
ENDDATA

MIN = @SUM(PLANT: COST*X);

@FOR(FARM(I):
   @SUM(CROP(J): X(I,J) ) <= ACRES(I) );
@FOR(CROP(J):
   @SUM(FARM(I): YIELD(I,J)*X(I,J) ) >= RQMT(J) );
END
```

LINGO

The generated model is

```
MIN     90 X( 1, CORN) + 90 X( 1, WHEAT) + 110 X( 2, CORN)
     + 80 X( 2, WHEAT)
SUBJECT TO
2]  X( 1, CORN) + X( 1, WHEAT) <=   100
3]  X( 2, CORN) + X( 2, WHEAT) <=   150
4]  100 X( 1, CORN) + 120 X( 2, CORN) >=   11000
5]  40 X( 1, WHEAT) + 35 X( 2, WHEAT) >=   6000
END
```

The solution:

Global optimal solution found at step: 4
Objective value: 23653.85

Variable	Value	Reduced Cost
X(1, CORN)	3.846154	0.0000000
X(1, WHEAT)	96.15385	0.0000000
X(2, CORN)	88.46154	0.0000000
X(2, WHEAT)	61.53846	0.0000000

Row	Slack or Surplus	Dual Price
1	23653.85	1.000000
2	0.00	2.307692
3	0.00	0.7692308
4	0.00	-0.9230769
5	0.00	-2.307692

Ranges in which the basis is unchanged:

Objective Coefficient Ranges

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X(1, CORN)	90.00000	0.2380952	INFINITY
X(1, WHEAT)	90.00000	INFINITY	0.2380952
X(2, CORN)	110.0000	INFINITY	0.2857143
X(2, WHEAT)	80.00000	0.2083333	INFINITY

Righthand Side Ranges

Row	Current RHS	Allowable Increase	Allowable Decrease
2	100.0000	28.75000	1.041667
3	150.0000	29.76190	1.190476
4	11000.00	142.8571	2875.000
5	6000.000	41.66667	1041.667

From LINDO:

THE TABLEAU

ROW	BASIS	C1	C2	W1	W2	SLK 2	SLK 3	SLK 4	SLK 5	
1	ART	0.0	0.0	0.0	0.0	17.692	14.231	1.1	2.7	-0.24E+05
2	C1	1.0	0.0	0.0	0.0	3.692	3.231	0.027	0.092	3.846
3	W2	0.0	0.0	0.0	1.0	3.077	3.692	0.031	0.077	61.538
4	C2	0.0	1.0	0.0	0.0	-3.077	-2.692	-0.031	-0.077	88.462
5	W1	0.0	0.0	1.0	0.0	-2.692	-3.231	-0.027	-0.092	96.154

Sensitivity Analysis

1. By what amount may the cost of planting corn on Farm #1 ***increase*** without changing the solution? ____________

2. Suppose that the farmer has decided to plant one acre of Farm #1 in vegetables, so that the amount of land available to corn and wheat is decreased. What will be his increased cost of filling the contract for corn and wheat? ____________ How would he change his plan to accomplish this? ________________

3. Suppose that in (2) the farmer has decided to plant two acres in vegetables rather than one acre. What will be his increased cost? ____________

4. The farmer asks you, “What is my marginal cost per bushel of corn to meet my contract to deliver corn?” *That is, what would be the cost of delivering an extra bushel of corn?* _______

5. “What is my average cost per bushel of wheat to meet my contract to deliver wheat?” __________