A processing system is composed of **two stations in tandem**.

- The arrival of jobs at station #1 is a Poisson process with a rate of 4/hour, but station #1 can accept jobs only when it is idle.
- The processing time at each station is exponentially distributed with a mean of 10 minutes.
- There is room in the system for only two jobs, one at each station.
- No queueing between stations or before the first station is permitted.

- A job which completes processing at the first station when the second station is busy will remain at the first station, **“blocking”** it, i.e., preventing it from accepting a new job.

**Compute the steady-state probabilities.**

**Compute the throughput rate for the system.**

The state of the system is 2-dimensional: denote the state of each station by

- 0) idle
- 1) busy

and, for station #1,

- b) “blocked”.

The possible states of the system are therefore

1. (0,0)
2. (1,0)
3. (0,1)
4. (1,1)
5. (b,1)
Transition Rate Matrix

<table>
<thead>
<tr>
<th></th>
<th>00</th>
<th>10</th>
<th>01</th>
<th>11</th>
<th>b1</th>
</tr>
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<td>-6</td>
</tr>
</tbody>
</table>

Steady-state equations:

\[
\begin{align*}
-4\pi_{00} + 6\pi_{01} &= 0 \\
4\pi_{00} - 6\pi_{10} + 6\pi_{11} &= 0 \\
6\pi_{10} - 10\pi_{11} + 6\pi_{51} &= 0 \\
4\pi_{01} - 12\pi_{11} &= 0 \\
6\pi_{11} - 6\pi_{51} &= 0 \\
\pi_{00} + \pi_{10} + \pi_{01} + \pi_{11} + \pi_{51} &= 0
\end{align*}
\]

Steady-state Distribution

<table>
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<tr>
<th>state</th>
<th>( \pi_i )</th>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>0.2963</td>
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<tr>
<td>3</td>
<td>0.2222</td>
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<tr>
<td>4</td>
<td>0.0741</td>
</tr>
<tr>
<td>5</td>
<td>0.0741</td>
</tr>
</tbody>
</table>

What is the average throughput of the system?

Jobs are completed at the rate 6/hr when system is in states 3, 4, & 5, having total probability 0.3704.

Therefore the throughput is 0.3704 \times 6/hr = 2.222/hr.