The "Farthest Insertion" heuristic algorithm constructs a tour, starting with an arbitrary node.

Each step begins with a subtour, and selects the node which is farthest from the set of nodes on the subtour to be added to the subtour.

After selecting the node \( k \) to be added, an edge \((i,j)\) is selected and the edges \((i,k)\) and \((k,j)\) then replace the edge \((i,j)\).

The edge \((i,j)\) is selected so as to minimize the increase in the length of the subtour, i.e.,

\[ d_{ik} + d_{kj} - d_{ij} \]

step 3: Replace arc \((i_{1},i_{2})\) in the tour \( T \) with the pair of arcs \((i_{1},j)\) and \((j,i_{2})\).

Let \( N = N' - \{ j \} \) and \( \hat{i} = j \).

step 4: If \( N' = \emptyset \), STOP. Else return to step 1.

---

**Example**

Random Symmetric TSP  
(seed: 133398)

---

Let's arbitrarily begin the tour with node \#1, i.e., \( T = [1] \)

We select the FARDEST node from the tour, i.e., node \#1. This is node \#10.
We compute the distance from each node to the nearest node already on the tour. The node selected to be inserted is that node which is FARTHEST from the tour, namely node #5.

We next need to decide between which 2 nodes on the tour to insert node #11. There are 3 possibilities:

1 → 11 → 10
10 → 11 → 5
5 → 11 → 1

We choose to insert node #11 in such a way that the increase in the tour length is minimized:

Increase in tour length is
90 + 40 - 95 = 35

Increase in tour length is
40 + 94 - 72 = 62

Minimum {35, 62, 122} = 35

and so node #11 is inserted between node #1 and node #10.

Increase in tour length is
94 + 90 - 62 = 122
Farthest Insertion Heuristic

Insert node 7
\[ T = (1, 11, 10, 5) \]

Insert node 12

Insert node 3

Insert node 12

Insert node 5

Farthest Insertion Tour: 1 12 4 5 6 7 8 9 10 11 1 3 2 4, with length 321

... etc.