System Reliability

This Hypercard stack was prepared by:
Dennis L. Bricker,
Dept. of Industrial Engineering,
University of Iowa,
Iowa City, Iowa 52242
e-mail: dbricker@icaen.uiowa.edu

author

System Reliability is the probability that the system will ***not fail*** to perform

- within specified limits
- for a specified period of time
- in a specified environment

Define the ***random variable***

T_{sys} = effective lifetime of system

with ***distribution function***

$F_{sys}(t) = P\{T_{sys} \leq t\}$

The ***reliability*** function is the complement of F_{sys}, i.e.,

$R_{sys}(t) = 1 - F_{sys}(t)$

$= P\{T_{sys} > t\}$

= P{system is properly functioning at time t}

System Reliability

Suppose that we know the reliabilities of the individual components of the system.

How do we estimate the reliability of the system?

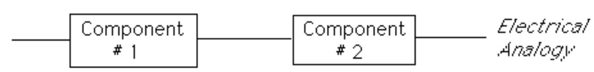
System Reliability

- Components in Series
- Components in Parallel (Redundancy)
- Partially Redundant System (m-out-of-n)
- *Examples: Systems with both series & parallel components*

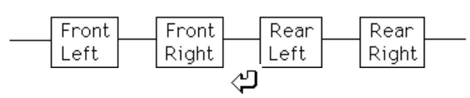
Standby System
- -with Perfect Switching
- -with Imperfect Switching
- -with Failures in Standby Mode
- - etc.

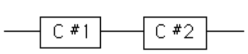
Series

- Any component may fail independently of any other
- When one component fails, the system (device) fails



Example: an automobile "fails" when any one of its tires fail





$R_s(t)$ = reliability of device

= P{ all compoents survive until time t}

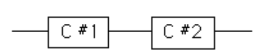
= P{component #1 survives until time t} x P{component #2 survives until time t}

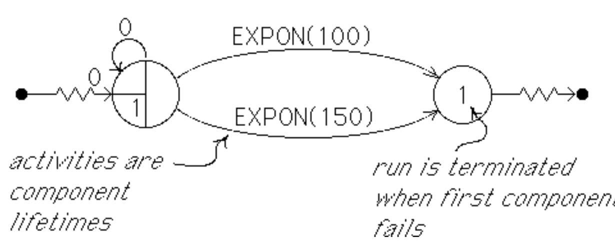
$= R_1(t) \times R_2(t)$

In general, for n components in series,

$R_s(t) = R_1(t) \times R_2(t) \times \ldots \times R_n(t)$

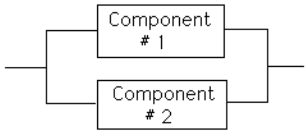
SLAM model of series system





Parallel

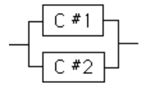
- Components operate simultaneously *(not "standby")*
- All components must fail in order that the system fails



Electrical Analogy

Example: a 2-engine plane fails only if both of its engines fail

Let $F_i(t) = 1 - R_i(t)$
= P{component #i fails before time t}

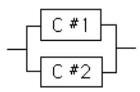


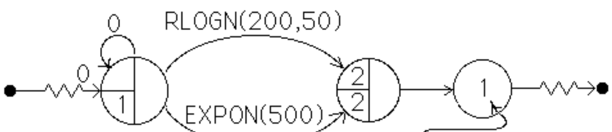
(cumulative distribution function of the component's lifetime)

$F_P(t)$ = P{parallel system fails before time t}
= P{all components fail before time t}
= P{component #1 fails before time t} x P{component #2 fails before time t}
$= F_1(t) \times F_2(t) = [1-R_1(t)] \times [1-R_2(t)]$

So $R_P(t) = 1 - F_P(t) = 1 - [1-R_1(t)] \times [1-R_2(t)]$

SLAM model of system with parallel components





ACCUMULATE node releases entity when both components have failed

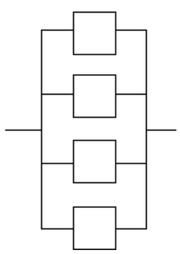
when first entity arrives, the run is terminated

Partially Redundant System

Suppose that **m** out of **n** identical components must function in order for the system to function. This represents "compromise" between the two extremes

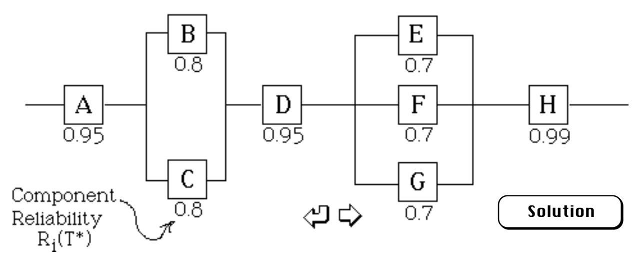
- series system, requiring all components
- parallel system, requiring only one component

Example

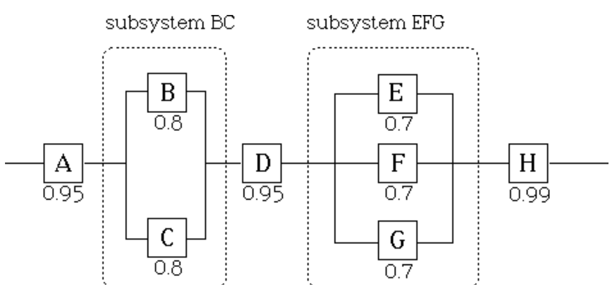


Suppose that the system consists of 4 identical components, and requires 3 of them to function in order for the system to properly function.

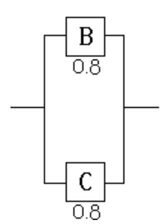
Suppose that a system is designed to fulfill its mission for a planned lifetime T*. Find the reliability of the system, given the component reliabilities:



Solution

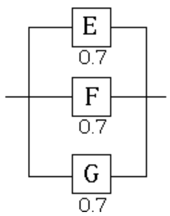


subsystem BC

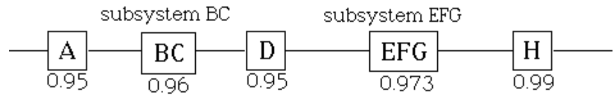


$R_{BC} = 1 - [1-R_B] \times [1-R_C]$
$= 1 - 0.2 \times 0.2$
$= 1 - 0.04$
$= 0.96$

subsystem EFG



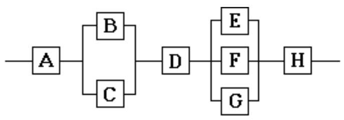
$$R_{EFG} = 1 - [\ 1-R_E\ ] \times [\ 1-R_F\ ] \times [\ 1-R_G\ ]$$
$$= 1 - 0.3 \times 0.3 \times 0.3$$
$$= 1 - 0.027$$
$$= 0.973$$

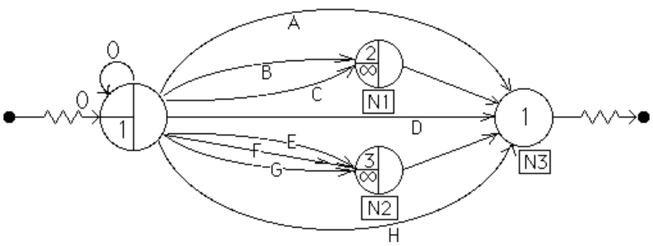


Reliability of series:

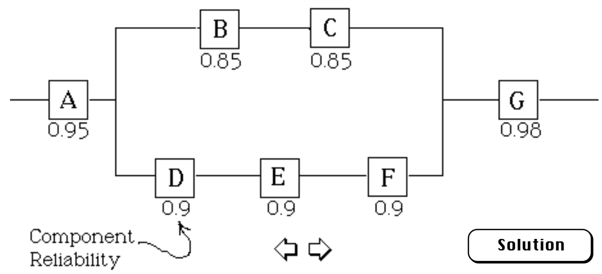
$$R_{system} = R_A \times R_{BC} \times R_D \times R_{EFG} \times R_H$$
$$= (0.95)(0.96)(0.95)(0.973)(0.99)$$
$$= 0.834577$$

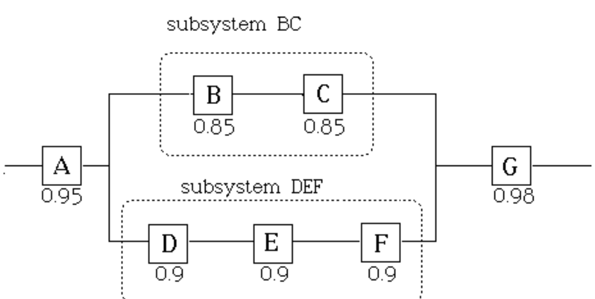
SLAM model

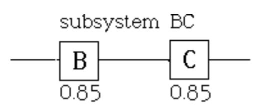




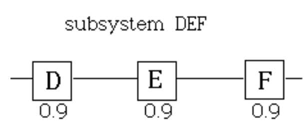
Find the reliability of the system:



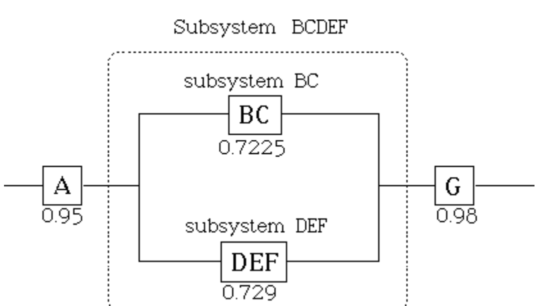


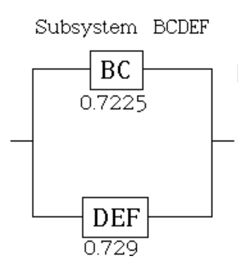


$$R_{BC} = R_B \times R_C$$
$$= 0.85 \times 0.85$$
$$= 0.7225$$

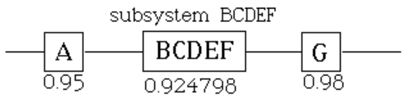


$$R_{DEF} = R_D \times R_E \times R_F$$
$$= 0.9 \times 0.9 \times 0.9$$
$$= 0.729$$



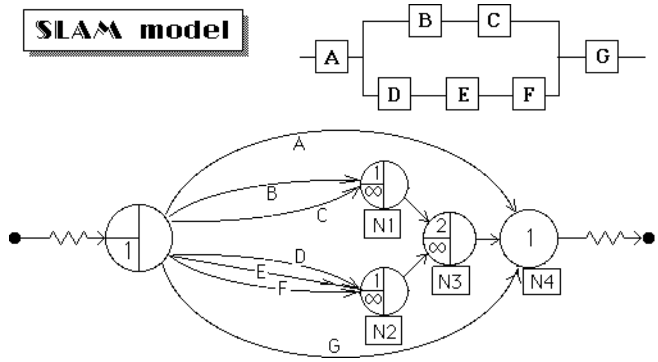


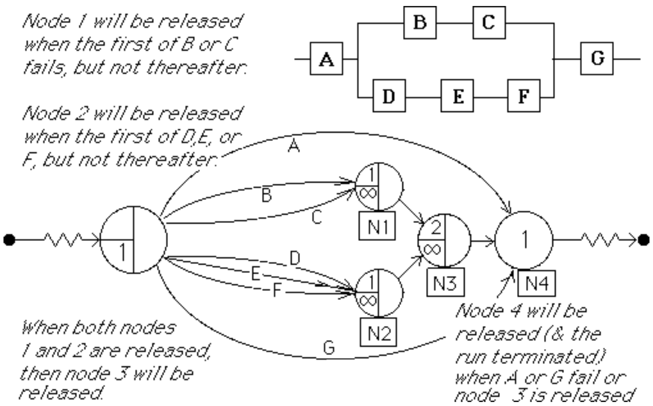
$$R_{BCDEF} = 1 - [1-R_{BC}] \times [1-R_{DEF}]$$
$$= 1 - (1-0.7225)(1-0.729)$$
$$= 1 - (0.2775)(0.271)$$
$$= 1 - 0.075202$$
$$= 0.924798$$



$$R_{system} = R_A \times R_{BCDEF} \times R_G$$
$$= (0.95)(0.924798)(0.98)$$
$$= 0.860987$$

SLAM model





Suppose that the component failure rates are constant, i.e., their lifetimes have exponential distributions:

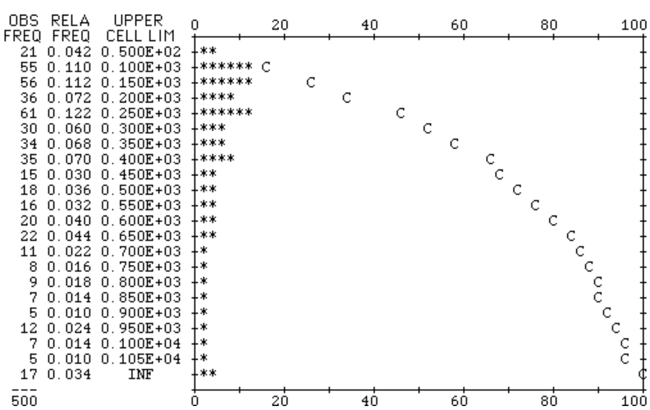
Component	Mean Lifetime	R(100)
A	1950	95%
B	615	85%
C	615	85%
D	950	90%
E	950	90%
F	950	90%
G	4950	98%

```
GEN,BRICKER,SYSTEM RELIABILITY,4/16/92,500, ,NO,
,NO,YES/500,72;
LIM, ,1,10;
INIT, , ,NO;
NETWORK;
        CREATE;
         ACTIVITY/1,EXPON(1950), ,N4;        COMPONENT A
         ACTIVITY/2,EXPON(615), ,N1;         COMPONENT B
         ACTIVITY/3,EXPON(615), ,N1;         COMPONENT C
         ACTIVITY/4,EXPON(950), ,N2;         COMPONENT D
         ACTIVITY/5,EXPON(950), ,N2;         COMPONENT E
         ACTIVITY/6,EXPON(950), ,N2;         COMPONENT F
         ACTIVITY/7,EXPON(4950), ,N4;        COMPONENT G
N1      ACCUM,1,99;        RELEASE FIRST ARRIVAL ONLY
         ACTIVITY, , ,N3;
```

SLAM code

STATISTICS FOR VARIABLES BASED ON OBSERVATION

	MEAN VALUE	STANDARD DEVIATION	COEFF. OF VARIATION	MINIMUM VALUE	MAXIMUM VALUE	NO.OF OBS
FAIL TIME	0.374E+03	0.300E+03	0.801E+00	0.579E+01	0.163E+04	500



OBS FREQ	RELA FREQ	UPPER CELL LIM	
21	0.042	0.500E+02	**
55	0.110	0.100E+03	******
56	0.112	0.150E+03	******
36	0.072	0.200E+03	****
61	0.122	0.250E+03	******
30	0.060	0.300E+03	***
34	0.068	0.350E+03	***
35	0.070	0.400E+03	****
15	0.030	0.450E+03	**
18	0.036	0.500E+03	**
16	0.032	0.550E+03	**
20	0.040	0.600E+03	**
22	0.044	0.650E+03	**
11	0.022	0.700E+03	*
8	0.016	0.750E+03	*
9	0.018	0.800E+03	*
7	0.014	0.850E+03	*
5	0.010	0.900E+03	*
12	0.024	0.950E+03	*
7	0.014	0.100E+04	*
5	0.010	0.105E+04	*
17	0.034	INF	**
500			

Using the estimated mean and standard deviation from the simulation runs, we can compute the shape and scale parameters of the Weibull probability distribution:

```
Mean: 374, Standard deviation: 300

Parameters of distribution:
  u=401.895 (scale), k=1.25471 (shape)
```

$$F(t) = 1 - e^{-(t/u)^k}$$
$$= 1 - e^{-(t/401.895)^{1.25471}}$$

t	F(t)	1-F(t)
50	0.0705536	0.929446
100	0.160197	0.839803
150	0.252016	0.747984
200	0.340715	0.659285
250	0.423745	0.576255
300	0.49987	0.50013
350	0.568608	0.431392
400	0.629939	0.370061
450	0.684124	0.315876
500	0.7316	0.2684
550	0.772899	0.227101
600	0.808594	0.191406
650	0.83927	0.16073
700	0.865496	0.134504
850	0.922661	0.0773395
1050	0.964444	0.0355564

$$F(t) = 1 - e^{-(t/401.895)^{1.25471}}$$

Next we calculate the probability of an observation falling in each interval, i.e.,

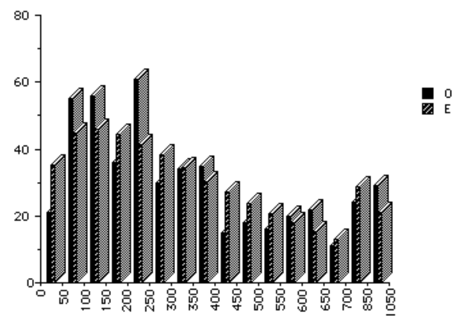
$$P_i = P\{ t_{i-1} \le X \le t_i \} = F(t_i) - F(t_{i-1})$$

and multiply by N=500 to get the expected number of observations in each cell.

i	upper limit of cell	P_i	E_i	O_i	D_i
1	50	0.0705	35.2767	21	5.7779
2	100	0.0896	44.8217	55	2.3112
3	150	0.0918	45.9095	56	2.2177
4	200	0.0886	44.3496	36	1.5719
5	250	0.0830	41.5147	61	9.1455
6	300	0.0761	38.0624	30	1.7078
7	350	0.0687	34.3691	34	0.0039
8	400	0.0613	30.6651	35	0.6127
9	450	0.0541	27.0927	15	5.3975
10	500	0.0474	23.7381	18	1.3870
11	550	0.0412	20.6491	16	1.0467
12	600	0.0356	17.8476	20	0.2595
13	650	0.0306	15.3380	22	2.8935
14	700	0.0262	13.1128	11	0.3404
15	850	0.0571	28.5823	24	0.7346
16	1050	0.0417	20.8915	29	3.1470
17	∞	0.0355	17.7781	17	0.0340
	SUM	1.000000	500	500	38.5898

$$D_i = \frac{(E_i - O_i)^2}{E_i}$$

Observed and Expected Frequencies



The number of degrees of freedom is

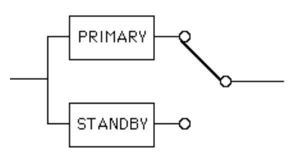
17 cells
−1 (dependence on # runs)
−2 (parameters estimated)
= 14

The Chi-Square Probability table indicates that
$P\{D \ge 23.685\} = 5\%$, $P\{D \ge 29.141\} = 1\%$,
$P\{D \ge 36.123\} = 0.1\%$, etc.

That is, if the assumed Weibull distribution were valid, there is less than 0.1% probability that we would observe the value D=38.5898.
It is therefore reasonable to reject this distribution!

Standby System with Perfect Switching

One or more components are in a standby mode ready to be activated when the operating component fails.

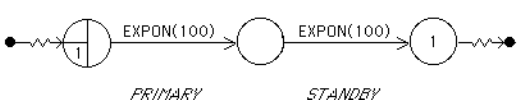


This requires a failure sensing/switching device which will bring the standby unit into operation

Assume this device does not fail!

SLAM model

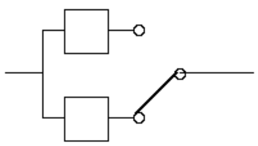
Suppose both the primary and standby component have lifetimes with exponential distribution, with expected time of failure of 100 days.



In this instance, the lifetime of the system is the sum of the lifetimes of the two units, which (because the sum of two exponentially distributed random variables has Erlang-2 distribution) is easily determined, and a simulation model may have little utility.

However, in most cases, it is quite difficult to determine the distribution of the sum of two random lifetime distributions, and simulation may be extremely useful!

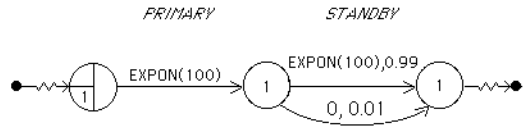
Standby System with Imperfect Switching



One or more components are in a standby mode ready to be activated when the operating component fails.
This requires a failure sensing/switching device which will bring the standby unit into operation
Suppose that this device fails to detect the failure with probability p_s

SLAM model

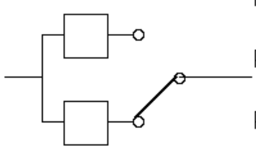
Suppose the switch fails with probability $p_s = 1\%$



With 1% probability, the entity arriving at the GOON node will exit via the activity with zero duration, and the simulation will immediately terminate!

Standby System with Failures in Standby Mode

Suppose that while in standby mode, the components may also fail

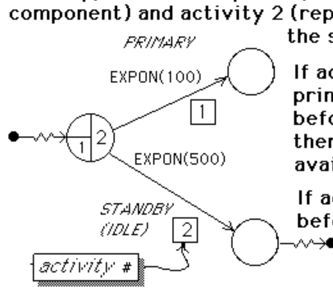


$R_1(t)$ = reliability of first component
$R_2^i(t)$ = reliability of standby unit while idle
$R_2^o(t)$ = reliability of standby unit in operation

SLAM model

Suppose expected lifetime while idle is 500 days:

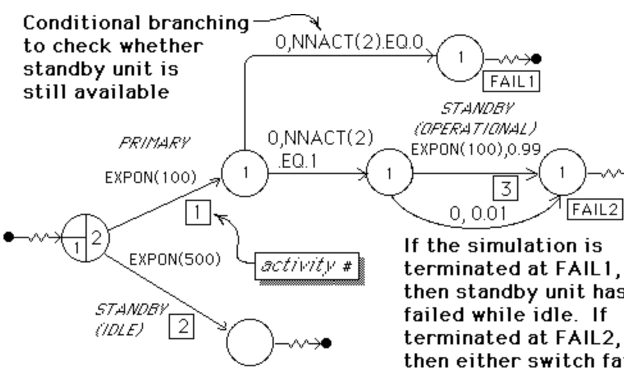
Initially, both activity 1 (representing lifetime of primary component) and activity 2 (representing idle lifetime of the standby unit) begin.



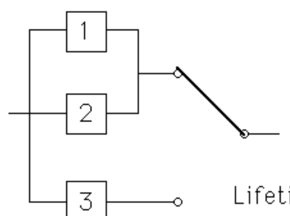
If activity 1 is completed (i.e., primary component dies) before activity 2 is completed, then the standby unit will be available for use.

If activity 2 is completed before activity 1, then when activity 1 is completed, there is no standby unit available & system fails.

Conditional branching to check whether standby unit is still available



If the simulation is terminated at FAIL1, then standby unit has failed while idle. If terminated at FAIL2, then either switch fails or standby unit while in use.



Suppose that components 1 & 2 operate as the primary system, with #3 used as a standby to be used when both #1 & #2 have failed.
The switching device is 99% reliable.
Lifetimes of the components have exponential distribution, with failure rates $\lambda_o = 0.01$/hr for #1,2,&3 in operation, and $\lambda_i = 0.001$ per hour for #3 when idle.