Dual ascent example

(Solved via dual ascent)

Sets (s) 23 24 25 removed from problem
(P = 16 17 25 > 9 - incumbent)
# sets remaining is 22

Dual ascent step

Selecting multiplier \( w(s) = 0 \) with subgradient = -1
Sets not in solution which cover \#4 are:
1 6 7 10 18 20 21 22
\( \delta \) enters when \( w(6) \) is 2
\( \delta \) enters when \( w(6) \) is 4
Updated \( w(6) = \max(0, (1-\epsilon) \times 2) + \alpha \times 2 \) where \( \epsilon = 0 \)
Anticipated improvement = 2

Point \#8 is arbitrarily selected
as one of the uncovered points.
Its multiplier should be increased
to provide incentive for a set to
cover it.

Dual ascent step

Selecting multiplier \( w(s) = 0 \) with subgradient = -1
Sets not in solution which cover \#3 are:
2 3 4 7 14
\( \delta \) enters when \( w(3) \) is 1
\( \delta \) enters when \( w(3) \) is 3
Updated \( w(3) = \max(0, (1-\epsilon) \times 3) + \alpha \times 2 \) where \( \epsilon = 0 \)
Anticipated improvement = 3

Dual ascent step

Selecting multiplier \( w(s) = 0 \) with subgradient = -1
Sets not in solution which cover \#6 are:
4 8 13 15
\( \delta \) enters when \( w(6) \) is 1
\( \delta \) enters when \( w(6) \) is 3
Updated \( w(6) = \max(0, (1-\epsilon) \times 5) + \alpha \times 5 \) where \( \epsilon = 0 \)
Anticipated improvement = 1

Dual ascent step

Selecting multiplier \( w(s) = 0 \) with subgradient = -1
Sets not in solution which cover \#6 are:
4 8 13 15
\( \delta \) enters when \( w(6) \) is 1
\( \delta \) enters when \( w(6) \) is 3
Updated \( w(6) = \max(0, (1-\epsilon) \times 5) + \alpha \times 5 \) where \( \epsilon = 0 \)
Anticipated improvement = 1

Current multipliers:
\[
\begin{array}{cccccc}
1 & 1 & 2 & 3 & 4 & 5
\end{array}
\]
\[
\begin{array}{cccccc}
6 & 7 & 8 & 9 & 10 & 11
\end{array}
\]

Solving Lagrangian relaxation:

*** Dual value is 0 ***
Sets in cover: \# 4
Points not covered: 1 2 3 4 5 6

Heuristic solution cost is 12
*** New incumbent: ***
Cover 1 2 4 7
with cost 12

Current multipliers:
\[
\begin{array}{cccccc}
1 & 1 & 2 & 3 & 4 & 5
\end{array}
\]
\[
\begin{array}{cccccc}
6 & 7 & 8 & 9 & 10 & 11
\end{array}
\]

Solving Lagrangian relaxation:

*** Dual value is 2 ***
Sets in cover: \# 1
Points not covered: 1 2 3 4 5 6

Sets (s) 16 17 19 22 removed from problem
(P = 13 14 15 16 > 9 - incumbent)
# sets remaining is 18

Current multipliers:
\[
\begin{array}{cccccc}
1 & 1 & 2 & 3 & 4 & 5
\end{array}
\]
\[
\begin{array}{cccccc}
6 & 7 & 8 & 9 & 10 & 11
\end{array}
\]

Solving Lagrangian relaxation:

*** Dual value is 6 ***
Sets in cover: \# 1 2 3
Points not covered: 5