Operations Research – In-class activity (LP Sensitivity Analysis)

Recent federal regulations strongly encourage the assignment of students to schools in a city so that the racial composition of any school approximates the racial composition of the entire city. Consider the case of the Greenville city schools. The city can be considered as composed of five areas with the following characteristics:

Area	Percent minority	Number of students
1	20%	1200
2	10%	900
3	85%	1700
4	60%	2000
5	90%	2500

The ruling handed down for Greenville is that a school can have neither more than 75% nor less than 30% minority enrollment. There are three schools in Greenville with the following capacities:

School	Capacity
Bond	3900
Pocahontas	3100
Pierron	2100

The objective is to design an assignment of students to schools so as to stay within the capacity of each school and satisfy the composition constraints, while minimizing the total distance traveled by students (and therefore the average distance traveled by students). The distances in kilometers between areas and schools are:

	Area				
School	1	2	3	4	5
Bond	2.7	1.4	2.4	1.1	0.5
Pocahontas	0.5	0.7	2.9	0.8	1.9
Pierron	1.6	2.0	0.1	1.3	2.2

There is an additional condition that no student can be transported more than 2.6 kilometers. Find the number of students which should be assigned to each school from each area. Assume that any group of students from an area have the same ethnic mix as the whole area.

Define variables:

Xij = number of students from area *i* assigned to school *j*

where *i* =1, 2, 3, 4, 5 and *j* =1, 2, 3, except for the cases i=1&j=1 and i=3&j=2 (because the distances traveled exceed the maximum allowed.)

LINDO output:

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 MIN      1.4 X21 + 2.4 X31 + 1.1 X41 + 0.5 X51 + 0.5 X12
  + 0.7 X22 + 0.8 X42 + 1.9 X52 + 1.6 X13 + 2 X23 + 0.1 X33
  + 1.3 X43 + 2.2 X53
  SUBJECT TO
2)   X12 + X13 =    1200        (all students from area 1 must be
                                        assigned to a school)
3)   X21 + X22 + X23 =    900   (all students from area 2  must be
                                        assigned to a school)
4)   X31 + X33 =    1700        (all students from area 3 must
                                        be assigned to a school)
5)   X41 + X42 + X43 =    2000  (all students from area 4  must be
                                       assigned to a school)
6)   X51 + X52 + X53 =    2500  (all students from area 5 must be
                                        assigned to a school)
7)   X21 + X31 + X41 + X51 <=   3900       (capacity of school 1)
8)   X12 + X22 + X42 + X52 <=   3100       (capacity of school 2)
9)   X13 + X23 + X33 + X43 + X53 <=   2100 (capacity of school 3)
10) - 0.55 X21 + 0.1 X31 - 0.15 X41 + 0.15 X51 <=   0
11)   0.2 X21 - 0.55 X31 - 0.3 X41 - 0.6 X51 <=   0
12) - 0.55 X12 - 0.65 X22 - 0.15 X42 + 0.15 X52 <=   0
13)   0.1 X12 + 0.2 X22 - 0.3 X42 - 0.6 X52 <=   0
14) - 0.55 X13 - 0.65 X23 + 0.1 X33 - 0.15 X43 + 0.15 X53 <=   0
15)   0.1 X13 + 0.2 X23 - 0.55 X33 - 0.3 X43 - 0.6 X53 <=   0
END

LP OPTIMUM FOUND AT STEP     12

        OBJECTIVE FUNCTION VALUE

        1)      5014.364

  VARIABLE        VALUE          REDUCED COST
       X21       542.727295          0.000000
       X31         0.000000          2.212727
       X41       148.181824          0.000000
       X51      2500.000000          0.000000
       X12       890.909119          0.000000
       X22       357.272736          0.000000
       X42      1851.818237          0.000000
       X52         0.000000          1.460000
       X13       309.090912          0.000000
       X23         0.000000          0.032727
       X33      1700.000000          0.000000
       X43         0.000000          0.069091
       X53         0.000000          1.830909


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -0.680000
        3)         0.000000         -0.880000
        4)         0.000000         -0.267273
        5)         0.000000         -0.980000
        6)         0.000000         -0.620000
        7)       709.090881          0.000000
        8)         0.000000          0.180000
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       9)        90.909088          0.000000
      10)         0.000000          0.800000
      11)      1435.909058          0.000000
      12)      1000.000000          0.000000
      13)       395.000000          0.000000
      14)         0.000000          1.672727
      15)       904.090881          0.000000

NO. ITERATIONS=      12

RANGES IN WHICH THE BASIS IS UNCHANGED:
                        OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
     X21        1.400000         0.600000         0.316666
     X31        2.400000         INFINITY         2.212727
     X41        1.100000         0.073077         0.138462
     X51        0.500000         1.460000         INFINITY
     X12        0.500000         0.920000         0.027692
     X22        0.700000         0.034615         0.600000
     X42        0.800000         0.138462         0.400000
     X52        1.900000         INFINITY         1.460000
     X13        1.600000         0.027692         0.920000
     X23        2.000000         INFINITY         0.032727
     X33        0.100000         2.212727         INFINITY
     X43        1.300000         INFINITY         0.069091
     X53        2.200000         INFINITY         1.830909

                       RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS          INCREASE         DECREASE
       2     1200.000000       709.090881       113.986023
       3      900.000000       607.692322       113.986023
       4     1700.000000        76.923080      1699.999878
       5     2000.000000       709.090881       113.986023
       6     2500.000000       493.939392      1809.090942
       7     3900.000000         INFINITY       709.090881
       8     3100.000000       113.986023       709.090881
       9     2100.000000         INFINITY        90.909088
      10        0.000000       271.363647        74.090912
      11        0.000000         INFINITY      1435.909058
      12        0.000000         INFINITY      1000.000000
      13        0.000000         INFINITY       395.000000
      14        0.000000       170.000000        49.999996
      15        0.000000         INFINITY       904.090881
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Solution:

	Area					
School	1	2	3	4	5	total
Bond	____	____	____	____	____	____
Pocahontas	____	____	____	____	____	____
Pierron	____	____	____	____	____	____

What is the average distance traveled by students, according to this plan? _____ km.

Suppose that a temporary classroom building are available which could be erected at any one of the school locations, which would increase the school's capacity by 100 students.

At which school should the building be erected? _____

How much decrease in the average distance traveled would result? _____ km.