Consider a typical integer programming problem:

**Problem (P):**

Find $z' = \min \{ c(x) | x \in X \subseteq Z^n \}$

where

- $Z$ is the set of nonnegative integers \{0,1,2,3,\ldots\}
- $Z^n$ is the set of n-dimensional vectors of nonnegative integers

**Problem P':**

Find $z = \min \{ f(x) | x \in X' \subseteq R^n \}$

is a **relaxation** of problem P if:

1. the feasible region $X'$ of P' contains the feasible region of P, i.e., $X \subseteq X'$
2. the objective value in P' is no worse than that of P for all $x$ in the domain of P, i.e., $c(x) \geq f(x)$ for all $x$ in $X$ (*for minimization*).

**Common relaxations:**

- Dropping integer restrictions (the **LP relaxation** of a (linear) MIP);
- Dropping some constraints.
- Aggregating constraints (**surrogate constraint**);
- Lagrangian relaxation
- Replacing cost function by linear underestimate

**Propositions:**

- If P' is a relaxation of P, then $z' \leq z$ (in case of *minimization*)
- If P' is infeasible, then P is infeasible
- If $x^*$ solves P' and is feasible in P (i.e., $x^* \in X$), and $f(x^*) = c(x^*)$, then $x^*$ solves P

These propositions imply that a relaxation can be used to fathom nodes of a search tree (**branch-&-bound method**).