Introduction to QUEUEING : M/M/1/N

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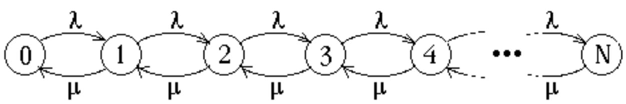
M/M/1/N

- *Arrival & Service processes are Memoryless, i.e.,*
  interarrival times have Exponential distribution with mean $1/\lambda$
  service times have Exponential distribution with mean $1/\mu$
- *Single server*
- ***Capacity** of queueing system is **finite:*** N
  (including customer currently being served)
- *Arriving customers **balk** when queue is full.*

M/M/1/N — Birth/Death Model



Steadystate distribution: $\frac{1}{\pi_0} = 1 + \rho + \rho + \rho^2 + \cdots + \rho^N$

finite geometric series, with sum: $\frac{1-\rho^{N+1}}{1-\rho}$

M/M/1/N — *Steadystate Distribution*

$$\pi_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$\pi_j = \rho^j \pi_0 = \rho^j \left(\frac{1-\rho}{1-\rho^{N+1}}\right)$$

where $\rho = \frac{\lambda}{\mu} \neq 1$

Note that ρ *is not restricted to be less than 1 for steady state to exist!*

Average Number of Customers in System

$$L = \sum_{j=0}^{N} j\,\pi_j$$

$$L = \frac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho^{N+1})(1-\rho)}$$

where $\rho = \frac{\lambda}{\mu} \neq 1$

M/M/1/N

Special Case: $\lambda = \mu$, i.e., $\rho = \frac{\lambda}{\mu} = 1$

Arrival rate = Service rate

$$\pi_j = \frac{1}{N+1}$$

$$L = \frac{N}{2}$$

All states are equally likely!

System is, on average, half-full!

Average Time in System per Customer

Little's Formula: $L = \underline{\lambda} W$

average arrival rate

$$\underline{\lambda} = \sum_{j=0}^{N-1} \lambda\,\pi_j = \lambda \sum_{j=0}^{N-1} \pi_j = \lambda\,(1-\pi_N)$$

since arrival rate is zero when there are N in system

$$W = \frac{L}{\underline{\lambda}} = \frac{L}{\lambda(1-\pi_N)}$$

for M/M/1/N queue only!