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PERT:
Project Evaluation & Review Technique

One of the shortcomings of CPM is the assumption that the durations of activities are deterministic, i.e., known with certainty.

PERT assumes that the duration of each activity is a random variable with known mean and standard variation, and derives a probability distribution for the project completion time.

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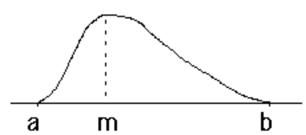
Assumptions of PERT:

- the duration of an activity is a random variable with BETA distribution
- the durations of the activities are statistically independent
- the critical path (computed assuming expected values of the durations) always requires a longer total time than any other path
- the Central Limit Theorem can be applied so that the sum of the durations of the activities on the critical path has approximately a NORMAL distribution

The BETA distribution

is **unimodal** with **finite endpoints**



Mean: $\mu = \dfrac{a + 4m + b}{6}$

Standard deviation: $\sigma = \dfrac{b - a}{6}$

(can be skewed either to right or left.)

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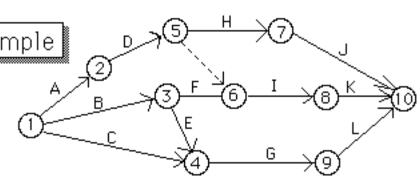
The user provides estimates of the parameters for each activity:

a: the optimistic estimate of completion time
b: the pessimistic estimate of completion time
m: estimate of the most likely completion time

From these parameters, the expected value and standard deviation of each activity's duration is computed.

Example



a *optimistic*
m *most likely*
b *pessimistic*

Activity	i	j	a	m	b
A	1	2	3	5	8
B	1	3	5	6	10
C	1	4	6	8	12
D	2	5	4	6	12
E	3	4	5	10	16
F	3	6	3	4	6
G	4	9	7	11	15
(dummy)	5	6	0	0	0
H	5	7	2	6	10
I	6	8	1	2	4
J	7	10	11	13	16
K	8	10	4	8	15
L	9	10	8	12	16

Example

a *optimistic*
m *most likely*
b *pessimistic*

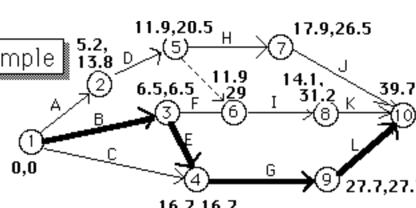
Calculate

$\mu = \dfrac{a+4m+b}{6}$

$\sigma = \dfrac{b-a}{6}$

Activity	i	j	a	m	b	μ	σ^2
A	1	2	3	5	8	5.2	0.69
B	1	3	5	6	10	6.5	0.69
C	1	4	6	8	12	8.3	1.00
D	2	5	4	6	12	6.7	1.78
E	3	4	5	10	16	10.2	3.36
F	3	6	3	4	6	4.2	0.25
G	4	9	7	11	15	11.0	1.78
(dummy)	5	6	0	0	0	0	0
H	5	7	2	6	10	6.0	1.78
I	6	8	1	2	4	2.2	0.25
J	7	10	11	13	16	13.2	0.69
K	8	10	4	8	15	8.5	3.36
L	9	10	8	12	16	12.0	1.78

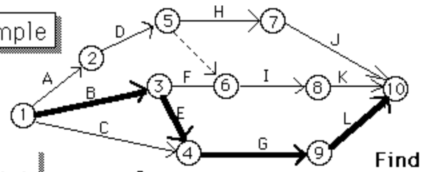
Example



Using μ as the duration of each activity, find the critical path.

Activity	i	j	μ
A	1	2	5.2
B	1	3	6.5 ✱✱✱ *critical*
C	1	4	8.3
D	2	5	6.7
E	3	4	10.2 ✱✱✱ *critical*
F	3	6	4.2
G	4	9	11.0 ✱✱✱ *critical*
(dummy)	5	6	0
H	5	7	6.0
I	6	8	2.2
J	7	10	13.2
K	8	10	8.5
L	9	10	12.0 ✱✱✱ *critical*

Example



Activity	i j	μ	σ^2	
B	1 3	6.5	0.69	*** *critical*
E	3 4	10.2	3.36	*** *critical*
G	4 9	11.0	1.78	*** *critical*
L	9 10	12.0	1.78	*** *critical*
	sum:		7.61	

Find the standard deviation of the sum of the durations on the critical path.

$\sigma_T = \sqrt{7.61}$

$= 2.759$

The completion time for the project is N(39.7, 2.759)

(Normal dist'n with mean 39.7 and std. deviation 2.759)

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The expected completion time of the project is 39.7 days.

What is the probability that it is completed within 42 days?

$$P\{T \le 42\} = P\{\frac{T-39.7}{2.759} \le \frac{42-39.7}{2.759}\} = P\{X \le 0.8336\}$$

$\doteq 79\ \%$

standard N(0,1) random variable

Assumptions of PERT:

- durations of activities are INDEPENDENT random variables with BETA distributions
- the critical path when durations are the mean values is ALWAYS the critical path
- the number of activities on the critical path is large enough to invoke the CENTRAL LIMIT THEOREM *(i.e., completion time has a NORMAL DISTRIBUTION)*