PAR, Inc. Golf Bags

Stochastic LP with Recourse

Benders' Method

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Dept. of Industrial Engineering
The University of Iowa

In our original problem, $X = R_+^{n_1}$, i.e., the first-stage feasible set is unbounded.

In order that we obtain bounded solutions from Benders' Master problem, we impose a constraint

$$X = \left\{ x : \sum_j x_j \le 1500, x \ge 0 \right\}$$

i.e., we limit the total number of golf bags produced to 1500 (which is large enough not to restrict the optimal solution!)

We also modify the problem so as to *minimize* the negative of the expected profit.

Two versions are illustrated:

- Multi-cut version: for each individual scenario k, dual solutions of the second-stage problems are used in order to form a piecewise-linear approximation of the function

  $Q_k(x)$ = minimum cost of second stage in scenario k if first-stage decisions x are selected.

  This means that K (=#scenarios) cuts are added to the master problem at each iteration.

- Uni-cut version: at each iteration, a single cut is added to the master problem in order to form a piecewise-linear approximation of the expected second-stage cost function $\sum_{k=1}^{K} p_k Q_k(x)$

MULTI-CUT VERSION

Iteration #1

We begin by choosing an initial trial solution for the first stage:

i	Variable	Value
1	STANDARD	0
2	DELUXE	0

Primal subproblem results:

Scenario #1
Optimal objective: ¯5664
Second-stage: nonzero variables

i	variable	value
1	ADD	900
4	OTF	192
7	slack_2	150
9	slack_4	45

Scenario #2
Optimal objective: ¯5024
Second-stage: nonzero variables

i	variable	value
1	ADD	828.571429
4	OTF	200.571429
7	slack_2	145.714286
9	slack_4	42.142857

Scenario #3
Optimal objective: ¯5104
Second-stage: nonzero variables

i	variable	value
1	ADD	857.142857
4	OTF	219.142857
7	slack_2	121.428571
9	slack_4	34.285714

Scenario #4
Optimal objective: ¯4464
Second-stage: nonzero variables

i	variable	value
1	ADD	785.714286
4	OTF	227.714286
7	slack_2	117.142857
9	slack_4	31.428571

--Primal subproblems summary

Second stage costs:

k	cost	p[k]
1	¯5664	0.3
2	¯5024	0.3
3	¯5104	0.3
4	¯4464	0.1

First stage cost: 0.00
Expected second stage cost: ¯5184.00
Total: ¯5184.00 *Initial incumbent!*

Lagrangian multipliers *(Dual variables π for second-stage constraints)*

1)	0	0	¯8	0
2)	0	0	¯8	0
3)	0	0	¯8	0
4)	0	0	¯8	0
Sum	0	0	¯32	0

Optimality cuts for Master problem:

$\theta_k \ge \lambda_k^1 X + \alpha_k^1$, where $\lambda_k^1 = -\pi_k^1 T_k, \alpha_k^1 = \pi_k^1 h_k$

Benders' Master Problem

$$\begin{aligned}
&\textit{Minimize}\ \ 10X_1 + 9X_2 + 0.3\theta_1 + 0.3\theta_2 + 0.3\theta_3 + 0.1\theta_4 \\
&s.t.\quad X_1 + X_2 \le 1500 \\
&8X_1 + 5.3333X_2 - 5664 \le \theta_1 \\
&8X_1 + 5.3333X_2 - 5024 \le \theta_2 \\
&8X_1 + 5.3333X_2 - 5104 \le \theta_3 \\
&8X_1 + 5.3333X_2 - 4464 \le \theta_4 \\
&X_1 \ge 0, X_2 \ge 0
\end{aligned}$$

Dual of Master Problem (solved for improved efficiency):

0	1	0	0	1500	5664	5024	5104	4464
¯10	0	1	0	¯1	¯8	¯8	¯8	¯8
¯9	0	0	1	¯1	¯5.3333	¯5.3333	¯5.3333	¯5.3333
0.3	0	0	0	0	1	0	0	0
0.3	0	0	0	0	0	1	0	0
0.3	0	0	0	0	0	0	1	0
0.1	0	0	0	0	0	0	0	1

Solution of Master Problem

value= ¯10684
X= 0 1500
First-stage cost: ¯13500
Estimated Q(X): 2336 2976 2896 3536
Total (estimated) expected value: ¯10684

This is an *underestimate* of the optimal cost, so we now know that the optimal cost is bounded

below by –10684 and above by –5184

Iteration #2

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	0
2	DELUXE	1500

Primal subproblem results:

Scenario #1
Optimal objective: 11546
Second-stage: nonzero variables

i	variable	value
2	OTCD	870
3	OTS	650
4	OTF	292
5	OTPI	240

Scenario #2
Optimal objective: 12716
Second-stage: nonzero variables

i	variable	value
2	OTCD	920
3	OTS	690
4	OTF	372
5	OTPI	250

Scenario #3
Optimal objective: 12616
Second-stage: nonzero variables

i	variable	value
2	OTCD	900
3	OTS	700
4	OTF	362
5	OTPI	255

Scenario #4
Optimal objective: 13786
Second-stage: nonzero variables

i	variable	value
2	OTCD	950
3	OTS	740
4	OTF	442
5	OTPI	265

--Primal subproblems summary
Second stage costs:

k	cost	p[k]
1	11546	0.3
2	12716	0.3
3	12616	0.3
4	13786	0.1

First stage cost: ¯13500.00
Expected second stage cost: 12442.00
Total: ¯1058.00 *(not a new incumbent!)*

Lagrangian multipliers *(Dual variables π of second-stage constraints)*

1)	¯5	¯6	¯8	¯4
2)	¯5	¯6	¯8	¯4
3)	¯5	¯6	¯8	¯4
4)	¯5	¯6	¯8	¯4
Sum	¯20	¯24	¯32	¯16

Using the dual variables from each of the subproblems, an additional constraint is generated: $\theta_k \geq \left(-\hat{\pi}_k T_k\right) X + \left(\hat{\pi}_k h_k\right)$

Scenario 1

Cut	Lambda		Alpha
1	8	5.333333	-5664.0
2	14.9	16.33333	-12954.0

Scenario 2

Cut	Lambda		Alpha
1	8	5.333333	-5024.0
2	14.9	16.33333	-11784.0

Scenario 3

Cut	Lambda		Alpha
1	8	5.333333	-5104.0
2	14.9	16.33333	-11884.0

Scenario 4

Cut	Lambda		Alpha
1	8	5.333333	-4464.0
2	14.9	16.33333	-10714.0

It has happened that in these first two iterations, in which the trial solutions (0,0) and (0,1500) are rather extreme decisions, the dual solutions are the same for each scenario, and since the matrix T does not vary by scenario for this problem, the coefficients of X in the cuts at each iteration are the same!

Solution of Master Problem

```
value= ¯7386.3333
X= 0 614.54545
First-stage cost: ¯5530.9091
Estimated Q(X): ¯2386.4242 ¯1746.4242 ¯1826.4242 ¯676.42424
Total (estimated) expected value: ¯7386.3333
```

This is an *underestimate* of the optimal cost, so we now know that the optimal cost is bounded

below by –7386 and above by –5184

Iteration #3

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	0.00000
2	DELUXE	614.54545

Primal subproblem results:

Scenario #1
Optimal objective: ¯858.133
Second-stage: nonzero variables

i	variable	value
1	ADD	298.303030
2	OTCD	193.357576
3	OTS	61.272727
5	OTPI	48.466667

Scenario #2
Optimal objective: ¯240.133
Second-stage: nonzero variables

i	variable	value
1	ADD	218.303030
2	OTCD	187.357576
3	OTS	61.272727
5	OTPI	50.466667

Scenario #3
Optimal objective: ¯271.133
Second-stage: nonzero variables

i	variable	value
1	ADD	228.303030
2	OTCD	174.357576
3	OTS	76.272727
5	OTPI	56.466667

Scenario #4
Optimal objective: 346.867
Second-stage: nonzero variables

i	variable	value
1	ADD	148.303030
2	OTCD	168.357576
3	OTS	76.272727
5	OTPI	58.466667

--Primal subproblems summary
Second stage costs:

k	cost	p[k]
1	¯858.13333	0.3
2	¯240.13333	0.3
3	¯271.13333	0.3
4	346.86667	0.1

First stage cost:	¯5530.91	
Expected second stage cost:	¯376.13	
Total:	¯5907.04	*New incumbent!*

Lagrangian multipliers (Dual variables π for 2^{nd} stage constraints)

1)	¯5	¯6	¯1.1	¯4
2)	¯5	¯6	¯1.1	¯4
3)	¯5	¯6	¯1.1	¯4
4)	¯5	¯6	¯1.1	¯4
Sum	¯20	¯24	¯4.4	¯16

Solution of Master Problem

value= ¯7270.1091
X= 387.41395 371.53125
First-stage cost: ¯7217.9207
Estimated Q(X): ¯583.18841 56.811594 ¯23.188406 1126.8116
Total (estimated) expected value: ¯7270.1091

This is an *underestimate* of the optimal cost, so we now know that the optimal cost is bounded

below by –7270 and above by –5907

Iteration #4

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	387.41395
2	DELUXE	371.53125

Primal subproblem results:

Scenario #1
Optimal objective: ¯248.78216
Second-stage: nonzero variables

i	variable	value
1	ADD	72.8985507
2	OTCD	63.7500000
5	OTPI	3.9140625
7	slack_2	60.2343750

Scenario #2
Optimal objective: 396.9135
Second-stage: nonzero variables

i	variable	value
2	OTCD	62.7210145
4	OTF	7.1014493
5	OTPI	6.6242074
7	slack_2	56.6836504

Scenario #3
Optimal objective: 248.21784
Second-stage: nonzero variables

i	variable	value
1	ADD	2.8985507
2	OTCD	44.7500000
5	OTPI	11.9140625
7	slack_2	45.2343750

Scenario #4
Optimal objective: 1166.9135
Second-stage: nonzero variables

i	variable	value
2	OTCD	92.7210145
4	OTF	77.1014493
5	OTPI	21.6242074
7	slack_2	6.6836504

--Primal subproblems summary

Second stage costs:

k	cost	p[k]
1	¯248.78216	0.3
2	396.91350	0.3
3	248.21784	0.3
4	1166.91350	0.1

First stage cost: ¯7217.92
Expected second stage cost: 235.60
Total: ¯6982.32 ***New incumbent!***

Lagrangian multipliers

1)	¯5	0	¯4.1	¯4
2)	¯5	0	¯8.0	¯4
3)	¯5	0	¯4.1	¯4
4)	¯5	0	¯8.0	¯4
Sum	¯20	0	¯24.2	¯16

We now know that the optimal cost is bounded

below by –7270 and above by –6982

Solution of Master Problem

value= ¯7109.7519
X= 547.10997 273.17647
First-stage cost: ¯7929.688
Estimated Q(X): 169.82097 1182.6087 729.82097 1952.6087
Total (estimated) expected value: ¯7109.7519

We now know that the optimal cost is bounded

below by –7109 and above by –6982

Iteration #5

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	547.10997
2	DELUXE	273.17647

Primal subproblem results:

Scenario #1
Optimal objective: 300.58824
Second-stage: nonzero variables

i	variable	value
2	OTCD	26.153453
4	OTF	21.227621
7	slack_2	98.797954
9	slack_4	11.994885

Scenario #2
Optimal objective: 1190.5882
Second-stage: nonzero variables

i	variable	value
2	OTCD	76.1534527
4	OTF	101.2276215
7	slack_2	58.7979540
9	slack_4	1.9948849

Scenario #3
Optimal objective: 1022.6087
Second-stage: nonzero variables

i	variable	value
2	OTCD	56.1534527
4	OTF	91.2276215
5	OTPI	3.0051151
7	slack_2	48.7979540

Scenario #4
Optimal objective: 1952.6087
Second-stage: nonzero variables

i	variable	value
2	OTCD	106.153453
4	OTF	171.227621
5	OTPI	13.005115
7	slack_2	8.797954

Multipliers for scenario 4 were previously generated!

--Primal subproblems summary
Second stage costs:

k	cost	p[k]
1	300.58824	0.3
2	1190.58824	0.3
3	1022.60870	0.3
4	1952.60870	0.1

First stage cost: ¯7929.69
Expected second stage cost: 949.40
Total: ¯6980.29 *Not a new incumbent!*

Lagrangian multipliers

1	¯5	0	¯8	0
2	¯5	0	¯8	0
3	¯5	0	¯8	¯4
4	¯5	0	¯8	¯4
Sum	¯20	0	¯32	¯8

Solution of Master Problem

value= ¯7065.3167
X= 472.0362   273.17647
First-stage cost: ¯7178.95
Estimated Q(X): ¯430.76 327.23 129.23 1059.23
Total (estimated) expected value: ¯7065.3167

We now know that the optimal cost is bounded

below by –7065   and   above by –6982

Iteration #6

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	472.03620
2	DELUXE	273.17647

Primal subproblem results:

Scenario #1
Optimal objective: ¯374.29864
Second-stage: nonzero variables

i	variable	value
1	ADD	53.846154
2	OTCD	11.294118
7	slack_2	109.411765
9	slack_4	14.117647

Scenario #2
Optimal objective: 327.23982
Second-stage: nonzero variables

i	variable	value
2	OTCD	23.6018099
4	OTF	26.1538462
7	slack_2	96.3348416
9	slack_4	9.5022624

Scenario #3
Optimal objective: 147.23982
Second-stage: nonzero variables

i	variable	value
2	OTCD	3.6018099
4	OTF	16.1538462
7	slack_2	86.3348416
9	slack_4	4.5022624

Scenario #4
Optimal objective: 1059.2308
Second-stage: nonzero variables

i	variable	value
2	OTCD	53.6018099
4	OTF	96.1538462
5	OTPI	5.4977376
7	slack_2	46.3348416

Multipliers for scenario 2 were previously generated!

Multipliers for scenario 4 were previously generated!

--Primal subproblems summary
Second stage costs:

k	cost	p[k]
1	¯374.29864	0.3
2	327.23982	0.3
3	147.23982	0.3
4	1059.23077	0.1

First stage cost: ¯7178.95
Expected second stage cost: 135.98
Total: ¯7042.97 *New incumbent!*

Lagrangian multipliers

1)	¯5	0	¯4.5	0
2)	¯5	0	¯8.0	0
3)	¯5	0	¯8.0	0
4)	¯5	0	¯8.0	¯4
Sum	¯20	0	¯28.5	¯4

Solution of Master Problem

```
value= ¯7046.2

X= 497.14286    252
First-stage cost: ¯7239.4286
Estimated Q(X): ¯342.85714  397.14286  ¯217.14286   1118
Total (estimated) expected value: ¯7046.2
```

We now know that the optimal cost is bounded

below by –7046.2   and   above by –7042.97

Iteration #7

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	497.14286
2	DELUXE	252.00000

Primal subproblem results:

```
Scenario #1
Optimal objective: ¯342.85714
Second-stage: nonzero variables
 i   variable     value

  1  ADD      42.857143
  7  slack_2 120.000000
  9  slack_4  18.000000
--------------------
Scenario #2
Optimal objective: 397.14286
Second-stage: nonzero variables
 i   variable     value

  2  OTCD     20.000000
  4  OTF      37.142857
  7  slack_2 101.428571
  9  slack_4  12.285714
--------------------
Scenario #3
Optimal objective: 217.14286
Second-stage: nonzero variables
 i   variable     value

  4  OTF     27.1428571
  7  slack_2 91.4285714
  9  slack_4  7.2857143
--------------------
Scenario #4
Optimal objective: 1118
Second-stage: nonzero variables
 i   variable      value

  2  OTCD     50.0000000
  4  OTF     107.1428571
  5  OTPI      2.7142857
  7  slack_2  51.4285714
--------------------

Multipliers for scenario 1 were previously generated!
Multipliers for scenario 2 were previously generated!
Multipliers for scenario 3 were previously generated!
Multipliers for scenario 4 were previously generated!
```



```
Primal subproblems summary
     Second stage costs:
                           k         cost   p[k]
                           1   ¯342.85714    0.3
                           2    397.14286    0.3
                           3    217.14286    0.3
                           4   1118.00000    0.1

First stage cost:                  ¯7239.43
Expected second stage cost:          193.23
Total:                             ¯7046.20
```

New Incumbent!

```
     Lagrangian multipliers
   1)   0 0  ¯8  0
   2)  ¯5 0  ¯8  0
   3)   0 0  ¯8  0
   4)  ¯5 0  ¯8 ¯4
 Sum ¯10 0 ¯32 ¯4
```

We now know that the optimal cost is bounded

below by –7046.2   and   above by –7046.2 *(converged!)*

Benders' Master Problem -- final (dual) tableau

b	z	1	2	3	1	2	3	4	5	6
0	1	0	0	1500	5664	5024	5104	4464	12954	11784
¯10	0	1	0	¯1	¯8	¯8	¯8	¯8	¯14.9	¯14.9
¯9	0	0	1	¯1	¯5.3333	¯5.333	¯5.333	¯5.333	¯16.333	¯16.333
0.3	0	0	0	0	1	0	0	0	1	0
0.3	0	0	0	0	0	1	0	0	0	1
0.3	0	0	0	0	0	0	1	0	0	0
0.1	0	0	0	0	0	0	0	1	0	0

7	8	9	1	2	3	4	5
11884	10714	8068.8	7450.8	7481.8	6863.8	6592.8	8424
¯14.9	¯14.9	¯8	¯8	¯8	¯8	¯8	¯11.9
¯16.333	¯16.333	¯11.733	¯11.733	¯11.733	¯11.733	¯8.7333	¯11.333
0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	1
1	0	0	0	1	0	0	0
0	1	0	0	0	1	0	0

6	7	8	9	1	2	3
6095.8	7654	8814	7924	8584	6336	8104
¯8	¯11.9	¯11.5	¯11.5	¯11.9	¯8	¯11.5
¯8.7333	¯11.333	¯10.333	¯10.333	¯11.333	¯8	¯10.333
0	0	1	0	0	1	0
0	0	0	1	0	0	0
1	0	0	0	1	0	1
0	1	0	0	0	0	0


```
<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>-<>
                    Optimality Cuts
For each scenario k, a set of linear functions of X has been
generated, each of which is an underestimate of the second-stage
cost as a function of X.
Hence the maximum provides an underestimating piece-wise linear
function of X, each of the form
              Max { (X)(Lambda[k]) + Alpha[k]}.
----------------------------------------
```


Scenario 1

Cut	Lambda	Alpha
1	8 5.3333333	¯5664.0
2	14.9 16.333333	¯12954.0
3	8 11.733333	¯8068.8
4	8 8.7333333	¯6592.8
5	11.5 10.333333	¯8814.0
6	8 8	¯6336.0
7	8 5.3333333	¯5664.0

Scenario 2

Cut	Lambda	Alpha
1	8 5.3333333	¯5024.0
2	14.9 16.333333	¯11784.0
3	8 11.733333	¯7450.8
4	11.9 11.333333	¯8424.0
5	11.5 10.333333	¯7924.0
6	11.5 10.333333	¯7924.0
7	11.5 10.333333	¯7924.0

Scenario 3

Cut	Lambda	Alpha
1	8 5.3333333	¯5104.0
2	14.9 16.333333	¯11884.0
3	8 11.733333	¯7481.8
4	8 8.7333333	¯6095.8
5	11.9 11.333333	¯8584.0
6	11.5 10.333333	¯8104.0
7	8 5.3333333	¯5104.0

Scenario 4

Cut	Lambda	Alpha
1	8 5.3333333	¯4464.0
2	14.9 16.333333	¯10714.0
3	8 11.733333	¯6863.8
4	11.9 11.333333	¯7654.0
5	11.9 11.333333	¯7654.0
6	11.9 11.333333	¯7654.0
7	11.9 11.333333	¯7654.0

Estimate of cost of trial solution

Suppose that the optimal solution (X_1=497.14286, X_2= 252) is not convenient to implement, and we wish to estimate the expected cost of the alternate solution (X_1=500, X_2= 250)

Without solving the subproblem for each scenario, we can use the approximations that we have computed to estimate the expected cost:

$$\sum_{k=1}^{K} p_k \underline{Q}_k(x_1, x_2) = \sum_{k=1}^{K} p_k \times \max_{j=1,\ldots} \left\{ \lambda_k^j x + \alpha_k^j \right\}$$

where $\lambda_k^1 = -\pi_k^1 T_k$ & $\alpha_k^1 = \pi_k^1 h_k$

Scenario 1 Computation

Cut#	Value
1	¯330.66667
2	¯1420.66667
3	¯1135.46667
4	¯409.46667
5	¯480.66667
6	¯336.00000
7	¯330.66667

Maximum: ¯330.66667
= *underestimate of cost*

Scenario 2 Computation

Cut#	Value
1	309.33333
2	¯250.66667
3	¯517.46667
4	359.33333
5	409.33333
6	409.33333
7	409.33333

Maximum: 409.33333
= *underestimate of cost*

Scenario 3 Computation

Cut#	Value
1	229.333334
2	¯350.666666
3	¯548.466667
4	87.533334
5	199.333334
6	229.333334
7	229.333334

Maximum: 229.33333
= *underestimate of cost*

Scenario 4 Computation

Cut#	Value
1	869.333334
2	819.333334
3	69.533333
4	1129.333334
5	1129.333334
6	1129.333334
7	1129.333334

Maximum: 1129.3333
= *underestimate of cost*

Estimated Second stage objective:

k	objective	p[k]
1	¯330.66667	0.3
2	409.33333	0.3
3	229.33333	0.3
4	1129.33333	0.1

First stage objective: ¯7250.00
Expected second stage objective: 205.33
Total: ¯7044.67

That is, the expected total cost will be no less than ¯7044.67 (expected total profit will be no greater than 7044.67).

This is nearly as much as the optimal expected profit, 7046.20.

To test how good this approximation is, let's solve another subproblem, with X_1=500, X_2=250:

Evaluation of trial solution

i	variable	X[i]
1	STANDARD	500
2	DELUXE	250

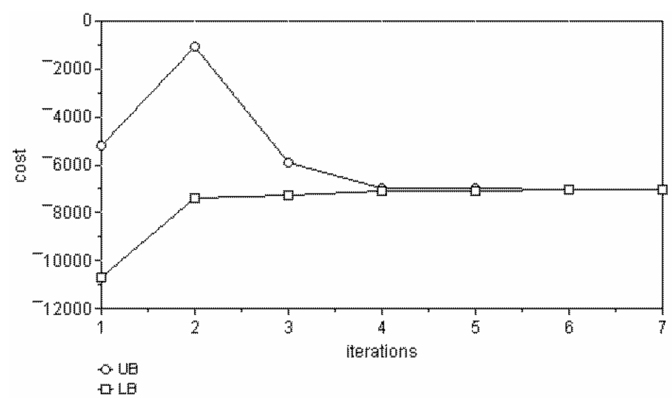
Second stage objective:

k	objective	p[k]
1	¯330.66667	0.3
2	409.33333	0.3
3	229.33333	0.3
4	1129.33333	0.1

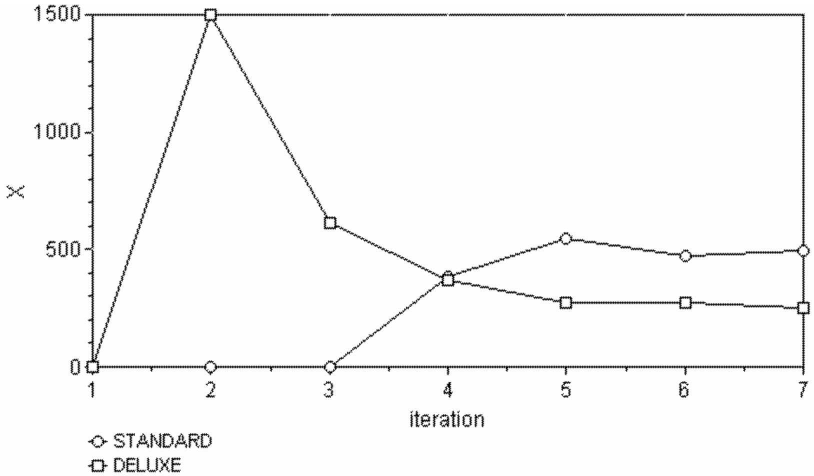
First stage objective: ¯7250.00
Expected second stage objective: 205.33
Total: ¯7044.67

So in fact this was a precise estimate!

Plot of Convergence of Upper & Lower Bounds



Plot of Convergence of Optimal Decisions



Uni-Cut Version

In the **"multi-cut"** version (above),

- an approximation $\underline{Q}_k(x)$ is constructed for the second-stage cost of *each* scenario k=1, 2, … K
- K supports are added to Benders' Partial Master Problem at each iteration.

In the **"uni-cut"** version,

- the "multi-cut" supports are aggregated to form an approximation $\underline{Q}(x)$ of the *expected* second-stage cost
- a single support is added to Benders' Partial Master Problem at each iteration.

Benders' Partial Master Problem *(final iteration)*

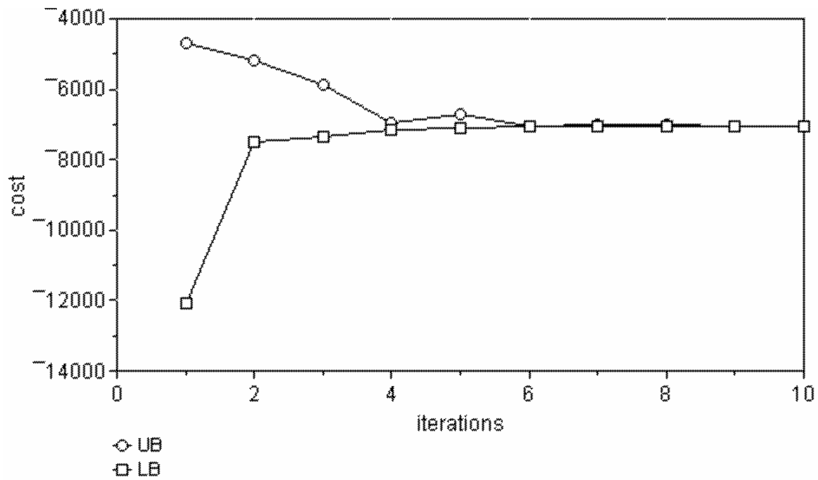
$$\textit{Minimize} \quad -10X_1 - 9X_2 + \theta$$

subject to:

$$\begin{aligned}
&X_1 + X_2 \le 1500\\
&\theta \ge 14.9X_1 + 16.3333X_2 - 12058\\
&\theta \ge 8X_1 + 5.3333X_2 - 5184\\
&\theta \ge 8X_1 + 11.7333X_2 - 7586.8\\
&\theta \ge 11.03X_1 + 11.0533X_2 - 8151.64\\
&\vdots\\
&\theta \ge 9.44X_1 + 7.43333X_2 - 6373\\
&\quad X_1 \ge 0,\ \ X_2 \ge 0
\end{aligned}$$

Cut		Lambda	Alpha
1)	14.9	16.3333	¯12058.00
2)	8	5.33333	¯5184.00
3)	8	11.7333	¯7586.80
4)	11.03	11.0533	¯8151.64
5)	11.5	10.3333	¯8174.00

Cut		Lambda	Alpha
6)	9.44	8.23333	¯6574.60
7)	10.49	9.73333	¯7474.60
8)	9.4	7.33333	¯6329.00
9)	9.44	7.43333	¯6373.00



UNI-CUT VERSION

Iteration #1

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	1500
2	DELUXE	0

Primal subproblem results:

Scenario #1
Optimal objective: 9396

i	variable	value
2	OTCD	420
3	OTS	150
4	OTF	792
5	OTPI	15

Scenario #2
Optimal objective: 10566

i	variable	value
2	OTCD	470
3	OTS	190
4	OTF	872
5	OTPI	25

Scenario #3
Optimal objective: 10466
Second-stage: nonzero variables

i	variable	value
2	OTCD	450
3	OTS	200
4	OTF	862
5	OTPI	30

Scenario #4
Optimal objective: 11636

i	variable	value
2	OTCD	500
3	OTS	240
4	OTF	942
5	OTPI	40

Primal subproblems summary

Second stage costs:

k	cost	p[k]
1	9396	0.3
2	10566	0.3
3	10466	0.3
4	11636	0.1

First stage cost: ¯15000
Expected 2nd stage cost: 10292
Total: ¯4708

Lagrangian multipliers

i	1	2	3	4
1	¯5	¯6	¯8	¯4
2	¯5	¯6	¯8	¯4
3	¯5	¯6	¯8	¯4
4	¯5	¯6	¯8	¯4
Sum	¯20	¯24	¯32	¯16

Solution of Master Problem

value= ¯12058
X= 0 0
First-stage cost: 0
Estimated Q(X): ¯12058

Iteration #2

Trial X for primal subproblems:

i	Variable	Value
1	STANDARD	0
2	DELUXE	0

Primal subproblem results:

Second stage costs:

k	cost	p[k]
1	¯5664	0.3
2	¯5024	0.3
3	¯5104	0.3
4	¯4464	0.1

First stage cost: 0
Expected 2nd stage cost: ¯5184
Total: ¯5184

Lagrangian multipliers

i	1	2	3	4
1	¯1.77636E¯15	0	¯8	0
2	¯1.77636E¯15	0	¯8	0
3	¯1.77636E¯15	0	¯8	0
4	¯1.77636E¯15	0	¯8	0
Sum	¯7.10543E¯15	0	¯32	0

Solution of Master Problem

value= ¯7475.33
X= 0 624.909
First-stage cost: ¯5624.18
Estimated Q(X): ¯1851.15

Iteration #3

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	0.000
2	DELUXE	624.909

Primal subproblem results:

k	cost	p[k]
1	¯736.533	0.3
2	¯118.533	0.3
3	¯149.533	0.3
4	468.467	0.1

First stage cost: ¯5624.18
Expected 2nd stage cost: ¯254.53
Total: ¯5878.72

Lagrangian multipliers

i	1	2	3	4
1	¯5	¯6	¯1.1	¯4
2	¯5	¯6	¯1.1	¯4
3	¯5	¯6	¯1.1	¯4
4	¯5	¯6	¯1.1	¯4
Sum	¯20	¯24	¯4.4	¯16

Solution of Master Problem

value= ¯7356.02
X= 397.708 375.438
First-stage cost: ¯7356.02
Estimated Q(X): 1.37837E¯13

Iteration #4

Trial X for primal subproblems:

i	Variable	Value
1	STANDARD	397.708
2	DELUXE	375.438

Primal subproblem results:

Second stage costs:

k	cost	p[k]
1	¯132.313	0.3
2	563.687	0.3
3	403.687	0.3
4	1344.000	0.1

First stage cost: ¯7356.02
Expected 2nd stage cost: 384.92
Total: ¯6971.10

Lagrangian multipliers

i	1	2	3	4
1	¯5	0	¯4.1	¯4
2	¯5	0	¯8.0	¯4
3	¯5	0	¯8.0	¯4
4	¯5	¯6	¯8.0	¯4
Sum	¯20	¯6	¯28.1	¯16

Multipliers for scenario 4 were previously generated!

Solution of Master Problem

value= ¯7142.84
X= 979.419 0
First-stage cost: ¯9794.19
Estimated Q(X): 2651.35

etc.

Iteration #6

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	465.010
2	DELUXE	272.493

Primal subproblem results:

Second stage costs:

k	cost	p[k]
1	¯435.9754	0.3
2	239.3771	0.3
3	69.3771	0.3
4	967.8741	0.1

First stage cost: ¯7102.54

Expected 2nd stage cost: 58.62

Total: ¯7043.92

Lagrangian multipliers

i	1	2	3	4
1	¯5.00000E0	0	¯4.5	0
2	¯5.00000E0	0	¯8.0	0
3	¯1.77636E¯15	0	¯8.0	0
4	¯5.00000E0	0	¯8.0	¯4
Sum	¯1.50000E1	0	¯28.5	¯4

Multipliers for scenarios 2&3 were previously generated!

Solution of Master Problem

value= ¯7065.32

X= 485.193 285.668

First-stage cost: ¯7422.94

Estimated Q(X): 357.62

Iteration #7

Trial X for primal subproblems:

i	Variable	Value
1	STANDARD	485.193
2	DELUXE	285.668

Primal subproblem results:

Second stage costs:

k	cost	p[k]
1	¯169.113	0.3
2	607.620	0.3
3	427.620	0.3
4	1357.365	0.1

First stage cost: ¯7422.94

Expected 2nd stage cost: 395.57

Total: ¯7027.37

Lagrangian multipliers

i	1	2	3	4
1	¯5	0	¯4.5	0
2	¯5	0	¯8.0	0
3	¯5	0	¯8.0	0
4	¯5	0	¯8.0	¯4
Sum	¯20	0	¯28.5	¯4

Multipliers for scenarios 1,2,3,4 were previously generated!

Solution of Master Problem

value= ¯7048.02

X= 575.254 197.322

First-stage cost: ¯7528.44

Estimated Q(X): 480.418

Iteration #8

Trial X for primal subproblems is

i	Variable	Value
1	STANDARD	575.254
2	DELUXE	197.322

Primal subproblem results:

Second stage costs:

k	cost	p[k]
1	¯9.58192	0.3
2	730.41808	0.3
3	550.41808	0.3
4	1440.41808	0.1

First stage cost: ¯7528.44

Expected 2nd stage cost: 525.42

Total: ¯7003.02

Lagrangian multipliers

i	1	2	3	4
1	¯1.77636E¯15	0	¯8	0
2	¯5.00000E0	0	¯8	0
3	¯1.77636E¯15	0	¯8	0
4	¯5.00000E0	0	¯8	0
Sum	¯1.00000E1	0	¯32	0

Multipliers for scenarios 1,2,3,4 were previously generated!

Solution of Master Problem

value= ¯7046.24

X= 498.983 250.712

First-stage cost: ¯7246.24

Estimated Q(X): 199.994

Iteration #9

Trial X for primal subproblems:

i	Variable	Value
1	STANDARD	498.983
2	DELUXE	250.712

Primal subproblem results:

Second stage costs:

k	cost	p[k]
1	¯335.006	0.3
2	404.994	0.3
3	224.994	0.3
4	1125.299	0.1

First stage cost: ¯7246.24

Expected 2nd stage cost: 201.02

Total: ¯7045.21

Lagrangian multipliers

i	1	2	3	4
1	¯1.77636E¯15	0	¯8	0
2	¯5.00000E0	0	¯8	0
3	¯1.77636E¯15	0	¯8	0
4	¯5.00000E0	0	¯8	¯4
Sum	¯1.00000E1	0	¯32	¯4

Multipliers for scenarios 1,2,3,4 were previously generated!

Solution of Master Problem

value= ¯7046.2

X= 497.143 252

First-stage cost: ¯7239.43

Estimated Q(X): 193.229

Iteration #10

Trial X for primal subproblems:

i	Variable	Value
1	STANDARD	497.143
2	DELUXE	252.000

Primal subproblem results:

Scenario #1
Optimal objective: ¯342.857
Second-stage: nonzero variables

i	variable	value
1	ADD	42.8571
7	slack_2	120.0000
9	slack_4	18.0000

Scenario #2
Optimal objective: 397.143
Second-stage: nonzero variables

i	variable	value
2	OTCD	20.0000
4	OTF	37.1429
7	slack_2	101.4286
9	slack_4	12.2857

Scenario #3
Optimal objective: 217.143
Second-stage: nonzero variables

i	variable	value
4	OTF	27.1429
7	slack_2	91.4286
9	slack_4	7.2857

Scenario #4
Optimal objective: 1118
Second-stage: nonzero variables

i	variable	value
2	OTCD	50.00000
4	OTF	107.14286
5	OTPI	2.71429
7	slack_2	51.42857

Second stage costs:

k	cost	p[k]
1	¯342.857	0.3
2	397.143	0.3
3	217.143	0.3
4	1118.000	0.1

First stage cost: ¯7239.43
Expected 2nd stage cost: 193.23
Total: ¯7046.20

Lagrangian multipliers

i	1	2	3	4
1	0	0	¯8	0
2	¯5	0	¯8	0
3	0	0	¯8	0
4	¯5	0	¯8	¯4
Sum	¯10	0	¯32	¯4

Multipliers for scenarios 1,2,3,4 were previously generated!

Solution of Master Problem

value= ¯7046.2
X= 497.143 252
First-stage cost: ¯7239.43
Estimated Q(X): 193.229

****

Converged at iteration #10!
X was generated by previous master problem!

Note that the number of iterations required is somewhat larger when the uni-cut version is used.