A Newsboy at Monte Carlo

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A newsboy orders newspapers which are delivered to him each morning at a cost of 15¢ each. He then sells them at his news stand for 50¢ each.

Daily demand is uncertain, and any excess newspapers are of no value.

Assuming that demand is normally distributed with mean 25 and standard deviation 5, how many newspapers should he order so as to maximize his average daily profit?

While this problem can be solved analytically, let's use Monte Carlo simulation and "trial & error" to search for the answer.

We will perform this simulation using an Excel® spreadsheet.

First we enter the problem data:

	A	B	C	D	E
1					
2	Demand distribution: Normal				
3	Mean:		25		
4	Std Deviation:		5		
5	Cost per newspaper:		$0.15		
6	Selling price per newspaper:			$0.50	
7	Order quantity:		30		
8					

We will create *seven* columns:

- A: random variable with N(0,1) distribution
- B: random demand: column A scaled to μ=25 and σ=5, and rounded to an integer
- C: sales (minimum of demand [column B] and order quantity)
- D: "Yes" or "No" indicating whether a shortage has occurred
- E: revenue (demand [column C] times selling price)
- F: cost (order quantity times cost)
- G: profit (revenue [column E] minus cost [column F])

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:		25				
4	Std Deviation:		5				
5	Cost per newspaper:		$0.15				
6	Selling price per newspaper:		$0.50				
7	Order quantity:		30				
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11							

We will approximate a N(0,1) random number by scaling a sum of uniformly-distributed random numbers (trusting the Central Limit Theorem):

$$Y = \frac{\sum_{i=1}^{n} X_i - 0.5n}{\left(\dfrac{n}{12}\right)^{\frac{1}{2}}}$$

SUM ▾ ✗ ✓ *fx* =(SUM(RAND(),RAND(),RAND(),RAND(),RAND(),RAND())-3)/0.707107

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:	25					
4	Std Deviation:	5					
5	Cost per newspaper:	$0.15					
6	Selling price per newspaper:		$0.50				
7	Order quantity:	30					
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	707107						
12							

Here we have used n=6, while n=12 is usually recommended!

Next we scale the N(0,1) random number so that it is N(25,5), and round it to the nearest integer:

SUM ▾ ✗ ✓ *fx* =ROUND($C$3+$C$4*A10,0)

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:		25				
4	Std Deviation:		5				
5	Cost per newspaper:		$0.15				
6	Selling price per newspaper:			$0.50			
7	Order quantity:		30				
8							
9							
10	N(0,1)#	Demand	Sales		Shortage?	Revenue	Cost	Profit
11	-0.3329						
12							

Note that absolute addresses are used for the mean & standard deviation, and a relative address for the N(0,1) random number.

The number of newspapers sold is the smaller of the demand and the order quantity:

SUM ▾ ✗ ✓ *fx* =MIN($C$7,B11)

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:	25					
4	Std Deviation:	5					
5	Cost per newspaper:	$0.15					
6	Selling price per newspaper:		$0.50				
7	Order quantity:	30					
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	0.87579	29	$7,B11)				
12							

A shortage has occurred if the demand exceeds the sales:

SUM ▾ ✗ ✓ *fx* =IF(B11>C11,"YES","NO")

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:		25				
4	Std Deviation:		5				
5	Cost per newspaper:		$0.15				
6	Selling price per newspaper:			$0.50			
7	Order quantity:		30				
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	0.52847	28	28	:S","NO")			
12							

Revenue is selling price times sales:

Formula bar: `=$D$6*C11`

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:		25				
4	Std Deviation:		5				
5	Cost per newspaper:		$0.15				
6	Selling price per newspaper:			$0.50			
7	Order quantity:		30				
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	-0.92358	20	20	NO	$6*C11		
12							

Cost is the cost per paper times the quantity ordered:

Formula bar: `=$C$5*$C$7`

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:		25				
4	Std Deviation:		5				
5	Cost per newspaper:		$0.15				
6	Selling price per newspaper:			$0.50			
7	Order quantity:		30				
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	-0.91389	20	20	NO	$10.00	C$5*$C$7	
12							

Finally, profit is revenue minus cost:

Formula bar: `=E11-F11`

	A	B	C	D	E	F	G
1							
2	Demand distribution: Normal						
3	Mean:		25				
4	Std Deviation:		5				
5	Cost per newspaper:		$0.15				
6	Selling price per newspaper:			$0.50			
7	Order quantity:		30				
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	0.58777	28	28	NO	$14.00	$4.50	=E11-F11
12							

We will simulate a 20-day period (5 days per week for 4 weeks) by copying the formulas:

	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	-1.08073	20	20	NO	$10.00	$4.50	$5.50
12	0.42853	27	27	NO	$13.50	$4.50	$9.00
13	0.53685	28	28	NO	$14.00	$4.50	$9.50
14	0.32154	27	27	NO	$13.50	$4.50	$9.00
15	-0.40174	23	23	NO	$11.50	$4.50	$7.00
16	0.10182	26	26	NO	$13.00	$4.50	$8.50
17	-0.82969	21	21	NO	$10.50	$4.50	$6.00
18	-0.63724	22	22	NO	$11.00	$4.50	$6.50
19	-0.66131	22	22	NO	$11.00	$4.50	$6.50
20	-0.35989	23	23	NO	$11.50	$4.50	$7.00
21	0.08854	25	25	NO	$12.50	$4.50	$8.00
22	0.63299	28	28	NO	$14.00	$4.50	$9.50
23	1.17011	31	30	YES	$15.00	$4.50	$10.50
24	-0.14265	24	24	NO	$12.00	$4.50	$7.50
25	0.496	27	27	NO	$13.50	$4.50	$9.00
26	-0.16167	24	24	NO	$12.00	$4.50	$7.50
27	0.26197	26	26	NO	$13.00	$4.50	$8.50
28	-0.38707	23	23	NO	$11.50	$4.50	$7.00
29	0.08018	25	25	NO	$12.50	$4.50	$8.00
30	-1.12241	19	19	NO	$9.50	$4.50	$5.00

We will add a formula to calculate the average daily profit:

	A	B	C	D	E	F	G
	SUM	▾ ✗ ✓ fx	=SUM(G11:G30)/20				
8							
9							
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	-1.08073	20	20	NO	$10.00	$4.50	$5.50
12	0.42853	27	27	NO	$13.50	$4.50	$9.00
13	0.53685	28	28	NO	$14.00	$4.50	$9.50
14	0.32154	27	27	NO	$13.50	$4.50	$9.00
15	-0.40174	23	23	NO	$11.50	$4.50	$7.00
16	0.10182	26	26	NO	$13.00	$4.50	$8.50
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18	-0.63724	22	22	NO	$11.00	$4.50	$6.50
19	-0.66131	22	22	NO	$11.00	$4.50	$6.50
20	-0.35989	23	23	NO	$11.50	$4.50	$7.00
21	0.08854	25	25	NO	$12.50	$4.50	$8.00
22	0.63299	28	28	NO	$14.00	$4.50	$9.50
23	1.17011	31	30	YES	$15.00	$4.50	$10.50
24	-0.14265	24	24	NO	$12.00	$4.50	$7.50
25	0.496	27	27	NO	$13.50	$4.50	$9.00
26	-0.16167	24	24	NO	$12.00	$4.50	$7.50
27	0.26197	26	26	NO	$13.00	$4.50	$8.50
28	-0.38707	23	23	NO	$11.50	$4.50	$7.00
29	0.08018	25	25	NO	$12.50	$4.50	$8.00
30	-1.12241	19	19	NO	$9.50	$4.50	$5.00
31							G30)/20

At any time, we can perform another 20-day simulation by pressing the F9 key:

	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
10	N(0,1)#	Demand	Sales	Shortage?	Revenue	Cost	Profit
11	1.06727	30	30	NO	$15.00	$4.50	$10.50
12	-0.67977	22	22	NO	$11.00	$4.50	$6.50
13	0.08515	25	25	NO	$12.50	$4.50	$8.00
14	-1.08906	20	20	NO	$10.00	$4.50	$5.50
15	0.44083	27	27	NO	$13.50	$4.50	$9.00
16	0.60701	28	28	NO	$14.00	$4.50	$9.50
17	0.25904	26	26	NO	$13.00	$4.50	$8.50
18	-0.30057	23	23	NO	$11.50	$4.50	$7.00
19	-0.01735	25	25	NO	$12.50	$4.50	$8.00
20	1.4511	32	30	YES	$15.00	$4.50	$10.50
21	-2.24874	14	14	NO	$7.00	$4.50	$2.50
22	0.06732	25	25	NO	$12.50	$4.50	$8.00
23	-0.34249	23	23	NO	$11.50	$4.50	$7.00
24	1.05039	30	30	NO	$15.00	$4.50	$10.50
25	-0.38358	23	23	NO	$11.50	$4.50	$7.00
26	0.65162	28	28	NO	$14.00	$4.50	$9.50
27	-1.17395	19	19	NO	$9.50	$4.50	$5.00
28	2.35603	37	30	YES	$15.00	$4.50	$10.50
29	0.72491	29	29	NO	$14.50	$4.50	$10.00
30	0.78013	29	29	NO	$14.50	$4.50	$10.00
31							$8.15
32							