A Manufacturing System with Product Inspection, Rejection, & Rework

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A machined part requires the following sequence of manufacturing steps:

- Machine A
- Inspection A
- Machine B
- Inspection B
- Machine C
- Inspection C
- Pack & Ship

During each machining step, parts could be ruined (perhaps because of a casting defect).

In the inspection step following each machine, the inspector may:

- pass the part to the next machine
- scrap the part if defective
- return the part to the preceding machine for rework

DATA

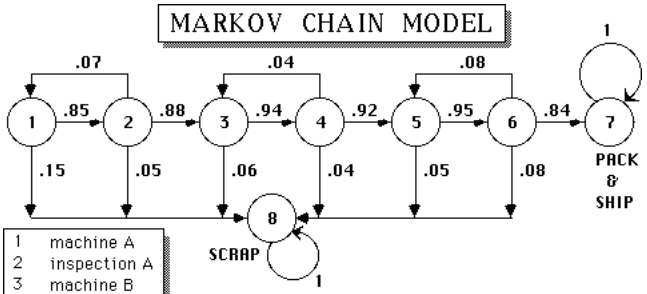
OPERATION	TIME RQMT. (man-hrs)	OPERATING COST ($/hr.)	SCRAP RATE(%)	% SENT BACK FOR REWORK
Machine A	5.0	12.00	15	
Inspection A	1.6	10.00	5	7
Machine B	3.0	12.00	6	
Inspection B	1.6	10.00	4	4
Machine C	2.7	15.00	5	
Inspection C	1.6	10.00	8	8
Pack & Ship	0.7	5.00		

Cost of blank part: $50
Salvage value of scrapped part: $12

Define a stochastic process for a part:

State	Location of Part	
1	Machine A	
2	Inspection station A	
3	Machine B	
4	Inspection station B	
5	Machine C	
6	Inspection station C	
7	Pack-&-Ship Dept.	*absorbing states*
8	Scrap bin	

MARKOV CHAIN MODEL



1	machine A
2	inspection A
3	machine B
4	inspection B
5	machine C
6	inspection C
7	pack-&-ship
8	scrap bin

TRANSITION PROBABILITY MATRIX:

	1	2	3	4	5	6	7	8
1	0	.85						.15
2	.07		.88					.05
3				.94				.06
4			.04		.92			.04
5						.95		.05
6					.08		.84	.08
7							1	
8								1

Partition the Matrix:

			transient states				*absorbing states*	
	1	2	3	4	5	6	7	8
1	0	.85						.15
2	.07		.88					.05
3				*Q* .94				*R* .06
4			.04		.92			.04
5						.95		.05
6					.08		.84	.08
7			*0*				1	*I* 1
8								

transient states / *absorbing states*

transitions from transient states to transient states $Q =$

$$\begin{bmatrix} 0 & 0.85 & 0 & 0 & 0 & 0 \\ 0.07 & 0 & 0.88 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.94 & 0 & 0 \\ 0 & 0 & 0.04 & 0 & 0.92 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.95 \\ 0 & 0 & 0 & 0 & 0.08 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$$

transitions from transient states to absorbing states $R =$

$$\begin{bmatrix} 0 & 0.15 \\ 0 & 0.05 \\ 0 & 0.06 \\ 0 & 0.04 \\ 0 & 0.05 \\ 0.84 & 0.08 \end{bmatrix}$$

EXPECTED NUMBER OF VISITS TO TRANSIENT STATES

$$E = (I - Q)^{-1}$$

	1	2	3	4	5	6
1:	1.06	.904	.826	.777	.773	.735
2:	.074	1.06	.972	.914	.91	.864
3:	0	0	1.04	.977	.972	.924
4:	0	0	.042	1.04	1.03	.983
5:	0	0	0	0	1.08	1.03
6:	0	0	0	0	.087	1.08

ABSORPTION PROBABILITIES

$$A = (I - Q)^{-1}R$$

	7	8
1:	.617	.383
2:	.726	.274
3:	.776	.224
4:	.826	.174
5:	.864	.136
6:	.909	.091

Parts arriving at machine A have a 61.7% probability of being successfully completed!

ESTIMATED MAN-HR RQMTS PER ENTERING PART

OPERATION	STATE	MAN-HR / ENTERING PART
MACHINE A	1	5.0 x 1.06 = 5.300
INSPECTION A	2	1.6 x .904 = 1.446
MACHINE B	3	3.0 x .826 = 2.478
INSPECTION B	4	1.6 x .777 = 1.243
MACHINE C	5	2.7 x .773 = 2.087
INSPECTION C	6	1.6 x .735 = 1.176
PACK & SHIP	7	0.7 x .617 = 0.432

TOTAL = 14.162 man-hrs

hrs/visit x # visits

ESTIMATED MAN-HR RQMTS PER COMPLETED PART

Each completed part requires an expected $1/0.617$ i.e., 1.6207 entering parts.

So we can multiply the man-hr requirements per entering part at each stage by the factor 1.6207 to get the expected man-hr requirements per completed part.

For example, the total man-hr. requirement (for all stages) will be 14.162 x 1.6207 = 22.95 man-hours

ESTIMATED MAN-HR RQMTS PER COMPLETED PART

OPERATION	STATE	MAN-HR / COMPLETED PART
Machine A	1	1.06 x 5 / 0.617 = 8.611
Inspection A	2	1.06 x 16/ 0.726 = 2.342
Machine B	3	1.04 x 3 / 0.776 = 4.016
Inspection B	4	1.04 x 1.6/ 0.826 = 2.014
Machine C	5	1.08 x 2.7/ 0.864 = 3.383
Inspection C	6	1.08 x 1.6/ 0.909 = 1.904
Pack-&-Ship	7	1 x 0.7/ 1.0 = 0.7

Total = 22.97 man-hrs

Expected Direct Costs per Completed Part

Materials: $50 x 1.6207 = $81.04

Scrap value recovered: $12 x 1.6207 x 0.383 = $7.45

OPERATIONS COST

OPERATION	HOURLY RATE	MAN-HRS	TOTAL COST
MACHINE A	12.00	8.613	103.40
INSPECTION A	10.00	2.343	23.43
MACHINE B	12.00	4.017	48.20
INSPECTION B	10.00	2.014	20.14
MACHINE C	15.00	3.383	50.75
INSPECTION C	10.00	1.905	19.05
PACK-&-SHIP	5.00	.700	3.50
		TOTAL =	$ 268.40

Total Direct Cost: $81.04 + $268.40 - $7.45 = $341.99