SunCo processes oil into aviation fuel and heating oil.
It costs $40 to purchase each barrel of oil, which is then distilled and yields 0.5 barrel of aviation fuel and 0.5 barrel of heating oil. Output from the distillation may be sold directly or processed in the catalytic cracker. If sold after distillation without further processing, aviation fuel sells for $60/barrel and heating oil for $40/barrel.

It takes 1 hour to process 1000 barrels of aviation fuel in the catalytic cracker, and these can be sold for $130/barrel.
It takes 45 minutes to process 1000 barrels of heating oil in the cracker, and these can be sold for $90/barrel.
Each day, at most 20,000 barrels of oil can be purchased, and 8 hours of catalytic cracker time are available.

Formulate an LP to maximize SunCo’s profits.

Maximize 40 HSOLD + 90 HCRACK + 60 ASOLD + 130 ACRACK - 40 OIL
subject to
OIL ≤ 20000
0.5 OIL = ASOLD + ACRACK
0.5 OIL = HSOLD + HCRACK
0.001 ACRACK + 0.00075 HCRACK ≤ 8
OIL ≥ 0, ASOLD ≥ 0, ACRACK ≥ 0, HSOLD ≥ 0, HCRACK ≥ 0

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>VALUE</th>
<th>REDUCED COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSOLD</td>
<td>100000</td>
<td>0.000000</td>
</tr>
<tr>
<td>HCRACK</td>
<td>0.0000</td>
<td>2.500000</td>
</tr>
<tr>
<td>ASOLD</td>
<td>20000</td>
<td>0.000000</td>
</tr>
<tr>
<td>ACRACK</td>
<td>80000</td>
<td>0.000000</td>
</tr>
<tr>
<td>OIL</td>
<td>20000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROW</th>
<th>SLACK OR SURPLUS</th>
<th>DUAL PRICES</th>
</tr>
</thead>
<tbody>
<tr>
<td>2)</td>
<td>0.000000</td>
<td>10.000000</td>
</tr>
<tr>
<td>3)</td>
<td>0.000000</td>
<td>-60.000000</td>
</tr>
<tr>
<td>4)</td>
<td>0.000000</td>
<td>-40.000000</td>
</tr>
<tr>
<td>5)</td>
<td>0.000000</td>
<td>70000.000000</td>
</tr>
</tbody>
</table>
RANGES IN WHICH THE BASIS IS UNCHANGED:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CURRENT COEF</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSOLD</td>
<td>40.0000000</td>
<td>INFINITY</td>
<td>2.500000</td>
</tr>
<tr>
<td>HCRACK</td>
<td>90.0000000</td>
<td>2.500000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>ASOLD</td>
<td>50.0000000</td>
<td>3.33333333</td>
<td>20.0000000</td>
</tr>
<tr>
<td>ACRACK</td>
<td>130.0000000</td>
<td>INFINITY</td>
<td>3.33333333</td>
</tr>
<tr>
<td>OIL</td>
<td>-40.0000000</td>
<td>INFINITY</td>
<td>10.0000000</td>
</tr>
</tbody>
</table>

RIGHHAND SIDE RANGES

<table>
<thead>
<tr>
<th>ROW</th>
<th>CURRENT</th>
<th>ALLOWABLE INCREASE</th>
<th>ALLOWABLE DECREASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20000.000000</td>
<td>INFINITY</td>
<td>4000.0010</td>
</tr>
<tr>
<td>3</td>
<td>.0000</td>
<td>20000.000003</td>
<td>INFINITY</td>
</tr>
<tr>
<td>4</td>
<td>.0000</td>
<td>10000.000000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>5</td>
<td>8.0000</td>
<td>2.0000</td>
<td>8.0000</td>
</tr>
</tbody>
</table>

Shoemakers of America forecasts the following demand for each of the next six months:

<table>
<thead>
<tr>
<th>Month</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
</tr>
<tr>
<td>2</td>
<td>6000</td>
</tr>
<tr>
<td>3</td>
<td>5000</td>
</tr>
<tr>
<td>4</td>
<td>9000</td>
</tr>
<tr>
<td>5</td>
<td>6000</td>
</tr>
<tr>
<td>6</td>
<td>5000</td>
</tr>
</tbody>
</table>

(Problem #18 of Chapter 4 Review Problems, page 184 of text by Winston.)

It takes a shoemaker 15 minutes to produce a pair of shoes. Each shoemaker works 150 hours/month, plus up to 40 hours/month overtime. A shoemaker is paid a regular salary of $2000/month, plus $50/hour for overtime. At the beginning of each month, Shoemakers can either hire or fire workers. It costs the company $1500 to hire a worker and $1900 to fire a worker.

The monthly holding cost per pair of shoes is 3% of the cost of producing a pair of shoes with regular-time labor. (The raw materials in a pair of shoes cost $20.) At the beginning of month 1, Shoemakers has 13 workers.

Formulate an LP that minimizes the cost of meeting (on time) the demands of the next six months.

Hints: Assume that, even though a shoemaker is paid for 150 hours of work per month, he may be idle part of the time if he is required in a later month, in order that the company avoid firing and hiring costs. You may find it useful to define the following decision variables for each month \( t, t=1,2,\ldots,6 \):

- \( W_t \): number of shoemakers in the work force during month \( t \)
- \( H_t \): number of shoemakers hired at the beginning of month \( t \)
- \( F_t \): number of shoemakers fired at the beginning of month \( t \)
- \( R_t \): number of pairs of shoes produced during regular time in month \( t \)
- \( O_t \): number of pairs of shoes produced during overtime in month \( t \)
- \( I_t \): number of pairs of shoes in inventory at the end of month \( t \)

\[ \begin{align*}
\text{MIN} & \quad 2000Y_1 + 2000Y_2 + 2000Y_3 + 2000Y_4 + 2000Y_5 + 2000Y_6 + 1500X_1 + 1500X_2 + 1500X_3 + 1500X_4 + 1500X_5 + 1500X_6 + 1900F_1 + 1900F_2 + 1900F_3 + 1900F_4 + 1900F_5 + 1900F_6 + 8.711 + 8.712 + 8.713 + 8.714 + 8.715 + 8.716 + 30.21 + 30.22 + 30.23 + 30.24 + 30.25 + 30.26 + 32.504 + 32.505 + 32.506 \\
\end{align*} \]
2) \[ -I_1 + E_1 + O_1 = 5900 \]
3) \[ I_1 - I_2 + E_2 + O_2 = 6000 \]
4) \[ I_2 - I_3 + E_3 + O_3 = 5900 \]
5) \[ I_3 - I_4 + E_4 + O_4 = 9900 \]
6) \[ I_4 - I_5 + E_5 + O_5 = 6300 \]
7) \[ I_5 + E_6 + O_6 = 5900 \]
8) \[ V_1 - R_1 + F_1 = 13 \]
9) \[ -V_1 + V_2 - R_2 + F_2 = 0 \]
10) \[ V_2 - V_3 - R_3 + F_3 = 0 \]
11) \[ -V_3 + V_4 - R_4 + F_4 = 0 \]
12) \[ V_4 - V_5 - R_5 + F_5 = 0 \]
13) \[ -V_5 + V_6 - R_6 + F_6 = 0 \]

The LP solution, unfortunately, is not integer!
(The number of persons hired &/or fired each month is noninteger!)

If we add the integer restrictions, and re-solve, using LINDO, the result is:

**OBJECTIVE FUNCTION VALUE**
1) 854080.000

As a result of adding the integer restriction, the cost is increased by

\[(854080 - 852716.62) = \$1363.38,\]

an increase of approximately 0.15%. 

(Exercise 30 of Review Problems, page 110 of text by Winston) “A paper recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are:
Two methods are available for processing the four inputs into pulp: de-inking and asphalt dispersion. It costs $20 to de-ink a ton of any input, a process which removes 10% of the pulp. It costs $15 to apply asphalt dispersion to a ton of material, a process which removes 20% of the pulp.

A total of 3000 tons, at most, can be run through the asphalt dispersion &/or de-inking process.
Grade 1 paper can be produced only with newsprint or book paper pulp.
Grade 2 paper can be produced only with book paper, tissue paper, or box board pulp.
Grade 3 paper can be produced only with newsprint, tissue paper, or box board pulp.

To meet its demands, the company needs:
- 500 tons of pulp for grade 1 paper
- 500 tons of pulp for grade 2 paper
- 600 tons of pulp for grade 3 paper

Variables

**BOX** = tons of purchased boxboard
**TISS** = tons of purchased tissue
**NEWS** = tons of purchased newsprint
**BOOK** = tons of purchased book paper

Variables, continued

**PBOX** = tons of pulp recovered from boxboard
**PTISS** = tons of pulp recovered from tissue
**PNEWS** = tons of pulp recovered from newsprint
**PBOOK** = tons of pulp recovered from book paper
**PBOX1** = tons of boxboard pulp used for grade 1 paper
**PBOX2** = tons of boxboard pulp used for grade 2 paper
**PBOX3** = tons of boxboard pulp used for grade 3 paper

MIN 5 BOX + 6 TISS + 8 NEWS + 10 BOOK + 20 BOX1 + 20 TISS1 + 20 NEWS1 + 20 BOOK1 + 15 BOX2 + 15 TISS2 + 15 NEWS2 + 15 BOOK2

SUBJECT TO

2) BOX + BOX1 + BOX2 <= 0
3) TISS + TISS1 + TISS2 <= 0
4) NEWS + NEWS1 + NEWS2 <= 0
5) BOOK + BOOK1 + BOOK2 <= 0
6) 0.135 BOX1 + 0.12 BOX2 - PBOX = 0
7) 0.18 TISS1 + 0.16 TISS2 - PTISS = 0
8) 0.27 NEWS1 + 0.24 NEWS2 - PNEWS = 0
9) 0.36 BOOK1 + 0.32 BOOK2 - PBOOK = 0

END

LP OPTIMUM FOUND AT STEP 23
OBJECTIVE FUNCTION VALUE
1) 1400000.00
The Boilen Oil Co. wishes to find the optimal mix of two possible blending processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>Input (barrels)</th>
<th>Output (gallons)</th>
<th>Crude A</th>
<th>Crude B</th>
<th>Gasoline X</th>
<th>Gasoline Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>50</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>30</td>
<td>80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Available crude: 120 bbls of A, 180 of B
Required blends: 2800 gal. of X, 2200 of Y
Profits: $0.10/gallon of X, $0.12/gallon of Y

Maximize \((3550-3150)A + (2280-2005)B + (3775-2825)C + (3750-2550)D\)
subject to
\[
\begin{align*}
4A + B + 6C & \leq 150 \\
A + 2B + 4C & \leq 80 \\
A \geq 0, & B \geq 0, C \geq 0, D \geq 0
\end{align*}
\]

What is the optimal production plan?