Questions

- What is the average stock-on-hand for this inventory system?
- What is the frequency of replenishments?
- What is the average number of days between stockouts?

Daily demand for an item is random, with the probability distribution:

<table>
<thead>
<tr>
<th>d</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(D=d)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

At the end of each day, the stock on hand is observed. If it exceeds \( s = 2 \) (the reorder point), no action is taken; otherwise, the inventory is replenished by an amount which brings the level up to \( S = 6 \) units at the beginning of the next day.

Questions

- If the initial stock-on-hand is 6,
  - what is the expected number of days until a stockout occurs?
  - what is the probability that the first stockout occurs 5 days hence?
  - what is the probability that a replenishment occurs 3 days hence?
  - what is the expected number of stockouts during the next 30 days?
  - what is the expected number of replenishments during the next 30 days?

Markov chain model

Simulation of the Markov chain
Powers of the transition probability matrix
Steady-state distribution
Expected number of visits
First-passage probabilities
Mean first-passage time

Define the state of the system according to the stock-on-hand (SOH) at the end of the day (before replenishment occurs):

\[
X_n = \begin{cases} 
1, & \text{SOH} = 0 \\
2, & \text{SOH} = 1 \\
3, & \text{SOH} = 2 \\
4, & \text{SOH} = 3 \\
5, & \text{SOH} = 4 \\
6, & \text{SOH} = 5 \\
7, & \text{SOH} = 6 
\end{cases}
\]

Transition Probabilities

\[
P_{ij} = P(X_n = j | X_{n-1} = i)
\]

If \( i < 3 \) (SOH > 2), no replenishment occurs:

\[
P_{ij} = \begin{cases} 
P(D = (i-j)) & \text{for } j > 1 \ (\text{SOH} > 0) \\
P(D = (i-j)) & \text{for } j = 1 \ (\text{SOH} = 0)
\end{cases}
\]

For example,
\[
P_{42} = P(D = 2) = 0.3 \\
P_{41} = P(D \geq 3) = P(D = 3) + P(D = 4) = 0.3 + 0.1 = 0.4
\]
Transition Probabilities

If \( s = 3 \) and \( h = 2 \), the SOH at the beginning of the next day is 5.

\[
P_{ij} = P(D = 6 - |j - 1|)\]

For example,

\[
P_{25} = P(D = 2) = 0.3
\]

Simulation of 30 days' operation

How many stockouts? replenishments?

Simulation results

10 simulations of 30 stages, beginning in state 7
(Stock-on-hand = 6)
Average Stock-on-Hand \( \sum_{i=1}^{7} (i-1)p_i \) 

The average cost/period in steady state is 2.7492

(Here, "cost" = SOH)