An international corporation has decided to let a 20-year contract for the transport of 4 million tons/year of bauxite and iron ore from a South American port to a U.S. port 3000 miles distant, and for the shipment of an equal tonnage of coal and other materials from the U.S. port back to the South American port.

Due to the magnitude of the project, the president of your shipping company has decided to make a bid and, if successful in winning the contract, to buy a new fleet of cargo ships for this job.

The president would like to know the minimum project cost for bidding purposes.

The optimal number of ships, load tonnage capacity, and cruising speed should also be determined.

"Rules of Thumb"

- weight of cargo ship (exclusive of power plant) = cargo tonnage
- ("Admiralty Formula"):

\[ \text{Shaft-hp} = (\text{displacement tonnage})^{\frac{2}{3}} \cdot (\text{Velocity})^{\frac{3}{2}} \cdot \frac{1}{K} \]

where unit of velocity is knots, and K = 400.

Costs

- initial acquisition costs
  - ship
  - power plant
- fuel & other operating costs

(Rough) cost estimates:
- Cost of ship (exclusive of power plant): $400/Ton
- Cost of power plant: $150/shaft-hp
- Cost of fuel: $0.003/shaft-hp-hr

Define variables

\[ X_1 = \text{# of ships} \]
\[ X_2 = \text{cargo load capacity (tons)} \]
\[ X_3 = \text{ship velocity (knots)} \]
\[ X_4 = \text{fraction of time en route} \]

Acquisition Costs

- Cost of ship (minus power plant) = $400/ton & weight of ship = 50% of cargo load capacity
  \[ 400 \times \frac{1}{2} X_2 = 200 X_2 \]

Displacement tonnage = ship weight + cargo weight

\[ = \frac{1}{2} X_2 + X_2 = \frac{3}{2} X_2 \]

Cost:

\[ 150 \left( \frac{3}{2} X_2 \right)^{\frac{2}{3}} \cdot \frac{X_3^{\frac{3}{2}}}{400} = 0.491389 X_2^{\frac{2}{3}} X_3^{\frac{3}{2}} \]
Fuel Cost

$0.003/\text{shaft-hp hr.}$

\[
0.003 \times \frac{(\frac{3}{2} X_2)^2 X_3^2}{400} = 9.03 \times 10^{-6} X_2^2 X_3^2 \text{ per hr.}
\]

* hrs/year = 8750
* hrs enroute/year = 8750 \(X_4\)

Cost: 

\[
9.82 \times 10^{-6} X_2^2 X_3^2 \times 8750 X_4 = 0.086 X_2^2 X_3^2 X_4 \text{ per year, per ship}
\]

Total present value of costs:

\[
0.491389 X_1 X_2^2 X_3^2 + 200 X_1 X_2 + 0.732 X_1 X_2 X_3 X_4
\]

power plants  \hspace{1cm} ships  \hspace{1cm} fuel

Present value of total fuel costs for next 20 years

Using interest rate of 10% per year, and assuming that fuel is purchased annually, the uniform-series present worth factor is 8.514

Present value of future fuel cost is

\[
0.732 X_2 X_3 X_4 \text{ per ship}
\]

Constraints

- 4 million tons/yr are to be transported in each direction.
- Because of limited docking facilities, at most 1000 tons/hr may be loaded or unloaded.

<table>
<thead>
<tr>
<th>Constraints</th>
<th></th>
</tr>
</thead>
</table>
| \(1.458 X_1 X_2 X_3 X_4 \geq 4 \times 10^6\) | \[ 2.74 \times 10^6 X_1 X_2 X_3 X_4 \leq 1 \]

Time required per (one-way) trip (in hrs):

- \(en\ route\): \[ \frac{3000 \text{ mi.}}{X_3 \text{ mpg/hr.}} \]
- \(loading\): \[ \frac{X_2 \text{ tons}}{1000 \text{ tons/hr.}} \]
- \(unloading\): \[ \frac{X_2 \text{ tons}}{1000 \text{ tons/hr.}} \]

Total time per one-way trip: 

\[ 3000 X_3^{-1} + 2 \times \frac{X_2}{1000} \]
Minimize
\[ 0.491389X_1X_2^2X_3^3 + 200 X_1X_2 + 0.732X_1X_2^2X_3^3X_4 \]
subject to
\[ 2.74 \times 10^5 X_1^3 X_2^5 X_3^3 \leq 1 \]
\[ X_4 + 6.667 \times 10^7 X_2X_3X_4 \leq 1 \]
\[ X_j > 0, \ j = 1, 2, 3, 4 \]

\# terms = 6
\# variables = 4

Degrees of difficulty: 6 - (1 + 4) = 1

Cargo Ship Design

Number of variables: 4
Number of polynomials: 3
Total number of terms: 6
Terms per polynomial: 3 1 2

Coefficients and exponent matrix:

\[
\begin{array}{cccc}
\text{exponents} & \text{coefficients} \\
1 & 1 & 6.00000000E-1 & 1 & 6.66667 & 2 & 0 \\
2 & 2 & 2.00000000E1 & 1 & 6.66667 & 2 & 1 \\
3 & 2 & 7.00000000E1 & 1 & 6.66667 & 2 & 1 \\
4 & 2 & 2.74000000E6 & -1 & -1 & -1 & -1 \\
5 & 3 & 1.00000000E0 & 0 & 0 & 0 & 0 \\
6 & 3 & 6.66667 & 0.000000E-7 & 0 & 1 & 1 \\
\end{array}
\]

\( t = \text{term number} \)
\( p = \text{polynomial} \)
\( c_t = \text{coefficient} \)

Iteration 1

LP Solution

<table>
<thead>
<tr>
<th>Col</th>
<th>P</th>
<th>Infeas</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 1.11111

Constraints

\# of ships 1

\[
\begin{array}{cccc}
\text{Weights of terms (c):} & \text{Col} & \text{P} & \text{Infeas} & \text{Lambda} \\
\hline
1 & 1 & 1 & 1 & 1 \\
2 & 1 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Objective function:

\[
\text{Primal:} \ 6.0928 \times 10^6 \quad \text{Dual:} \ 6.8281 \times 10^6 \\
\text{Duality Gap:} \ 1.700917 \times 10^{-1} = 29.29\% 
\]

Type-1 grid points (10 11) added for polynomials 1 5
GP_CargoShip

LP Dual solution

Constraints

<table>
<thead>
<tr>
<th>i</th>
<th>xi(i)</th>
<th>Phi</th>
<th>Infeas</th>
<th>Lamda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

Heights of terms (p):

<table>
<thead>
<tr>
<th>h</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Objective functions:

Prim. 7.15172E7  Dual: 7.15172E7

Duality Gap: 3.842E-05 = 0.000000 percent

Constraints

<table>
<thead>
<tr>
<th>i</th>
<th>xi(i)</th>
<th>Phi</th>
<th>Infeas</th>
<th>Lamda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

# of ships

cargo load (tons)

ship speed (knots)

fraction of time enroute

(Sum of absolute differences between K & Infeas is 0.0012E00)

Objective function = 7.15172E7

Prim. feasible solution

Dual Solutions: Cargo Ship

i.e., the ships' power plants will cost 12.96% of the total cost, the ships will cost 70.07% of the total cost, and the fuel will cost 16.95% of the total cost.

MOI: Dakota, 3 of 4, 1999