Generalized Linear Programming (GLP) 

Formulation of the Geometric Programming Dual Problem

Undesirable Properties of GP Dual:

- Objective function is nondifferentiable if \( \delta_i = 0 \) & \( \lambda_k = 0 \) for any \( i \) & \( k \)
- If \( \lambda_k = 0 \), then \( \delta_i = 0 \) \( \forall i \in [k] \)
- It is possible that the dual solution does not provide sufficient information to compute the optimal primal solution.
- For small \( \delta_i \), computation of the terms \( \delta_i \) in \( \delta_i \) introduce substantial numerical errors.

Define functions

\[ G_k(\rho) = \sum_{j \in [k]} \left[ \rho_j \ln c_j - \rho_j \ln \rho_j \right] \]

subject to

\[ \sum_{j \in [k]} a_{ij} \delta_i = 0, \ j = 1, \ldots, M \]

orthogonality

\[ \delta_i \geq 0, \lambda_k \geq 0 \]

Make a change of variable:

\[ \delta_j = \rho_j \lambda_k \text{ for } j \in [k] \]

so that \( \rho_j = \frac{\delta_j}{\lambda_k} \) if \( j \in [k] \) & \( \lambda_k > 0 \)

Note that

\[ \sum_{i \in [k]} \delta_i = \lambda_k \implies \sum_{j \in [k]} \rho_j = 1 \quad \forall k \]

Maximize

\[ \sum_{k=0}^{P} G_k(\rho) \lambda_k \]

subject to

\[ \sum_{k=0}^{P} A_{ki}(\rho) \lambda_k = 0, \ i = 1, \ldots, N \]

\[ \lambda_0 = 1 \]

Geometric Programming Dual Problem

\[ \sum_{j \in [k]} \rho_j = 1, \ k = 0, 1, \ldots, K \]

\[ \lambda_k \geq 0, \rho_j \geq 0, \ \forall k, j \]

That is,

Maximize \[ \sum_{k=0}^{P} \gamma_k \lambda_k \]

subject to

\[ \sum_{k=0}^{P} \alpha_k \lambda_k = 0, \ j = 1, \ldots, m \]

\[ \lambda_0 = 1 \]

\[ \lambda_k \geq 0, \ k = 1, \ldots, p \]

\[ (\gamma_k, \alpha_k) \in S_k, \ k = 0, \ldots, p \]

where

\[ S_k = \{ (\gamma, \alpha) \mid \exists \rho \geq 0, \sum_{i \in [k]} \rho_i = 1 \text{ such that } \gamma = G_k(\rho), \alpha_i = A_{ki}(\rho) \} \]
This class of problems, which are linear programming problems in which the columns are also to be selected, was called Generalized Linear Programs by George Dantzig, and was used in solving chemical equilibrium problems.

If $t$ is optimal in the primal, and $(\rho, \lambda)$ is optimal in the dual, then $\rho_j > 0$ and $g_k(t) > 0$

and

$$
\rho_j = \sum_{i=1}^{N} \frac{c_j \prod_{i=1}^{t} \rho_i}{g_i(t)}
$$

whether the constraint $k$ is tight or slack!

The dual solution thereby always provides sufficient information to compute the primal solution!

$$(\gamma_k, \alpha_k) \in S_k \implies \exists \mu_{kn} \geq 0, n \in N_k$$

such that

$$
\sum_{n \in N_k} \mu_{kn} = 1
$$

$$
\gamma_k = \sum_{n \in N_k} \mu_{kn} \hat{c}_k^n
$$

$$
\alpha_k = \sum_{n \in N_k} \mu_{kn} \hat{d}_k^n
$$

Any element of a convex set can be represented by a convex combination of the extreme points of the set!

The GLP dual may then be written

$$
\text{Maximize } \sum_{k=0}^{P} \left[ \sum_{n \in N_k} \mu_{kn} \gamma_k^n \right] \lambda_k
$$

$$
\sum_{k=0}^{P} \left[ \sum_{n \in N_k} \mu_{kn} \alpha_k^n \right] \lambda_k = 0, j=1, \ldots, m
$$

$$
\sum_{n \in N_k} \mu_{kn} \lambda_0 = 1
$$

or, by defining $u_{kn} = \mu_{kn} \lambda_k$

$$
\text{Maximize } \sum_{k=0}^{P} \sum_{n \in N_k} \hat{c}_k^n u_{kn}
$$

subject to

$$
\sum_{k=0}^{P} \sum_{n \in N_k} \hat{d}_k^n u_{kn} = 0, j=1, \ldots, m
$$

$$
\sum_{n \in N_k} u_{kn} = 1
$$

which is an "ordinary" LP with infinitely many variables (semi-infinite LP)

**Step 0**
For each $k$, select one or more extreme points of

**Step 1**
Generate an LP with columns and variables corresponding to the sets of extreme points

**Step 2**
Compute the simplex multipliers of the orthogonality constraints and normality constraint.

**Step 3**
For each $k=0, 1, \ldots, p$, choose $(\gamma_k, \alpha_k)$ so as to maximize the relative profit:

$$
\text{maximize } \gamma - \sum_{i=1}^{m} c_i \gamma_i - w_{k-1}
$$

if $k=0$

$$
\text{maximize } \gamma - \sum_{i=1}^{m} c_i \gamma_i
$$

if $k>0$
For each \((y_k, a_k)\) whose relative profit exceeds some tolerance, add the corresponding column to the LP tableau.

If no columns can be added, stop; else, return to Step 1.

Maximizing the Relative Profit

That is, if we compute the primal (approximate) solution \(x_i = e^{|a_i|}, j=1, \ldots, m\) (i.e., exponentiate the simplex multipliers of the orthogonality conditions)

\[
\rho_n = \frac{c_n \prod x_n^{a_n}}{\sum_{j \in [k]} c_j \prod x_j^{a_j}} = \frac{\text{term}_n}{\text{polynomial}_k} > 0
\]

\(\forall n \in [k]\)

For the \(\rho_n\) thus obtained, i.e.,

\[
\rho_n = \frac{c_n \prod x_n^{a_n}}{\sum_{j \in [k]} c_j \prod x_j^{a_j}} \quad \forall n \in [k]
\]

the relative profit function is nonpositive if & only if

\[
g_0(x) = b_0(x^*)
\]

\[
g_k(x) \leq 1
\]

Multi-Item EOQ

Demand for three items is known, with the rate of demand constant over time, \(D_i/\text{year}\). Holding cost of item \(i\) is \(H_i/\text{unit/\text{year}}\), and each replenishment incurs a cost of \(A_i\).

Let \(Q_i = \text{order quantity of item } i\).

\[
D_i/Q_i = \text{average } i \text{ replenishments per year}
\]

\[
Q_i/2 = \text{average inventory level}
\]

Often there are additional constraints on the ordering policy, e.g.,

- a limit on the number of replenishments/year
  \[
  \sum_{i} \frac{D_i}{Q_i} \leq N
  \]
- a limit on the maximum volume (if all orders were to arrive simultaneously)
  \[
  \sum x_i Q_i \leq V
  \]
- a limit on the average investment in inventory
  \[
  \sum \frac{1}{2} C_i Q_i \leq B
  \]

Maximizing the Relative Profit

For each \(k\), the column having maximum relative profit is \((y_k, a_k)\) where

\[
y_k = G_k(p), \quad a_k = A_k(p)
\]

and

\[
\rho_k = \frac{c_k \prod a_k}{\sum_{j \in [k]} c_j \prod a_j}
\]

For the \(\rho_k\) thus obtained, i.e.,

\[
\rho_k = \frac{c_k \prod a_k}{\sum_{j \in [k]} c_j \prod a_j} \quad \forall k \in [k]
\]

the relative profit function is nonpositive if & only if

\[
g_0(x) = b_0(x^*)
\]

\[
g_k(x) \leq 1
\]

Annual cost of item \(i\) is

\[
\frac{A_i D_i}{Q_i} \quad \text{ordering cost}
\]

\[
\frac{1}{2} H_i Q_i \quad \text{holding cost}
\]

The classic EOQ ("economic order quantity") formula of Wilson specifies the order quantity which minimizes this annual cost:

\[
Q_i^* = \sqrt{\frac{2 A_i D_i}{H_i}}
\]

Example

<table>
<thead>
<tr>
<th>i</th>
<th>(A_i)</th>
<th>(D_i)</th>
<th>(H_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>1000</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>2000</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>2000</td>
<td>3</td>
</tr>
</tbody>
</table>

\(\hat{Q} = 100\)

Minimize \(\sum_{i=1}^3 \left[ \frac{A_i D_i}{Q_i} + \frac{1}{2} H_i Q_i \right]\)

subject to \(\sum_{i=1}^3 Q_i \leq \hat{Q}\)

\(Q_i > 0, i=1,2,3\)
Number of variables: 3
Number of polynomials: 2
Total number of terms: 6
Terms per polynomial: 3

Coefficients and exponents matrix:

<table>
<thead>
<tr>
<th>t</th>
<th>p</th>
<th>C</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>50000</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>20000</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>160000</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.01</td>
<td>0</td>
</tr>
</tbody>
</table>

i.e., add a column for each polynomial constraint whose infeasibility exceeds 0.0005 and for the objective function if the duality gap exceeds 0.05%.

Frequencies for:
- Discarding unused grid points: 10
- Reporting solutions: 1
- Types of grid points to be generated: 0 1

----------Exponentiating LP Dual solution----------

\[x = 12.71, 50.339, 40.671\]
Weights (p):
0.37741, 0.0021155, 0.32741, 0.01070, 0.32741, 0.0050773
0.12195, 0.40774, 0.5924

Objective functions:

Primal: 12016
Dual: 3924
Duality gap: 506.15 - 296.49 percent

Constraints:

Value: 1.0422
Infeasibility: 0.042196
Lambda: 1

Type-1 grid points (#1 16) added for polynomials 1 2
CPU time: 14.6 sec.

----------Exponentiating LP Dual solution----------

\[x = 100.1840819, 69.692\]
Weights (p):
0.05553, 0.001239, 0.54832, 0.002407, 0.41592, 0.001512
0.72207, 0.1359, 0.1429

Objective functions:

Primal: 19839
Dual: 19864
Duality gap: 506.17 - 81.71 percent

Constraints:

Value: 1.396
Infeasibility: 0.286
Lambda: 0.96488

Type-1 grid points (#17 18) added for polynomials 1 2
CPU time: 56.6 sec.

----------Exponentiating LP Dual solution----------

\[x = 27.139, 15.341, 18.816\]
Weights (p):
0.15864, 0.004583, 0.47785, 0.0074588, 0.34979, 0.00489
0.34948, 0.34958, 0.58157

Objective functions:

Primal: 11845
Dual: 10712
Duality gap: 1155.9 - 10.58 percent

Constraints:

Value: 1.011
Infeasibility: 0.010558
Lambda: 0.99466

Type-1 grid points (#13 20) added for polynomials 1 2
Iteration 4
----------

LP Solution
---

<table>
<thead>
<tr>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2858417</td>
</tr>
<tr>
<td>16</td>
<td>0.3360693</td>
</tr>
<tr>
<td>12</td>
<td>0.1287346</td>
</tr>
<tr>
<td>4</td>
<td>0.0388989</td>
</tr>
<tr>
<td>19</td>
<td>0.0131911</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.12666

----------

Iteration 5
----------

LP Solution
---

<table>
<thead>
<tr>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3103979</td>
</tr>
<tr>
<td>16</td>
<td>0.7087758</td>
</tr>
<tr>
<td>20</td>
<td>0.1468802</td>
</tr>
<tr>
<td>22</td>
<td>0.1123762</td>
</tr>
<tr>
<td>15</td>
<td>1.0000000</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.059451

----------

Iteration 6
----------

LP Solution
---

<table>
<thead>
<tr>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3128606</td>
</tr>
<tr>
<td>16</td>
<td>0.5066616</td>
</tr>
<tr>
<td>14</td>
<td>0.4646642</td>
</tr>
<tr>
<td>23</td>
<td>0.3106708</td>
</tr>
<tr>
<td>19</td>
<td>0.0807039</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.012446

----------

Iteration 7
----------

LP Solution
---

<table>
<thead>
<tr>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3550822</td>
</tr>
<tr>
<td>25</td>
<td>0.2266617</td>
</tr>
<tr>
<td>14</td>
<td>0.3977612</td>
</tr>
<tr>
<td>22</td>
<td>0.3703110</td>
</tr>
<tr>
<td>19</td>
<td>0.4173373</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0045076

----------

Exponentiating LP Dual solution
---

X = 22.62 84.828 14.841
Weights (α): 0.11212 0.0038896 0.18539 0.041015 0.7125 0.0414712
0.22791 0.85713 0.11466

Objective functions:
Fracal: 10151 Dual: 1078 Dual: 10962
Feasibility Gap: 4250.4 - 40.252 percent

Constraints:
Value 1.3909
Infeasibility 0.07089
Lambda 0.90745

Type-1 grid points (#21 12) added for polynomials 1 2
CPU time: 88.35 sec.

----------

Exponentiating LP Dual solution
---

X = 24.384 55.342 26.841
Weights (α): 0.197 0.0041144 0.2048 0.011673 0.60292 0.0033947
0.22881 0.51922 0.25187

Objective functions:
Fracal: 11810 Dual: 11952
Feasibility Gap: 806.61 - 7.2430 percent

Constraints:
Value 1.0657
Infeasibility 0.02666
Lambda 0.94614

Type-1 grid points (#23 24) added for polynomials 1 2
CPU time: 108.74 sec.

----------

Exponentiating LP Dual solution
---

X = 15.692 46.668 35.622
Weights (α): 0.22436 0.002746 0.371 0.0101 0.39824 0.0046257
0.19151 0.48551 0.34985

Objective functions:
Fracal: 11951 Dual: 11234
Feasibility Gap: 316.93 - 2.8211 percent

Constraints:
Value 1.0179
Infeasibility 0.047625
Lambda 0.98474

Type-1 grid points (#25 26) added for polynomials 1 2
CPU time: 145.9 sec.

----------

Exponentiating LP Dual solution
---

X = 31.619 40.674 37.025
Weights (α): 0.19700 0.002864 0.42138 0.0087554 0.37209 0.0047819
0.22855 0.40394 0.34761

Objective functions:
Fracal: 11654 Dual: 11538
Feasibility Gap: 30.955 - 0.70194 percent

Constraints:
Value 1.0072
Infeasibility 0.0371968
Lambda 0.96476

Type-1 grid points (#27 28) added for polynomials 1 2
CPU time: 170.3 sec.
### Iteration 3

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Col</th>
<th>Pos</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.93529821</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>1</td>
<td>0.22359289</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>0</td>
<td>0.44129977</td>
</tr>
<tr>
<td>26</td>
<td>1</td>
<td>1</td>
<td>0.52479451</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0.77640512</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0024712

### Iteration 5

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Col</th>
<th>Pos</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.36147572</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>1</td>
<td>0.01748770</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>1</td>
<td>0.67594862</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0.20826530</td>
</tr>
<tr>
<td>51</td>
<td>1</td>
<td>1</td>
<td>0.98125230</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0016411

### Iteration 7

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Step</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Objective Functions:**

**Primal:** 11664  **Dual:** 11659

**Feasibility Gap:** 21.245 = 0.18549 percent

**Constraints:**

**Value:** 1.0069  
**Infeasibility:** 0.00488652  
**Lambda:** 0.59457

Type-I grid points (42930) added for polynomials 1-2

**CP time:** 103.9 sec.

### Iteration 9

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Col</th>
<th>Pos</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.36147572</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
<td>1</td>
<td>0.37672041</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>1</td>
<td>0.53972463</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>1</td>
<td>0.41029466</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0.67279559</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0006506

### Iteration 11

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Col</th>
<th>Pos</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.36147572</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
<td>1</td>
<td>0.37672041</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>1</td>
<td>0.53972463</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>1</td>
<td>0.41029466</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0.67279559</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0006506

### Iteration 15

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Col</th>
<th>Pos</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.36147572</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
<td>1</td>
<td>0.37672041</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>1</td>
<td>0.53972463</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>1</td>
<td>0.41029466</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0.67279559</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0006506

### Iteration 18

**LP SOLUTION**

<table>
<thead>
<tr>
<th>Col</th>
<th>Pos</th>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.36147572</td>
</tr>
<tr>
<td>31</td>
<td>1</td>
<td>1</td>
<td>0.37672041</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>1</td>
<td>0.53972463</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>1</td>
<td>0.41029466</td>
</tr>
<tr>
<td>27</td>
<td>1</td>
<td>1</td>
<td>0.67279559</td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.0006506

## Exponentiating LP Dual solution

<table>
<thead>
<tr>
<th>X</th>
<th>19.6165</th>
<th>42.9251</th>
<th>46.415</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>1.0000</td>
<td>0.00488652</td>
<td>0.59457</td>
</tr>
</tbody>
</table>

**Objective Functions:**

**Primal:** 11664  **Dual:** 11659

**Feasibility Gap:** 12.35 = 0.11660 percent

**Constraints:**

**Value:** 1.0005  
**Infeasibility:** 0.0011619  
**Lambda:** 0.958

Type-I grid points (42334) added for polynomials 1-2

**CP time:** 403.8 sec.
Weights (g):
0.1987 0.003697 0.4135 0.008384 0.1704 0.00476
0.2155 0.4144 0.3701

Objective functions:
Primal: 11670 Dual: 11660
Duality Gap: 5.621 = 0.00821 percent

Constraints:
Value | Infeasibility 0.000474
Lambda 0.9654

--- Terminated at iteration 12 ---
Converged: Tolerances are satisfied
CPU time: "455.8 sec.
Frequency of use of each type grid point (p):
Type | Frequency 28 34

Primal Solution: Multi-Iter EOQ
5/02/98 11:31
Solution reported as optimal
Objective function: 11670

<table>
<thead>
<tr>
<th>1</th>
<th>X(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.57</td>
</tr>
<tr>
<td>2</td>
<td>45.48</td>
</tr>
<tr>
<td>3</td>
<td>37.01</td>
</tr>
</tbody>
</table>

Constraints
X | P | Lambda |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.9654</td>
</tr>
</tbody>
</table>

Dual Solution: Multi-Iter EOQ
Heights of terms (g):
0.19876 0.3972 3.9528 4.1352 0.008384 0.1704 0.00476 0.2155 0.4144 0.3701

Dual Solution: Multi-Iter EOQ

| 1 | 1.99721 | 1.9972 | 1.9972 | 1.9972 | 1.9972 | 1.9972 | 1.9972 |
| 2 | 2.1556 | 2.1556 | 2.1556 | 2.1556 | 2.1556 | 2.1556 | 2.1556 |

Log Gap vs Iteration
Log Infeasibility vs Iteration
Exponentiating the Simplex Multipliers yields:

\[
X = 0.7132276 100 0.10031327 \quad \text{order quantities}
\]

Weights (\(w\)):

\[
0.17542 \quad 0.02527 \quad 0.4163 \quad 0.052038 \quad 0.033046 \quad 0.0019047 \\
0.03395 \quad 0.59947 \quad 0.00659
\]

Objective functions:

\[\text{Final: 4604.1, Dual: 2000}\]

\[\text{Duality gap: 2004.1} = 140.2\%\]

Constraints:

\[
\begin{array}{c|c|c}
\text{Value} & \text{Infeasibility} & \text{Cost} \\
1.6691 & 0.6691 & \\
\end{array}
\]

Type-1 grid points (#15 16) added for polynomials 1 2
GLP Dual for GP 7/17/98

### Iteration 1

<table>
<thead>
<tr>
<th>LP Solution</th>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0</td>
<td>6 6000000000</td>
<td></td>
</tr>
<tr>
<td>0 3 2</td>
<td>0 0000000000</td>
<td></td>
</tr>
<tr>
<td>3 1 0</td>
<td>1 0000000000</td>
<td></td>
</tr>
<tr>
<td>4 0 0</td>
<td>2 0000000000</td>
<td></td>
</tr>
<tr>
<td>5 2 1</td>
<td>3 0000000000</td>
<td></td>
</tr>
<tr>
<td>6 1 0</td>
<td>4 0000000000</td>
<td></td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.001904713295

### Iteration 2

<table>
<thead>
<tr>
<th>LP Solution</th>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0</td>
<td>6 6000000000</td>
<td></td>
</tr>
<tr>
<td>1 0 0</td>
<td>5 542857143</td>
<td></td>
</tr>
<tr>
<td>1 1 0</td>
<td>1 0000000000</td>
<td></td>
</tr>
<tr>
<td>2 2 1</td>
<td>2 0000000000</td>
<td></td>
</tr>
<tr>
<td>3 1 0</td>
<td>3 0000000000</td>
<td></td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.000523666023

### Iteration 3

<table>
<thead>
<tr>
<th>LP Solution</th>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0</td>
<td>6 6000000000</td>
<td></td>
</tr>
<tr>
<td>1 0 0</td>
<td>5 542857143</td>
<td></td>
</tr>
<tr>
<td>1 1 0</td>
<td>1 0000000000</td>
<td></td>
</tr>
<tr>
<td>2 2 1</td>
<td>2 0000000000</td>
<td></td>
</tr>
<tr>
<td>3 1 0</td>
<td>3 0000000000</td>
<td></td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.000523666023

### Iteration 4

<table>
<thead>
<tr>
<th>LP Solution</th>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0</td>
<td>6 6000000000</td>
<td></td>
</tr>
<tr>
<td>1 0 0</td>
<td>5 542857143</td>
<td></td>
</tr>
<tr>
<td>1 1 0</td>
<td>1 0000000000</td>
<td></td>
</tr>
<tr>
<td>2 2 1</td>
<td>2 0000000000</td>
<td></td>
</tr>
<tr>
<td>3 1 0</td>
<td>3 0000000000</td>
<td></td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.000523666023

### Iteration 5

<table>
<thead>
<tr>
<th>LP Solution</th>
<th>Col Posy Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 0</td>
<td>6 6000000000</td>
<td></td>
</tr>
<tr>
<td>1 0 0</td>
<td>5 542857143</td>
<td></td>
</tr>
<tr>
<td>1 1 0</td>
<td>1 0000000000</td>
<td></td>
</tr>
<tr>
<td>2 2 1</td>
<td>2 0000000000</td>
<td></td>
</tr>
<tr>
<td>3 1 0</td>
<td>3 0000000000</td>
<td></td>
</tr>
</tbody>
</table>

Determinant of the basis matrix = 0.000523666023

---

### Exponentiating the Simplex Multipliers Yields:

**Iteration 1**

- **Objective Function:**
  - **Frical:** 5550
  - **Dual:** 2990
  - **Duality Gap:** 2990 - 197.8 percent

- **Constraints:**

<table>
<thead>
<tr>
<th>Value</th>
<th>Infeasibility</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Added grid points (17 18) for polyomials 1 2

---

### Exponentiating the Simplex Multipliers Yields:

**Iteration 2**

- **Objective Function:**
  - **Frical:** 5900
  - **Dual:** 4945.49
  - **Duality Gap:** 614.51 - 18.34 percent

- **Constraints:**

<table>
<thead>
<tr>
<th>Value</th>
<th>Infeasibility</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.25</td>
<td></td>
<td>0.625</td>
</tr>
</tbody>
</table>

- Added grid points (19 20) for polyomials 1 2

---

### Exponentiating the Simplex Multipliers Yields:

**Iteration 3**

- **Objective Function:**
  - **Frical:** 4444.2
  - **Dual:** 4041.7
  - **Duality Gap:** 401.17 = 6.214 percent

- **Constraints:**

<table>
<thead>
<tr>
<th>Value</th>
<th>Infeasibility</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.005</td>
<td></td>
<td>0.052</td>
</tr>
</tbody>
</table>

- Added grid points (21 22) for polyomials 1 2

---

### Exponentiating the Simplex Multipliers Yields:

**Iteration 4**

- **Objective Function:**
  - **Frical:** 4444.2
  - **Dual:** 4041.7
  - **Duality Gap:** 401.17 = 6.214 percent

- **Constraints:**

<table>
<thead>
<tr>
<th>Value</th>
<th>Infeasibility</th>
<th>Lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.02</td>
<td></td>
<td>0.027</td>
</tr>
</tbody>
</table>

- Type-1 grid points (23 24) added for polyomials 1 2